BRIGHTNESS PRESERVATION IN HISTOGRAM EQUALIZATION BASED CONTRAST ENHANCEMENT TECHNIQUES

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Abstract

Histogram equalization (HE) is a simple yet effective image enhancement technique. However, it tends to change the brightness of an image significantly, causing annoying artifacts and unnatural contrast enhancement. Brightness preserving bi-histogram equalization (BBHE) and Dualistic Sub-Image Histogram Equalization (DSIHE) have been proposed to overcome the problem but they still fail under certain condition. This paper proposes a novel extension of BBHE referred to as Minimum Mean Brightness Error Bi-Histogram Equalization (MMBEBHE) whose main objective is to minimize the difference between input and output imageís mean. Simulation results show that MMBEBHE is better than BBHE and DSIHE. Besides, this paper also proposes an efficient, integer-based implementation of MMBEBHE. Nevertheless, MMBEBHE also has its limitation. Hence, this paper further proposes a generalization of BBHE referred to as Recursive Mean-Separate Histogram Equalization (RMSHE). RMSHE is featured with scalable brightness preservation. Simulation results show that RMSHE works more elficiently compared to HE, BBHE, DSIHE and MMBEBHE.

Keywords: **Bi-histogram equalization, dualistic sub-image, histogram equalization, mean separate, minimum mean brightness error, recursive**

Introduction

Histogram equalization (HE) is a very popular technique for enhancing the contrast of an image (Umbaugh, 1998). It maps the gray levels based on the probability distribution of the input gray levels. It flattens and stretches the dynamic range of an imageís histogram to improve imageís contrast. HE has been applied in various fields such as medical image processing and radar image processing (Kim, 1997).

An image's mean brightness after HE tends to be around the middle gray level regardless of the input mean. While this is useful in certain application such as face detection (to reduce brightness variance), it is not desirable in some applications such as consumer electronics (digital camera etc.) where preserving the original brightness is necessary. Brightness preserving Bi-histogram equalization

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(BBHE) has been proposed to overcome the aforementioned problems. It has been shown that this technique does preserve the original brightness to a certain extends (Kim, 1997). Equal Area Dualistic Sub-Image Histogram Equalization (DSIHE) has been proposed and claimed to outperform BBHE in term of preserving image's brightness and entropy. BBHE and DSIHE are similar except that DSIHE choose to separate the histogram using threshold level with cumulative probability density equal to 0.5. DSIHE is claimed to yield output image with maximum entropy (Wan *et al.,* 1999). However, BBHE and DSIHE will also fail to enhance an image under certain condition.

This paper presents solutions to the limitation of HE, BBHE and DISHE. In what follows, the mathematical formulation of HE and BBHE is reviewed in section 2. The extension of BBHE, namely - Minimum Mean Brightness Error Bi-Histogram Equalization (MMBEBHE) will be presented in section 3 together with the formulation of the efficient integer-based implementation of MMBEBHE. Section 4 lists a few experimental results to illustrate the performance of MMBEBHE. Next, the generalization of BBHE, namely - Recursive Mean Separate Histogram Equalization (RMSHE) will be presented in RMSHE together with the mathematical analysis on RMSHE. Section 6 lists a few experimental results to illustrate the actual performance of RMSHE. Section 7 serves as the conclusion of this paper.

Histogram Equalization

This section covers brief details of HE and BBHE techniques as well as their mathematical analysis in preserving brightness. The content is mostly from (Kim, 1997) and (Kim and Cho, 1999).

Typical Histogram Equalization

Let's suppose X={X(*i, j*)}, denotes a digital image, where X(*i, j*) denotes the gray level of pixel at (*i, j*). The total number of the image pixels is *N*. The image intensity is digitized into *L* gray levels that are $\{X_0, X_1, ..., X_{L-1},\}$ and hence, $\forall X(i, j) \in \{ X_0, X_1, ..., X_{L-1},\}$. Suppose n_k denotes the total number of pixels with gray level X_k, then

$$p(X_k) = \frac{n_k}{N}, k = 0,\ 1,\ ...,\ L-1 \quad (1)$$

The relationship between $p(X_k)$ and X_k is defined as the probability density function (PDF). In fact, a plot of n_k vs. X_k is known as the histogram of **X**. Based on the PDF; the cumulative density function (CDF) is defined as

$$c(X_k) = \sum_{j=0}^{k} p\ (X_j),\ k = 0,\ 1,\ ...,\ L-1 \quad (2)$$

Note that $c(X_{L-1}) = 1$ by definition. HE is a scheme that maps the input image into the entire dynamic range, (X_0, X_{L-1}), by using the CDF as a transform function. Let's define a transform function based on the CDF as

$$f(X_k) = X_0 + \left(X_{L-1} - X_0\right)c\left(X_k\right) \quad (3)$$

Suppose the output image of the HE,

$$\mathbf{Y} = \{Y(i, j)\}, \text{ then}$$
$$\mathbf{Y} = f(\mathbf{X}) = \{f(X(i, j)) \mid \forall X(i, j) \in \mathbf{X}\} \quad (4)$$

HE enhances the contrast of an image through expanding the dynamic range. Besides, HE also produces an image, whose gray levels have a uniform density. In other words, it maximizes the entropy of an image (Wan *et al.,* 1999).
Nevertheless, an image whose gray levels has a uniform density, i.e.,

$$p(X) = 1 \ / \ (X_{L-1} - X_0) \quad (5)$$

will also have its mean equal to the middle gray level:

$$\mathbf{E(Y)} = \int_{X_0}^{X_{L-1}} xp(x)dx = \int_{X_0}^{X_{L-1}} \frac{X}{X_{L-1} - X_0} dx$$
$$= (X_{L-1} + X_0)/2 \quad (6)$$

E(.) denotes a statistical expectation. In other words, HE can introduce a significant change in the brightness of an image, which hesitate the direct application of HE in consumer electronics. For instance, Figure 4 (a) and

Figure. 4 (c) shows original image *arctic hare* and the resultant image of the HE that are composed of 256 gray levels. The equalized image is much darker than the input image.This is also unnatural enhancement in most part of the image. This is a direct consequence of the excessive change in brightness by HE when image has a high density over high gray levels. Figure. 5 (a) is the given original image girl and the Figure. 5 (c) is the result of HE. Figure. 5 (c) shows annoying artifacts in the background. It also shows that the contrast of the hair in the figure has also become poorer. The face and the cloth clearly show unnatural enhancement. The fundamental reason behind such limitation is that HE does not take the mean brightness of an image into account. BBHE has been designed to overcome such limitation

Brightness Preserving Bi-Histogram Equalization

Let's X_m denotes the mean of an image $\mathbf{X}$ and assumes that $X_m \in \{X_0, X_1, ..., X_{L-1}\}$. Based on its mean, the image is decomposed into two sub-images $\mathbf{X}_A$ and $\mathbf{X}_B$ as

$$\mathbf{X} = \mathbf{X}_A \cup \mathbf{X}_B \tag{7}$$

where

$$\mathbf{X}_A = \{X(i,j) | X(i,j) \leq X_m, \forall X(i,j) \in \mathbf{X}\} \tag{8}$$

and

$$\mathbf{X}_B = \{X(i,j) | X(i,j) > X_m, \forall X(i,j) \in \mathbf{X}\} \tag{9}$$

The sub-image $\mathbf{X}_A$ is composed of $\{X_0, X_1, ..., X_m\}$ and the sub-image $\mathbf{X}_B$ is composed of $\{X_{m+1}, X_{m+2}, ..., X_{L-1}\}$.

Next, let's define the respective PDF of the sub-images $\mathbf{X}_A$ and $\mathbf{X}_B$ as

$$p_A(X_k) = \frac{n_k}{N_A}, \; k = 0, 1, ..., m, \tag{10}$$

and

$$p_B(X_k) = \frac{n_k}{N_B}, \; m+1, m+2, ..., L-1 \tag{11}$$

N_A and N_B are the total number of pixels in $\mathbf{X}_A$ and $\mathbf{X}_B$, respectively. Note that $N = N_A .+ N_B$ The respective CDF for $\mathbf{X}_A$ and $\mathbf{X}_B$ are then defined as

$$c_A(X_k) = \sum_{j=0}^{k} p_A(X_j) \tag{12}$$

and

$$c_B(X_k) = \sum_{j=m+1}^{k} p_B(X_j) \tag{13}$$

Note that $c_A(X_m) = 1$ and $c_B(X_{L-1}) = 1$ by definition.

Similar to the case of HE where a CDF is used as a transform function, let's define the respective transform function for the sub-images $\mathbf{X}_A$ and $\mathbf{X}_B$ as:

$$f_A(X_k) = X_0 + (X_m - X_0)\, c_A(X_k) \tag{14}$$

and

$$f_B(X_k) = X_{m+1} + (X_{L+1} - X_{m+1})\, c_B(X_k) \tag{15}$$

The decomposed sub-images are equalized independently based on these transform functions. The compositions of the resulting equalized sub-images constitute the output of the BBHE. That is, the output image of BBHE, $\mathbf{Y}$, is finally expressed as

$$\mathbf{Y} = \{Y(i,j)\} = f_A(X_A) \cup f_B(X_B), \tag{16}$$

It is easy to see that $f_A(\mathbf{X}_A)$ equalizes the sub-image $\mathbf{X}_A$ over the range (X_0, X_m) whereas $f_B(\mathbf{X}_B)$ equalizes the sub-image $\mathbf{X}_B$ over the range (X_{m+1}, X_{L-1}). As a consequence, the input image $\mathbf{X}$ is equalized over the entire dynamic range (X_0, X_{L-1}) with the constraint that the sample less than the input mean are mapped to (X_0, X_m) and the samples greater than the mean are mapped to (X_{m+1}, X_{L-1}).

Analysis On The Brightness Change By the BBHE (Kim, 1997)

□The mean brightness of the output of th BBHE can be expressed a

$$\begin{aligned} \square E\,(\mathbf{Y}\square \;& \square E\,(\mathbf{Y} \mid \mathbf{X} \leq X_m) p(\mathbf{X} \leq X_m) \\ & \square E\,(\mathbf{Y} \mid \mathbf{X} > X_m) p(\mathbf{X} > X_m \\ \square \;& \square E\,(\mathbf{Y} \mid \mathbf{X} > X_m) \\ & \square E\,(\mathbf{Y} \mid \mathbf{X} > X_m) \qquad \square(17 \end{aligned}$$

$\square p(\mathbf{X} \leq X_m) = p(\mathbf{X} > X_m) =$ is used since $\mathbf{X}$ i assumed to have a symmetric distribution aroun Xm. With similar discussion used to obtain (6) it can easily shown tha

$$E\,(\mathbf{Y} \mid \mathbf{X} \leq X_m) = (X_0 + X_m) / 2 \qquad (18)$$

and

$$E\,(\mathbf{Y} \mid \mathbf{X} > X_m) = (X_m + X_{L-1}) / 2 \qquad (19)$$

The use of (18) and (19) in (17) results in

$$E\,(\mathbf{Y}) = (X_m + X_G) / 2 \qquad (20)$$

where

$$X_G = (X_0 + X_{L-1}) / 2 \qquad (21)$$

Equation (20) implies that the output mean of the BBHE is a function of the input mean brightness X_m. This clearly shows that the BBHE has the feature of preserving the original brightness, which is not found in typical HE.

Minimum Mean Brighness Error Bi-Histogram Equalization (MMBE BHE)

This paper proposes to use an objective measurement for measuring how well the brightness is preserved. It's referred to as Absolute Mean Brightness Error (AMBE), defined as the absolute difference between the input and the output mean as follow:

$$AMBE = \mid E(\mathbf{X}) - E(\mathbf{Y}) \mid \qquad (22)$$

Lower AMBE indicates that the brightness is preserved better. This paper proposes MMBEBHE, which perform bi-histogram equalization using the threshold level that minimizes the AMBE.

MMBEBHE is formally defined by the following procedures:

1. Calculate the AMBE for each of the possible threshold levels.
2. Find the threshold level, X_T that yield minimum AMBE,
3. Separate the input histogram into two based on the X_T found in step 2 and equalized them independently as in BBHE

Step 2 and 3 are straightforward process. Step 1 would require considerable amount of computation if one full BBHE process were required to compute the AMBE for each of the possible threshold level. The computation complexity would then be proportional to the square of the total number of gray levels. A 16-bits/pixel image would require up to 2^{32} operation. This could become a major drawback of MMBEBHE in real time implementation.

This paper proposes a fast implementation of MMBEBHE. Start from equation (23), which is the general definition for the mean brightness of the output image from bi-histogram equalization based techniques such as BBHE, DSIHE and MMBEBHE:

$$\begin{aligned} E(\mathbf{Y}) &= E\,(\mathbf{Y} \mid \mathbf{X} \leq X_T)\, p\,(\mathbf{X} \leq X_T) + E\,(\mathbf{Y} \mid \mathbf{X} > X_T)\, p\,(\mathbf{X} > X_T) \\ &= E_1(X_T)p_1(X_T) + E_2(X_T)\, p_2(X_T) \end{aligned} \qquad (23)$$

where X_T is the chosen threshold, $E\,(X_T)$ and $p(X_T)$ are the mean brightness and CDF of each image involved. Now take the differentiation on both sides:

$$\begin{aligned} \Delta E\,(\mathbf{Y} &= \Delta E_1(X_T)\, p_1\,(X_T) + E_1(X_T)\,\Delta\, p_1\,(X_T) \\ &\quad \Delta E_2\,(X_T)\, p_2\,(X_T) + E_2\,(X_T)\,\Delta\, p_2\,(X_T \end{aligned} \qquad (24$$

If one considers only the change in value of $E(\mathbf{Y}$ when $X_T \rightarrow X_{T+1}$, the

$$\Delta E\,(\mathbf{Y}) = E_{T+1}(\mathbf{Y}) - E_T(\mathbf{Y} \qquad (25$$

By using definitions of E_1 and E_2 like the one shown in (18) and (19), and the fact tha $p_1(X_T) + p_2(X_T) = 1$, one can rewrite equation (24 when $X_T \rightarrow X_{T+1}$ or when ΔT = 1, a

$$\begin{aligned} \Delta E(\mathbf{Y} &= \tfrac{1}{2} + [E_1(X_T) - E_2(X_T)]\, p(X_{T+1} \\ &= \tfrac{1}{2} - \tfrac{1}{2}\,(X_{L-1} - X_0 + 1)\, p(X_{T+1} \\ &= \tfrac{1}{2}\,[1 - Lp(X_{T+1}) \end{aligned} \qquad (26$$

From equation (25), one can relate the Mea Brightness Error (MBE) as follows

$$\begin{aligned} \Delta E(\mathbf{Y} &= [E_{T+1}(\mathbf{Y}) - E(\mathbf{X})] - [E_T(\mathbf{Y}) - E(\mathbf{X}) \\ &= MBE_{T+1} - MBE \end{aligned} \qquad (27$$

Combining (26) and (27) will giv

$$MBE_{T+1} = MBE_T + \tfrac{1}{2}\,[1 - Lp(X_{T+1}) \qquad (28$$

Note that $MBE_0 = X_G - E(\mathbf{X})$. Equation (28 involves $p(X_{T+1})$ which are floating poin numbers. Since the application requires onl finding the threshold level with minimu AMBE, the scaled MBE would be sufficient Scaled MBE involves only integer numbers Let's denote $F(X_i)$ as the number of pixel for gra level X_i and N as the total number of pixel. The it follows that:

$$(2N)MBE_{T+1} = (2N)\,[MBE_{T+}\tfrac{1}{2}\,[1- Lp(X_{T+1})]]$$
$$\mathrm{SMBE}_{T+1} = \mathrm{SMBE}_{T} + [N- LF(X_{T+1})] \qquad (29)$$

where $SMBE = (2N)MBE$. The number gray levels, L are often of base 2. In such cases, the multiplication with L could be further reduced to basic shift operation. In order to find the absolute value of SMBE, a comparator is required. Check each SMBE and if it is negative, negate the value. From equation (29), it's clear that the complexity of computation has been reduced from a square function to a linear function of the number of gray levels, L.

Simulation Results of MMBEBHE

Table 1 contains the AMBE of the test images after being enhanced by the HE, BBHE, DSIHE and MMBEBHE respectively. Image *F16* has been used in (Kim, 1997) to demonstrate the success of BBHE. In this particular case, the threshold level used by the BBHE and MMBEBHE are both close to zero. This indicates that the threshold level should be chosen base on the resulting AMBE and not fixed to the input mean. Table 1 also shows that HE, BBHE and DSIHE do not guarantee minimum mean error. Graph 1 shows the |SMBE| vs. Threshold Level for image *Arctic hare* shown in Figure 4. The graph also clearly shows that BBHE and DSIHE do not guarantee minimum mean error.

In Figure 4 the output images of HE, BBHE and DSIHE clearly show the presence of unnatural contrast enhancement; their mean brightness are much darker compared to the original image. In contrast, the output image of MMBEBHE (Figure 4 (b)) has brighter mean brightness and more natural enhancement. Yet, there are images that require far more brightness preservation than MMBEBHE can provide to avoid annoying artifacts. This is clearly shown in image *Girl* (Figure 5). The coming section will present a generalization of bi-histogram equalization to overcome this limitation.

Recursive Mean-Separate Histogram Equalization (RMSHE)

The design of BBHE indicates that one of the ways to preserve brightness is by having mean-separation before performing the equalization process. Mean-separation refers to separating an image into two sub-image based on the mean of the input image. In fact, this is equivalent to separating an image's histogram into two portions based on the mean of the input image. After mean separation, the two portions of the histogram are equalized independently. Recursive Mean-Separate Histogram Equalization (RMSHE) proposed in this paper is basically a generalization of HE and BBHE in the aspect of brightness preservation. This idea is illustrated more clearly in the following figures.

Table 1. List of the selected threshold level, X_T and the resulting AMBE for HE, BBHE, DSIHE and MMBEBHE.

	HE	BBHE		DSIHE		MMBEBHE	
	AMBE	X_T	AMBE	X_T	AMBE	X_T	AMBE
U2	96.7	31	13.3	23	41.5	40	6.24
Arctic hare	90.5	239	24.2	244	37.9	224	13.5
Copter	63.4	148	18.1	155	28.0	142	3.5
F16	48.7	176	0.35	197	14.6	173	0.02
Hands	99.5	27	17.5	0	18.3	9	15.4

Figure 1 shows histogram before and after HE. No mean-separation is performed before equalization and no brightness preservation is observed. This is indicated in equation (6) where output mean is always the middle gray level. In fact, HE is equivalent to RMSHE level 0 ($r = 0$).

Figure 2 shows histogram before and after BBHE. One mean-separation is performed before equalization and some level of brightness preservation is observed. This is indicated by equation (20) where input mean, X_m has equal weight as the middle gray level, X_G in the output mean. In fact, BBHE is equivalent to RMSHE with $r = 1$. It follows that to achieve higher brightness preservation, one may perform the mean separation recursively; that is to separate each portions of the histogram based on their respective means.

Supposed that **X** is further separated into 4 portions based on the mean of the two portions of the histogram, X_{ml} and X_{mu} as shown in Figure 3.

Let's define X_{ml} and X_{mu} :

$$X_{ml} = \frac{\int_{X_0}^{X_m} xp(x)dx}{\int_{X_0}^{X_m} p(x)dx} = 2\int_{X_0}^{X_m} xp(x)dx \qquad (30)$$

$$X_{mu} = \frac{\int_{X_m}^{X_{L-1}} xp(x)dx}{\int_{Xm}^{X_{L-1}} p(x)dx} = 2\int_{Xm}^{X_{L-1}} xp(x)dx \qquad (31)$$

where

$$\int_{X_0}^{X_m} p(x)dx = \int_{X_m}^{X_{L-1}} p(x)dx = \tfrac{1}{2} \qquad (32)$$

since **X** is assumed to have a symmetric distribution around X_m (Kim, 1997).

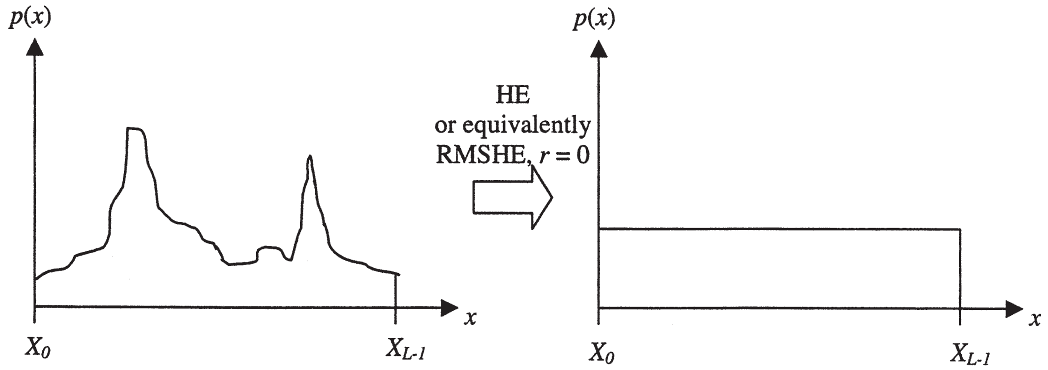

Figure 1. Histogram before and after HE or equivalently, RMSHE, $r = 0$.

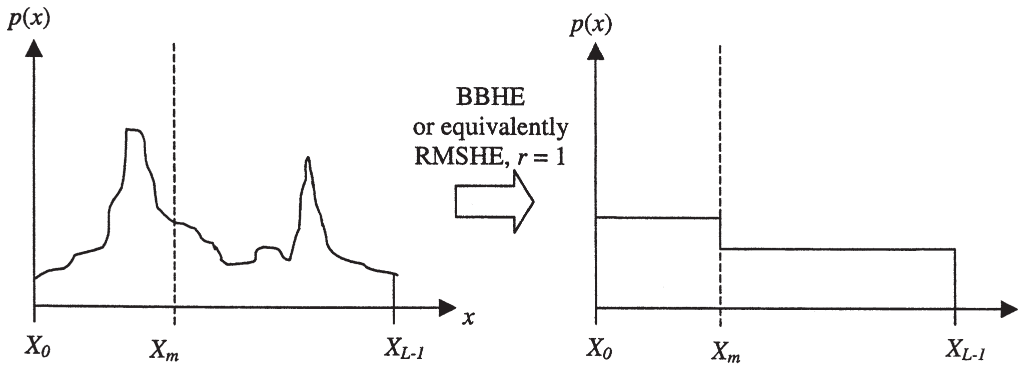

Figure 2. Histogram before and after BBHE or equivalently, RMSHE, $r = 1$.

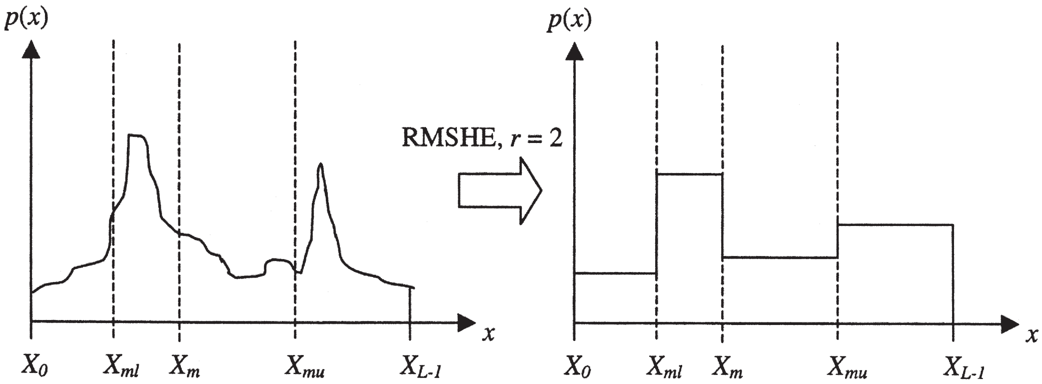

Figure 3. Histogram before and after RMSHE, r = 2.

Figure 3 shows the histogram before and after equalizing the four portions of the histogram independently. This is the result of RMSHE with r = 2. The following shows the formulation of the output mean.

$$\begin{aligned} E(\mathbf{Y}) &= E(\mathbf{Y} \mid \mathbf{X} \le X_{ml})\, p(X \le X_{ml}) + \\ &\quad E(\mathbf{Y} \mid X_{ml} < \mathbf{X} \le X_m)\, p(X_{ml} < \mathbf{X} \le X_m) \\ &\quad + E(\mathbf{Y} \mid X_m < \mathbf{X} \le X_{mu})\, p(X_m < \mathbf{X} \le X_{mu}) \\ &\quad + E(\mathbf{Y} \mid \mathbf{X} > X_{mu})\, p(\mathbf{X} > X_{mu}) \\ &= \tfrac{1}{4}\{ E(\mathbf{Y} \mid \mathbf{X} \le X_{ml}) \\ &\quad + E(\mathbf{Y} \mid X_{ml} < \mathbf{X} \le X_m) \\ &\quad + E(\mathbf{Y} \mid X_m < \mathbf{X} \le X_{mu}) \\ &\quad + E(\mathbf{Y} \mid \mathbf{X} > X_{mu})\} \end{aligned} \tag{33}$$

where $p(\mathbf{X} \le X_{ml}) = p(X_{ml} < \mathbf{X} \le X_m) = p(X_m < \mathbf{X} \le X_{mu}) = p(\mathbf{X} > X_{mu}) = \tfrac{1}{4}$ is used since $\mathbf{X}$ is assumed to have a symmetric distribution around X_m.

With similar discussion to obtain (6)

$$\begin{aligned} E(\mathbf{Y}) &= \tfrac{1}{4}\{ [(X_0 + X_{ml})/2] + [(X_{ml} + X_m)/2] + [(X_m + X_{mu})/2] + [(X_{mu} + X_{L-1})/2] \} \\ &= \tfrac{1}{4}\{ X_G + 3X_m \} \end{aligned} \tag{34}$$

since from (30) and (31),

$$\frac{X_{mu} + X_{ml}}{2} = X_m \tag{35}$$

Two mean-separations before equalization results in higher level of brightness preservation as shown in equation (34) where the weight of input mean, X_m has increased to three times as much as the weight of middle gray level, X_G. Following the above discussion, it is very reasonable to expect that the brightness preservation will increase as the number of recursive mean-separations increases. Based on the output mean for RMSHE with r = 0, 1 and 2 shown above, it is not difficult to generalized the output mean $E(\mathbf{Y})$ for RMSHE with r as follows:

$$\begin{aligned} E(\mathbf{Y}) &= ((2^r - 1)X_m + X_G)/2^r \\ &= X_m + [(X_G - X_m)/2^r] \end{aligned} \tag{36}$$

Equation (36) indicates that as r grows larger, $E(\mathbf{Y})$ will eventually converge to the input mean, X_m. In other words, it allows scalable brightness preservation range from 0% (output of HE) - 100% (getting back the original image). In the applications of consumer electronics, the variety of image involve are too wide to be covered with only a specific level of brightness preservation. Therefore, the scalability in this algorithm is the most desirable property to allow adjustment of level of brightness preservation base on individual image's characteristic.

Simulation Results of RMSHE

In order to demonstrate the performance of the proposal algorithm, simulation results of RMSHE on image *Girl* is presented in Figure 5. The result of HE, BBHE DSIHE, MMBEBHE (Figure 5 (c), (d), (e) and (f)) show unpleasant

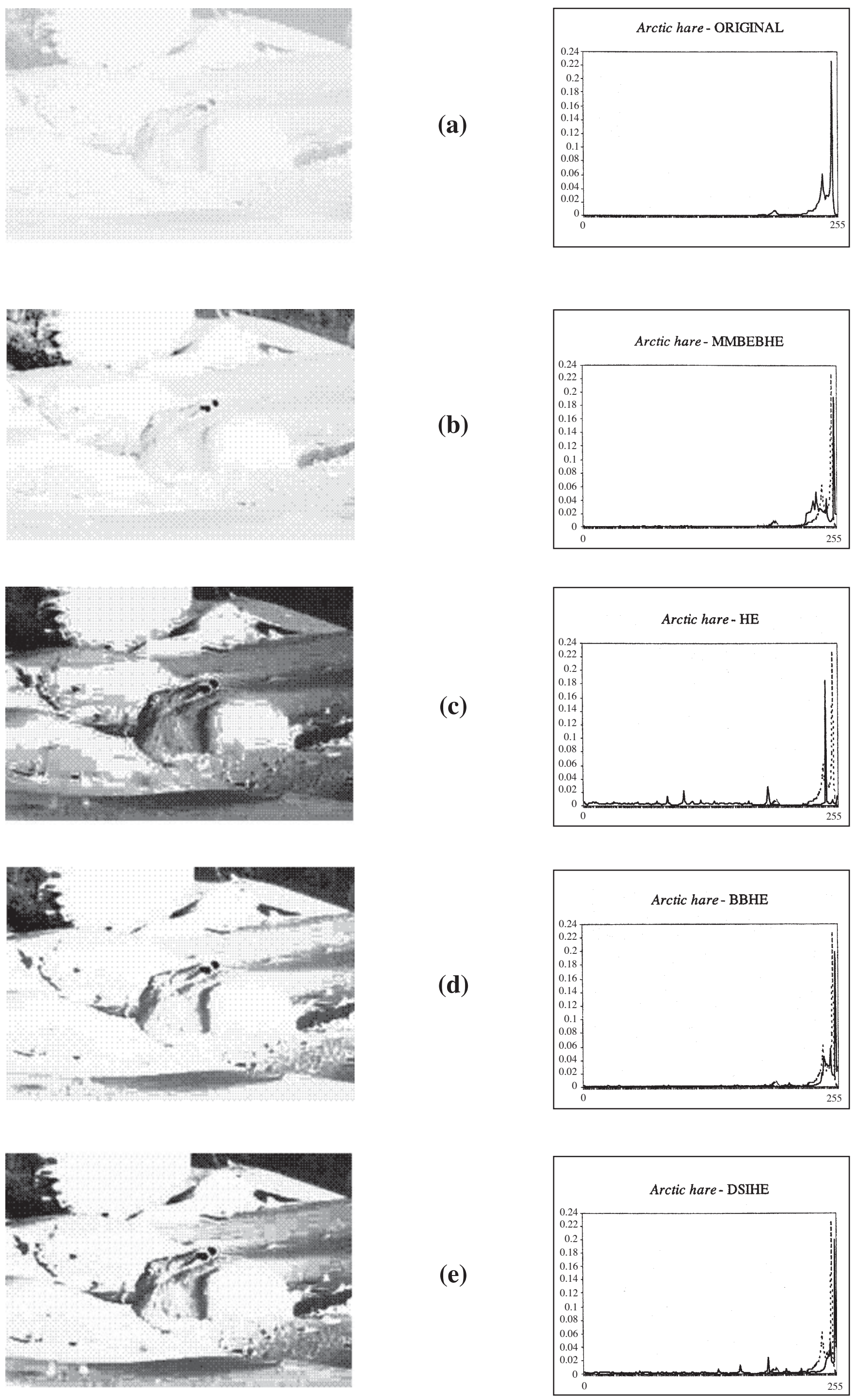

Figure 4. **(a) Original image of *Arctic hare*; (b) Result of MMBEBHE; (c) Result of HE; (d) Result of BBHE and (e) Result of DSIHE.**

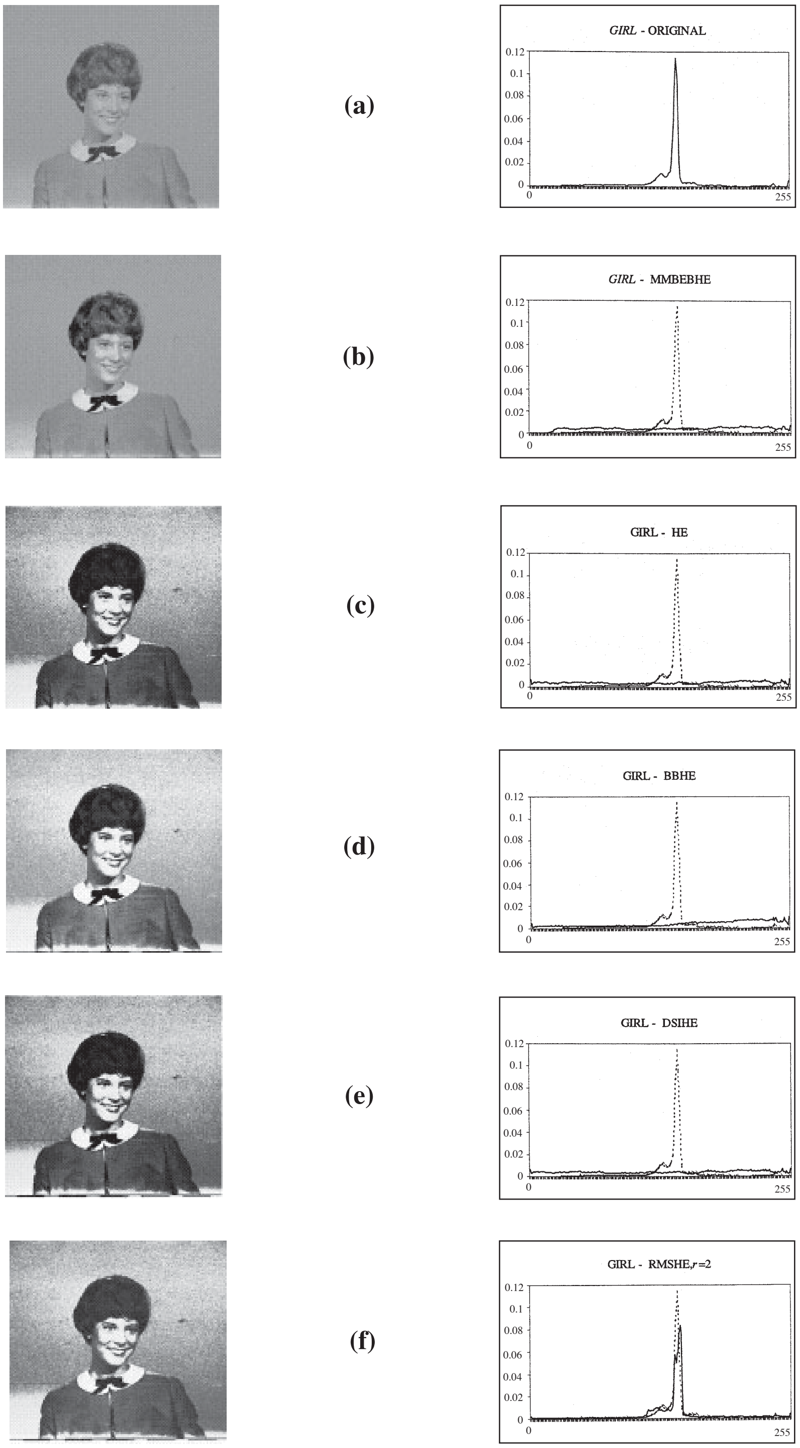

Figure 5. (a)Original image *girl*; (b) Result of RMSHE r = 2; (c) Result of HE; (d) Result of BBHE; (e) Result of DSIHE and (f) Result of MMBEBHE.

artifacts in the background, decrease of contrast in the hair and also unnatural enhancement of the face. All these artifacts are not seen at all in the results of RMSHE with r = 2 as shown in Figure 5 (b).

Conclusions

This paper proposes MMBEBHE which extend BBHE such that the mean error between the input and output image is minimized This paper has also formulated an efficient integer-based implementation of MMBEBHE such that the computation complexity is reduced from a square function to a linear function of the total number of gray levels. Simulation results have shown that MMBEBHE does preserve the brightness better. Sample images which are failed to be enhanced by HE, BBHE and DSIHE, have been enhanced properly by MMBEBHE.

Nevertheless, MMBEBHE also fails to enhance certain image. Therefore, this paper further proposes a generalization of BBHE referred to as RMSHE. RMSHE separates the input histogram based on the mean recursively. Mathmatical analysis on the output mean of RMSHE shows that the output mean will converge to the input mean as the number of recursive mean-separation increses. This implies RMSHE allows scalable preservation range from 0% (output of HE) - 100% (getting back the original image). Simulation results show that the images which cannot be enhanced well by HE, BBHE DSIHE and MMBEBHE, can be properly enhanced by RMSHE.

Future work suggested is to look for proper mechanisme to automate the selection of the level of RMSHE, *r* that give optimum output. This paper also suggests to look into the effective implementation of the RMSHE, in the similar fashion of how the Quantized Mean-Separate HE (Kim, 1999) has been proposed as a cost reduced implementation for the BBHE. This paper also suggests to look into the extension of this algorithm to color image, in the similar fashion of what has been proposed in (Kim, 2000) for extension of the BBHE to color image.

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