DISCRETE-TIME FEEDBACK ERROR LEARNING

Sirisak Wongsura[*] and Waree Kongprawechnon

Received:

Abstract

In this study, a theoretical foundation on the Discrete-Time Feedback Error Learning (DTFEL) method is established. This is analogous to the original continuous-time version Feedback Error Learning (FEL) method which is proposed as a control model of cerebellum in the field of computational neuroscience. The FEL is an adaptive control method for two-degree-of-freedom control schemes. In this control scheme the adaptive controller must become an inverse system of the plant. DTFEL is proposed because now the controller is computer-based, which means that the digital control scheme is used instead of the analog one. Many control algorithms are also computed in discrete-time format. Furthermore, some systems are themselves inherently discrete and, certainly for these systems, it is useful to have results available in discrete version for controlling. The ready-to-use controller is the key target of this study.

Keywords: Discrete-time system, Feedback Error Learning, strictly positive real, learning control

Introduction

Recently, as a control method for the case where the plant is unknown, many schemes on adaptive control have been studied. Adaptive control is a fascinating field of study and research. It is also of increasing practical importance since adaptive techniques are being used more and more in industrial control systems. However, there are still many unsolved theoretical and practical issues.

This study presents a formulation and formal stability analysis of the Discrete-Time Feedback Error Learning (DTFEL) method. It is developed from the continuous-time feedback error learning (FEL) method (Gomi and Kawato, 1993). This research is to study and design the controller for a class of discrete-time systems from a viewpoint of the adaptive control theory. Originally, FEL method was proposed from a biological perspective to establish a computational model of the cerebellum for learning motor control with internal models in the central nervous system (CNS) (Kawato *et al*., 1987). The research presented here is inspired by the insight of the close relationship between FEL method and adaptive control algorithms which is gained during our recent development of a new adaptive control framework with advanced statistical learning. From a control theoretic viewpoint, FEL method can be conceived of as an adaptive control technique (Miyamura and Kimura, 2002). Stability analysis of FEL for a class of linear systems and a two-link planar robot arm

[*] *Sirinchorn International Institute of Technology, Thammasat University, Pathumthani, 12121, Thailand, Tel:0-2501-3505-20 ext.1813, Fax: 0-2501-3524, E-mail: sir_isak@yahoo.com*

in a horizontal plane are presented by Miyamura and Kimura (Miyamura and Kimura, 2002) and Ushida and Kimura (Ushida and Kimura, 2002), respectively. However, the considered plant dynamics are confined to a restricted class of continuous-time linear systems. Nowadays, the controller is computer-based which is usually not suitable to apply the theoretical knowledge of the FEL method directly. That means, to apply to real applications, the algorithm of the original FEL method must be discretized and programmed digitally beforehand. This may lead to a serious situation because the stability properties, analyzed in continuous-time case, are not guaranteed for the discretized system. Furthermore, in some systems, they are originally be discrete systems. It might be better if we control those systems by discrete-time controller. This study is aimed to establish a theoretical foundation on the DTFEL method which can be applied to the systems directly with discrete-time nature.

This paper is organized as follows: First, all mathematical and control theories, required to analyze the stability of DTFEL system, are summarized. There are some new proved knowledge presented in this section. Second, the structure of the control system are presented. Then, the stability properties of DTFEL are discussed. A Lyapunov stability analysis of DTFEL is also provided. Next, the numerical simulated examples to illustrate the theoretical stability properties of DTFEL is presented. Last, the conclusion of this study is drawn.

Notation:

Throughout this study, a fairly standard notation is used. The overview is as follow.

$\gamma_{\min}[P]$	the smallest eigenvalue of P.
$\|\|A\|\| = \sqrt{tr(A^T A)} = \sqrt{\sum_{i,j} a_{ij}^2}$	the Frobenius norm.
$(A, B, C, D) = D + C(zI-A)^{-1}B$	a minimal realization
p.r.	positive real
s.p.r.	strictly positive real
PE	persistently exciting

Mathematical Preliminaries

In this section, the mathematical requirements to analyze the DTFEL method in the next section are discussed. The main and most important area is to study the strictly positive real system. Also, there are some new theorems proved in this section.

Definition 1 (Tao and Ioannou, 1990) A square matrix $H(z)$ of real rational functions is a positive real (p.r.) matrix if

(d1) $H(z)$ has elements analytic in $|z| > 1$.

(d2) $H^T(z^*)+H(z)$ is positive, semidefinite and Hermitian for $|z| > 1$.

Condition (d2) can be replaced by

(d3) The poles of the elements of $H(z)$ on $|z| = 1$ are simple and the associated residue matrices of $H(z)$ at these poles are 0.

(d4) $H(e^{j\theta})+H(e^{-j\theta})$ is a positive semidefinite Hermitian matrix for all real θ for which $H(e^{j\theta})$ exists.

Definition 2 (Tao and Ioannou, 1990) A rational transfer matrix $H(z)$ is a strictly positive real (s.p.r.) matrix if $H(\rho z)$ is p.r. for some $0 < \rho < 1$.

Given Definition 2, a necessary and sufficient condition in the frequency domain for s.p.r. transfer matrices in the class $\aleph$ can be defined as following.

Definition 3 (Tao and Ioannou, 1990) An $n \times n$ rational matrix $H(z)$ is said to belong to class $\aleph$ if $H(z) + H^T(z^{-1})$ has rank n almost everywhere in the complex z-plane.

Theorem 1 (Tao and Ioannou, 1990) Consider the $n \times n$ rational matrix $H(z) \in \aleph$ given in Definition 3. Then, $H(z)$ is a s.p.r. matrix if and only if

(a) All elements of $H(z)$ are analytic in $|z| \geq 1$,

(b) $H(e^{j\theta})+H^T(e^{-j\theta})>0, \forall\theta\in[0,2\pi]$

Lemma 1 (Discrete-time version of Kalman-Yakubovich-Popov) (Tao and Ioannou, 1990) Assume that the rational transfer matrix *H(z)* has poles that lie in $|z| < \gamma$, where $0 < \gamma < 1$ and *(A, B, C, D)* is a minimal realization of *H(z)*. Then, $H(\gamma z)$is s.p.r., if and only if real matrices $P = P^T$,>0, Q, andexist such that

$$A^T PA - P = -QQ^T - (1-\gamma^2)P,$$
$$APB = C^T - QK,$$
$$K^T K = D + D^T - B^T PB.$$

Remark

If *L(z)* is a stable transfer function, there exists sufficiently large *K* such that $\frac{a}{k}\left(L(z) + K\right)^{-l}$is s.p.r. Consider the linear discrete-time varying system given by

$$x(k+1) = A(k)x(k) + B(k)u(k), \quad y(k) = C(k)x(k), \tag{1}$$

with *A(k)*, *B(k)*, *C(k)* being appropriately dimensioned matrices.

Lemma 2 (Jagannathan, 1996) Define $\psi(k_1, k_0)$ as the state-transition matrix corresponding to A(k) for the system (1), i.e., $\psi(k_1, k_0) = \prod_{k=k_0}^{k_1-1}$ A(k). Then, if $\psi(k_1,k_0)\|\leq 1,\ \forall k_1,k_0 \geq 0$, the system (1) is exponentially stable.

Lemma 3 (Jagannathan, 1996) If $A(k) = I-\alpha\phi(k)\phi^T(k)$ in (1), where $0 < \alpha < 2$ and $\phi(k)$ is a regressor vector of past inputs and outputs, then $\| \phi(k_1,k_0) \| < 1$ is guaranteed if there is an $L > 0$ such that $\sum_{k=k_0}^{k_1+L-1} \phi(k)\phi^T(k) > 0$ for all k. Then, Lemma 2 guarantees the exponential stability of the system (1).

Definition 4 (Jagannathan, 1996) An input sequence *x(k)* is said to be persistently exciting (PE) if $\gamma > 0$ and an integer $k_l \geq 1$ such that

$$\gamma_{\min}\left[\sum_{k=k_0}^{k_1+L-1} \phi(k)\phi^T(k)\right] > \gamma, \forall k_0 \geq 0. \tag{2}$$

Note: PE is exactly the stability condition needed in Lemma 3.

Theorem 2 A difference equation

$$z(k+1) = \left(I - \xi(k)L(z)\xi^T(k)\right)z(k) \tag{3}$$

is asymptotically stable for any time-varying vector *(k)* which satisfies the PE condition, if *L(z)* is s.p.r..

Proof

To prove this theorem, consider the following discrete-time state-space equation of a scalar pulse-transfer function $L(z) = \frac{Y(z)}{U(z)} = c^T \left(zI - A\right)^{-1} b + d$,

$$x(k+1) = Ax(k) + bu(k),$$
$$y(k) = c^T x(k) + du(k).$$

By using this state-space equation form, the difference equation in Eqn. (3) can then be represented as

$$x(k+1) = Ax(k) + b\xi^T z(k), \tag{4}$$

$$y(k) = c^T x(k) + d\xi^T z(k), \tag{5}$$

$$z(k+1) = z(k) - \xi^T y \tag{6}$$

Assume that *L(z)* is s.p.r.. The theorem can then be proved by using Lyapunov stability analysis.

Consider a Lyapunov function

$$V(k) = x^T(k)Px(k) + \| z(k)\|^2 \tag{7}$$

From Eqn.(4)-(6) and Lemma 1,

$$x^T(k+1)Px(k+1) - x^T(k)Px(k)$$
$$= \left(Ax + b\xi^T z\right)^T P\left(Ax + b\xi^T z\right) - x^T Px$$
$$= \left(x^T A^T P + z^T \xi b^T P\right)\left(Ax + b\xi^T z\right) - x^T Px$$
$$= x^T A^T PAx + z^T \xi b^T PAx + x^T A^T Pb\xi^T z + z^T \xi b^T Pb\xi^T z - x^T Px$$
$$= x^T \left(A^T PA - P\right)x + z^T \xi b^T PAx + x^T A^T Pb\xi^T z + z^T \xi b^T Pb\xi^T z$$
$$= x^T \left(-qq^T - \varepsilon L\right)x + z^T \xi b^T PAx +$$

$$
\begin{aligned}
& x^T A^T P b \xi^T z + z^T \xi b^T P b \xi^T z \\
&= -x^T q q^T x - x^T \varepsilon L x + z^T \xi \left(A^T P b\right)^T x + \\
&\quad x^T A^T P b \xi^T z + z^T \xi \left(b^T P b\right) \xi^T z \\
&= -x^T q q^T x - x^T \varepsilon L x + z^T \xi \left(\frac{c}{2} + v q\right)^T x + \\
&\quad x^T A^T P b \xi^T z + z^T \xi \left(d - v^2\right) \xi^T z \\
&= -x^T q q^T x - x^T \varepsilon L x + \frac{1}{2} z^T \xi c^T x + \\
&\quad z^T \xi q^T v x + x^T A^T P b \xi^T z + z^T \xi d \xi^T z \\
&\quad - z^T \xi v^2 \xi^T z \\
&= -x^T \varepsilon L x - \Big[(q^T x)^T q^T x - \left(v \xi^T z\right)^T \left(q^T x\right) \\
&\quad - \left(q^T x\right)^T \left(v \xi^T z\right) + \left(v \xi^T z\right)^T \left(v \xi^T z\right) \Big] \\
&\quad + \frac{1}{2} z^T \xi c^T x + x^T A^T P b \xi^T z + z^T \xi d \xi^T z \\
&\quad - x^T q v \xi^T z \\
&= -x^T \varepsilon L x - \left|q^T x - v \xi^T z\right|^2 + \frac{1}{2} z^T \xi c^T x \\
&\quad + x^T A^T P b \xi^T z + [z^T \xi \left(d \xi^T z + c^T x\right)] \\
&\quad - \left[z^T \xi c^T x \right] - x^T q v \xi^T z \\
&= -x^T \varepsilon L x - \left|q^T x - v \xi^T z\right|^2 - \frac{1}{2} z^T \xi c^T x \\
&\quad + x^T A^T P b \xi^T z + [z^T \xi \left(y\right)] - x^T q v \xi^T z \\
&= -x^T \varepsilon L x - \left|q^T x - v \xi^T z\right|^2 - \frac{1}{2} z^T \xi c^T x \\
&\quad + x^T \left(A^T P b - v q\right) \xi^T z + z^T \xi y \\
&= -x^T \varepsilon L x - \left|q^T x - v \xi^T z\right|^2 - \frac{1}{2} z^T \xi c^T x \\
&\quad + x^T \left(\frac{c}{2}\right) \xi^T z + z^T \xi y \\
&= -x^T \varepsilon L x - \left|q^T x - v \xi^T z\right|^2 + z^T \xi y
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
& \| z(k+1) \|^2 - \| z(k) \|^2 \\
&\quad = \Big[\| z \|^2 - y^T \xi z - \\
&\quad\quad z^T \xi^T y + y^T \xi \xi^T y \Big] - \| z(k) \|^2 \\
&\quad = -z^T \xi^T y - y^T \xi z + y^T \xi \xi^T y
\end{aligned}
$$

Then,

$$
\begin{aligned}
\Delta V(k) &= V(k+1) - V(k) \\
&= \left[x^T(k+1) P x(k+1) - x^T(k) P x(k) \right] \\
&\quad + \left[\| z(k+1) \|^2 - \| z(k) \|^2 \right] \\
&= \left[-x^T \varepsilon L x - \left|q^T x - v \xi^T z\right|^2 + z^T \xi y \right] \\
&\quad + \left[-z^T \xi^T y - y^T \xi z + y^T \xi \xi^T y \right] \\
&= -x^T \varepsilon L x - \left|q^T x - v \xi^T z\right|^2 - y^T \xi z \\
&\quad + y^T \xi \xi^T y \\
&\le 0 \qquad \text{if} \quad d^T \| z \|^2 \ge \| y \|^2 \qquad (8)
\end{aligned}
$$

From Eqns. (7) and (8), *x(k)* and $\xi^T(k) z(k)$ converge to 0. From this result and Eqns. (4)-(6), for sufficiently large *k*,

$$
z(k+1) = z(k) - d \xi(k) \xi^T(k) z(k) \qquad (9)
$$

Since *L(z)* is s.p.r., *d > 0*. From the assumption, $\xi(k)$ satisfies the PE condition (2). Hence, due to Lemma 1, the system described by (9) is asymptotically stable which implies that *z(k)* converges to 0. Hence, Theorem 2 has been proved.

Note that a special case of Theorem 2, where *L(z)* = 1, corresponds to Eqns (3).

The requirement in Eqns (8) can be translated as "the direct input-output transmission gain *d* is positive and sufficiently large". This clarifies the essential differences between continuous-time and discrete-time cases. This is a special feature of discrete-time systems which makes the requirement relatively complicated.

Similar requirements frequently occur in literature relating to some discrete-time control systems (Kongprawechnon and Kimura, 1998; de la Sen, 2000).

Analysis of the Discrete-Time Feedback Error Learning

Feedforward Adaptive Control Method Without Feedback Element

The discussion of the feedback error learning method (henceforth, it is simply referred as the Kawato scheme) from the viewpoint of adaptive control is the main objective of this section. Figure 1 illustrates the block diagram of the Kawato scheme.

To briefly explain the key concept of DTFEL, consider its architecture shown in Figure 1. The objective of control is to minimize the error *e(k)* between the command signal *r(k)* and the plant output *y(k)*. The input *u(k)* to the plant *P* is composed of the output $u_{ff}(k)$ of feedforward controller K_2 and $u_{fb}(k)$ of the feedback controller K_1. If *P* is known and P^{-1} exists and is stable, choosing $K_2=P^{-1}$ makes the tracking perfect. Indeed, from the relations $u_{ff}=P^{-1}r$, $u_{fb}=K_1(r-y)$ and $y=P(u_{ff}+u_{fb})$, it is easily to see that $y=r$. However, in most systems, *P* is unknown, so some adaptive schemes for K_2 are employed so that K_2 converges to P^{-1}. Thus, the novelty of the DTFEL method lies in its way to learn the inverse model of *P*. That is the parameters be adapted so that $K_2=P^{-1}$.

Throughout this section, the following assumptions are applied:

Assumptions

(A1) The plant *P* is stable and has stable inverse P^{-1}.

(A2) The upper bound of the order of *P* is known.

(A3) $l_0 = \lim_{z\to\infty} P(z)$ is assumed to be positive.

(A4) Input signal is bounded and satisfies the PE condition.

The assumption (A1) is rather restrictive in the context of control system design. This may be relaxed without significant difficulty, but in this study, this assumption is kept in order to focus on the intrinsic nature of the Kawato scheme. In the context of motor control, this assumption is not restrictive because the plant is always a neuro-muscular system with low order. This lets the computed torque method, which is essentially equivalent to constructing an inverse model, to be applicable.

If l_o is negative in (A3), the subsequent results are valid by taking *-P(z)* instead of *P(z)*. Hence, (A3) is relaxed to the assumption that the sign of l_o is known. For the sake of the simplicity of exposition, however, (A3) is retained. From the assumption (A4), it is obvious that also $\xi(k)$ satisfies the PE condition.

Parameterization of Unknown Systems

To handle adaptation, it is important to decide how to parameterize the adaptive system. Throughout this study, the following parameterization of the unknown system k_2 is utilized:

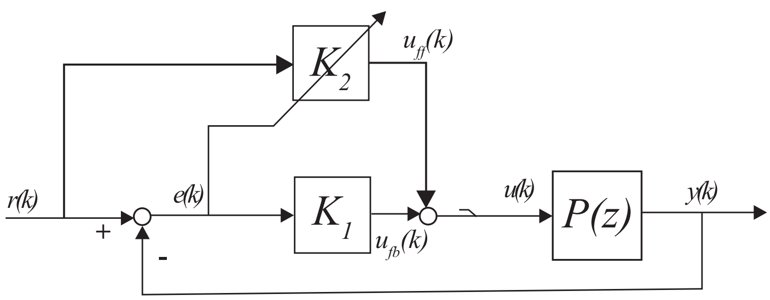

Figure 1. Discrete-time feedback error learning scheme

$$\xi_1(k+1) = F\xi_1(k) + gr(k) \qquad (10)$$

$$\xi_2(k+1) = F\xi_2(k) + gu(k) \qquad (11)$$

$$u(k) = c^T(k)\xi_1(k) + d^T(k)\xi_2(k) \qquad (12)$$

$$+ l(k)r(k),$$

where F is any stable matrix and g is any vector with $\{F,g\}$ being controllable. The block diagram of this parameterization is shown in Figure 2. In Eqns. (10)-(12), *c(k), d(k)* and *l(k)* are unknown parameters to be estimated. *u(k)* and *r(k)* are the output and the desired output of this system, respectively. It is easy to see that appropriate selection of parameters $c(k)=c_0$, $l(k)=l_0$ and can yield an arbitrary transfer function from *r(k)* to *u(k)*.

To see this, let the matrix and vector be in a controllable canonical form:

$$F = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ddots & 1 \\ -f_1 & -f_2 & -f_3 & \dots & -f_n \end{bmatrix}, g = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \qquad (13)$$

From Eqns. (10) - (13), the transfer function from to is given by

$$\frac{U(z)}{R(z)} = \left(\frac{U(z)-d^T\xi_2}{R(z)}\right)\left(\frac{U(z)\cdot}{U(z)-d^T\xi_2}\right)$$

$$= \left(\frac{U(z)-d^T\xi_2}{R(z)}\right)\left(\frac{1}{1-d^T\dfrac{\xi_2}{U(z)}}\right) \qquad (14)$$

$$= \frac{l_0 + c_0^T(zI-F)^{-1}g}{1-d_0^T(zI-F)^{-1}g}$$

$$= \frac{l_o z^n + (f_n l_o + c_n)z^{n-1} + \dots + (f_1 l_o + c_1)}{z^n + (f_n - d_n)z^{n-1} + \dots + (f_1 - d_1)},$$

Therefore, any transfer function of degree less than or equal to n can be constructed by selecting parameters c_0, d_0 and l_0 appropriately. The advantage of the parameterization (10) - (12) is that the unknown parameters enter linearly in the system description. The continuous version of this parameterization was firstly used in adaptive observer (Narendra and Valavani, 1989).

Adaptation Law

The same parameterization of the adaptive feedforward controller K_2 as in Eqns. (10) - (12) is taken.

$$\xi_1(k+1) = F\xi_1(k) + gr(k) \qquad (15)$$

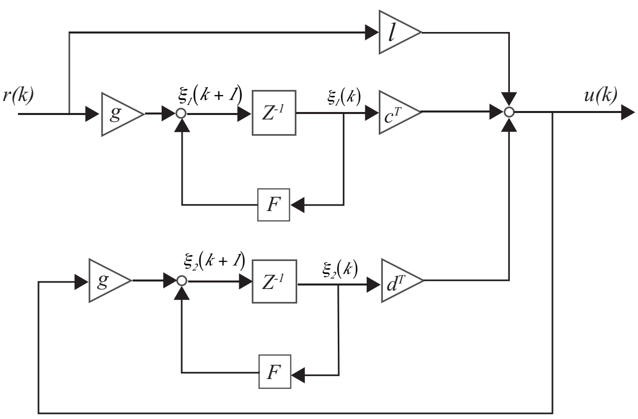

Figure 2. Parameterization of

$$\xi_2(k+1) = F\xi_2(k) + gu(k) \quad (16)$$

$$u_{ff}(k) = c^T(k)\xi_1(k) + d^T(k)\xi_2(k) + l(k)r(k) \quad (17)$$

$$u(k) = u_{ff}(k) + K_1 e(k), \quad (18)$$

where F is stable and $\{F, g\}$ is controllable and $e(k)$ is the error signal defined as

$$e(k) = r(k) - u(k)$$

In the ideal situation, K_2 is identical to P^{-1}. In that case, $e(k)=0, u(k)=u_{ff}(k)=u_0(k)=P^{-1}(z)r(k)$. The true values c_0, d_0 and l_0 of $c(k)$, $d(k)$ and $l(k)$, respectively, satisfy

$$\frac{l_0 + c_0^T(zI-F)^{-1}g}{1-d_0^T(zI-F)^{-1}g} = P^{-1}(z) \quad (19)$$

as given in Eqns (14).
The cost function for adaptation is defined as

$$J(k) = \frac{1}{2}\sum_{i=0}^{k} e^2(i) \quad (20)$$

The unknown parameters $c(k)$, $d(k)$ and $l(k)$ must be updated so that the error signal $e(k)$ decreases. $\xi(k)$ is defined as

$$\xi(k) := [\xi_1(k)^T \quad \xi_2(k)^T \quad r(k)]^T \quad (21)$$

The usual gradient method gives rise to the updating rule. Then, the adaptation law of parameters is obtained as

$$\theta(k) := [c(k)^T \quad d(k)^T \quad l(k)]^T, \quad (22)$$

$$\theta(k+1) = \theta(k) + \frac{\alpha}{K_1} e(k)\xi(k) \quad (23)$$

Note: This is adapted from the continuous-time adaptation algorithm by using the gradient method presented by Miyamura (2000).

By using such the above parameterization algorithm and adaptation law, together with some control theorems proved previously, the convergence of DTFEL system can be proved easily.

Convergence Proof

The error signal can be rewritten as

$$e(k) = r(k) - P(z)u(k)$$

Hence,

$$u(k) = u_d(k) - P^{-1}(z)e(k),$$

$$u_d(k) = P^{-1}(z)r(k)$$

Then, the adaptive controller is written as

$$\xi_1(k+1) = F\xi_1(k) + gr(k) \quad (24)$$

$$\xi_2(k+1) = F\xi_2(k) + g\left(u_d(k) - P^{-1}(z)e(k)\right) \quad (25)$$

$$u_{ff}(k) = c^T(k)\xi_1(k) + d^T(k)\xi_2(k) + l(k)r(k) \quad (26)$$

$$\left.\begin{aligned} c(k+1) &= c(k) + \frac{\alpha}{K_1}e(k)\xi_1(k), \\ d(k+1) &= d(k) + \frac{\alpha}{K_1}e(k)\xi_2(k), \\ l(k+1) &= l(k) + \frac{\alpha}{K_1}e(k)r(k) \end{aligned}\right\} \quad (27)$$

Assume that the true system is written as

$$z_1(k+1) = Fz_1(k) + gr(k), \quad (28)$$

$$z_2(k+1) = Fz_2(k) + gu_d(k), \quad (29)$$

$$u_d(k) = c_0^T(k)z_1(k) + d_0^T(k)z_2(k) + l_0(k)r(k) \quad (30)$$

Then,

$$\begin{aligned} u_{ff}(k) - u_d(k) = &\left(c(k)-c_0\right)^T\xi_1(k) + \left(d(k)\right. \\ &\left.- d_0\right)^T\xi_2(k) + \left(l(k)-l_0\right)r(k) \\ &- d_0^T(zI-F)^{-1}gP^{-1}(z)e(k) \end{aligned} \quad (31)$$

Here, the following asymptotic relations are used

$$\xi_1(k) \to z_1(k)$$

$$\xi_2(k) \to z_2(k) - d_0^T(zI-F)^{-1}gP^{-1}(z)e(k)$$

The relation (31) is written as

$$u_{ff}(k)-u_d(k) = \psi(k)^T\xi(k)-d_0^T\left(zI-F\right)^{-1}gP^{-1}(z)e(k),$$

where

$$\psi(k):=\theta(k)-\theta_0=\begin{bmatrix} c(k)-c_0 \\ d(k)-d_0 \\ l(k)-l_0 \end{bmatrix} \quad (32)$$

From the relations

$$u(k)=u_{ff}(k)+K_1e(k),$$

$$-\left[P^{-1}(z)e(k)+K_1e(k)\right]=\psi(k)^T\xi(k)-d_0^T\left(zI-F\right)^{-1}gP^{-1}(z)e(k), \quad (33)$$

which results in

$$\left(G(z)+K_1\right)e(k)=\psi(k)^T\xi(k), \quad (34)$$

$$G(z):=\left(1-d_0^T\left(zI-F\right)^{-1}g\right)P^{-1}(z) \quad (35)$$

On the other hand, from (32),

$$\psi(k+1)-\psi(k)=\theta(k+1)-\theta(k) = \frac{\alpha}{K_1}\xi(k)e(k) \quad (36)$$

It should be noted that the relation (19) implies that

$$G(z)=l_0+c_0^T\left(zI-F\right)^{-1}g \quad (37)$$

Combining (34) and (36),

$$\psi(k+1)-\psi(k)=\frac{\alpha}{K_1}\xi(k)e(k) = \frac{\alpha}{K_1}\xi(k)\left(G(z)+K_1\right)^{-1}\xi(k)^T\psi(k) \quad (38)$$

$$\psi(k+1)=\psi(k)-\frac{\alpha}{K_1}\xi(k)\left(G(z)+K_1\right)^{-1}\xi(k)^T\psi(k),$$

which is the same form as (3), i.e.

$$\psi(k+1)=\left(I-\xi(k)L_0(z)\xi(k)^T\right)\psi(k),$$

where is equal to

$$L_0(z):=\left(G(z)+K_1\right)^{-1}\frac{\alpha}{K_1}$$

According to Theorem 2, the difference equation (38) is asymptotically stable, if $L_0(z)$ givdÃ by (39) is s.p.r., K_1 is chosen such that $G(z)+K_1$ is s.p.r. Such K_1 always exists from Definition 2 of s.p.r. (See Remark following Lemma 1). If $G(z)+K_1$ is s.p.r., so is $L_0(z)$. Thus, the following fundamental result has been established:

Theorem 3 *Under the assumptions (A1)-(A4), the feedback error learning method (24)-(27) is converging, i.e., the controller K_2 converges to $P^{-1}(z)$.*

$$P(z)=\frac{z+0.2}{z+0.3}$$

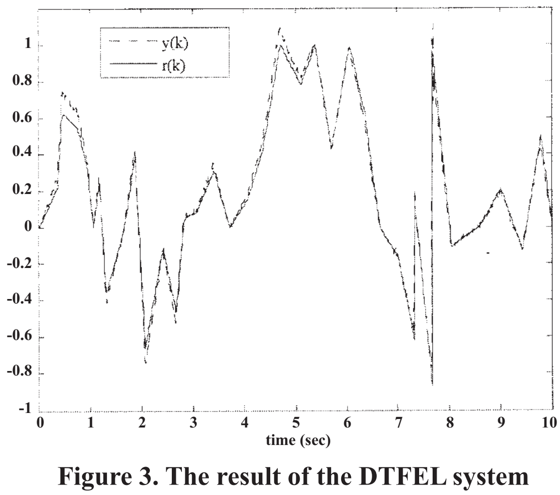

Figure 3. The result of the DTFEL system

Results

In this section, the simulation results are to demonstrate the effectiveness of the theoretical results obtained in this study. The pulse-transfer function of the plant is

Note that this plant has a stable inverse. In Figure 3, the tracking performance between the input signal *r(k)* and the output signal *y(k)* is shown. The *error e(k)=y(k)-r(k)* shown in Figure 4. These Figures show rhe convergence of the signal and the comparison of the tracking performance of the system before adaptation, from 0 sec to about 5.7 sec, and after adaptation, from 5.7 sec to 10 sec. It should be noted that the error pulses between 7 and 8 sec can be considered as the unusual performance of the input. It is interesting that the system can still be stable. Note also that the learning rate is set to be very low to show the result clearly. In fact, the adaptation rate is very fast.

Conclusion

In this study, the "Discrete-Time Feedback Error Learning" (DTFEL) method is demonstrated.

The results are direct analogues to those in continuous-time systems. However, these extensions are by no means straightforward, and clarified the essential difference between continuous-time and discrete-time cases. The extensions are far from trivial. These discrete-time versions turn out to be more complicated than their continuous counterparts. The mathematical requirements to analyze the stability of the DTFEL method, where the plant has stable inverse, are studied and proved. Then the simulation results using MATLAB® are shown. Although the system with DTFEL seems to be welled control, the requirement of this method is very restricted in the real applications where most plants are not stable or invertedly stable. So, the extension of DTFEL method, especially for controlling unstable or invertedly unstable plants, should be one of the possible further research in this area. Furthermore, the stability analysis of DTFEL system with time-delay should be studied.

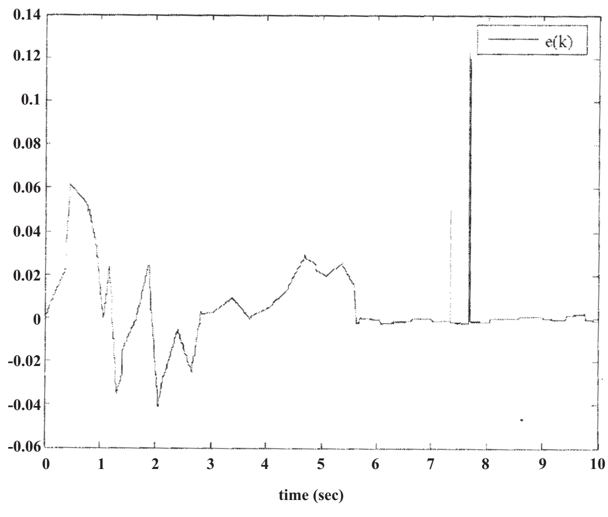

Figure 4. The error of the DTFEL system

References

de la Sen, M. (2000). Preserving positive realness through discretization. Proceedings of the American Control Conference; June 28-30, 2000; Chicago, Illinois, p. 1, 144-1,148.

Gomi, H., and Kawato, M. (1993). Neural network control for a closed-loop system using feedback-error-learning. Neural Net works, 6:933-946.

Jagannathan, S. (1996). Discrete-time adaptive control of feedback linearizable nonlinear systems. IEEE Proceedings of the 35th Conference on Decision and Control; December 11-13, 1996; Kobe, Japan, p. 4,747-4,752.

Kawato, M., Furukawa, K., and Suzuki, R. (1987). A hierarchical neural network model for control and learning of voluntary movement. Biological Cybernetics, 57:169-185.

Kongprawechnon, W., and Kimura, H. (1998). J-lossless factorization and -control for discrete-time systems. International J. of Control, 70(3):423-446.

Miyamura, A. (2000). Theoretical analysis on the feedback error learning method, [Master's thesis]. Department of Complexity Science and Engineering, University of Tokyo, Tokyo, Japan, total number of pages.

Miyamura, A., and Kimura, H. (2002). Stability of feedback error learning scheme. Elsevier, System & Control Letters, 45:303-316.

Narendra, K.S., and Valavani, L.S. (1989). Stable Adaptive Systems. book edition. Prentice Hall, International Editions, Eaglewood Cliffs, New Jersey, USA, total number of pages.

Tao, G., and Ioannou, P.A. (1990). Neccessary and sufficient conditions for strictly positive real matrices. IEE Proceedings G: Circuits, Devices and Systems, 137(5): 360-366.

Ushida, S., and Kimura, H. (2002). Adaptive control of nonlinear system with time delay based on the feedback error learning method. Proceedings of the 2002 IEEE International Conference on Industrial Technology; December 11-14, 2002; Bangkok, Thailand, p. 360-366.