FINITE-WAVE VECTOR EFFECT OF PLASMON DISPERSION ON CRITICAL TEMPERATURE IN $La_{1.85}$ $Sr_{0.15}$ CuO_4

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Abstract

Using the plasmon exchange model in the framework of the Eliashberg theory for strong-coupling superconductors, the critical temperature of $La_{1.85}$ $Sr_{0.15}$ CuO_4 has been calculated. The finite-wave vector effect on acoustic plasmon dispersion has been included in the expression for the effective interaction between charge carriers. This effect is shown to enhance the critical temperature significantly as compared with the result without this term. The critical temperature is sensitively dependent on the spacer dielectric constant ε_M which is not known precisely. The Coulomb repulsion strength μ^* has been tested around the commonly used value of 0.1. It is found that their proper values of them are $\varepsilon_M \approx 8.0$ and $\mu^* \approx 0.1$.

Keywords: Plasmon exchange model, finite-wave vector effect, acoustic plasmons, critical temperature

Introduction

After the discovery of high-temperature superconductivity in cuprates (Bednorz and Müller, 1986), various kinds of theoretical models for the mechanism of high-temperature superconductivity were proposed. Even today there is no consensus among theoretical physicists as to how to develop a more detailed theoretical description of the cuprates. It is known that all cuprate high-temperature superconductors (HTS) have a layered structure. The layers are composed of Cu-O planes (or sheets) separated from each other by planes of various other oxides and rare earths. The Cu-O layer is assumed to form a two-dimensional electron gas (2DEG) and the electrons in a given layer can interact with each other within the same layer as well as from layer to layer via an effective interaction. An isolated layer has only one plasmon mode with a dispersion relation $\omega_p \propto q^{1/2}$. Interlayer interaction leads to a noticeable modification of the pure two-dimensional (2D) dispersion relation, namely, to the formation of plasmon bands.

Indeed, it is a well-known fact that the spectrum of a layered electron gas contains low-energy electronic collective modes, often called acoustic plasmons, with a dispersion relation $\omega\rho \propto q$. That such modes could not be observed experimentally at finite q so far is related to the fact that the only technique known to date to determine the plasmon energy as a function of its wave-vector (i.e., electron energy loss spectroscopy), has a resolution of 0.2 - 0.5 eV at best (Nücker *et al.,* 1989; Stöckli *et al.,* 2000). It remains thus an experimental challenge to measure collective charge excitations down to very low energies at finite q. It is also worth noting that the largest contribution of acoustic plasmons to physical quantities such as

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condensation energy is expected to come from finite but rather small values of q with respect to the Fermi wave-vector. To study the effect of acoustic plasmons on superconductivity requires thus to probe finite q's. Recently, the effect of temperature dependence and the inclusion of the leading higher order in wave-vector q on the plasmon dispersion relation in layered superconductors has been reported (Hompanya and Phatisena, 2005). At a low temperature limit the slope of the layered plasmon dispersion was shown to increase significantly due to the inclusion of the higher order in q while the thermal enhancement is nearly negligible.

The influence of acoustic modes on superconducting properties has been studied within the strong-coupling phonon-plasmon scheme (Bill *et al.,* 2000). It was shown that the density of states is peaked at $q_z = \pi$ and $q_z = 0$, where q_z is the wave-vector perpendicular to the planes. The optical branch ($q_z = 0$) has a smaller attractive interaction than the acoustic branch. Therefore, the largest collective-mode contribution to T_c is provided by the lowest acoustic branch. Screening of the Coulomb interaction in a layered conductor is incomplete due to the nature of layering (Visscher and Falicov, 1971; Fetter, 1974). The response to a charge fluctuation is time dependent and the frequency dependence of the screened Coulomb interaction becomes important. The additional impact of dynamic screening on pairing in layered superconductors has been evaluated (Bill *et al.,* 2003). The plasmonsí contribution in conjunction with the phonon mechanism was used. The presence of only phonons is assumed to be sufficient to overcome the static Coulomb repulsive interaction and the dynamic screening acts as an additional factor. The full temperature, frequency and wave-vector dependence of the dielectric function was used to calculate T_c of three classes of layered superconductors. In metal-intercalated halide nitrides the contribution arising from acoustic plasmons is dominant while the contribution of phonons and acoustic plasmons is of the same order in layered organic superconductors and the contribution of acoustic plasmons is significant but not dominant in high-temperature oxides.

In this paper the Eliashberg theory for strong-coupling superconductors as modified by McMillan (McMillan, 1968) and Kresin (Kresin, 1987) will be used to calculate the superconducting transition temperature, T_c. The plasmon exchange model will be reconsidered and the effective interaction between electrons is described within the random phase approximation (RPA). This model was previously used to calculate the T_c in HTS (Longe and Bose, 1992). Here, the finite-wave vector effect on the plasmon dispersion relation in a layered superconductor will be included in our calculation. The appropriated values of the dielectric constant ε_M and the effective Coulomb repulsion μ^* for the cuprate superconductor $La_{1.85}Sr_{0.15}CuO_4$ will be determined. Numerical results and discussions will be presented.

Acoustic Plasmon Exchange Model

The simple layered electron gas system consists of two conducting sheets along the z-axis separated by a dielectric spacer with the dielectric constant ε_M and with the interlayer distance L. The description of layered conductors can be made by neglecting the small interlayer hopping in a first approximation. The electrons in a Cu-O plane interact via the Coulomb interaction with charge carriers both within and between the planes. The effective interaction between the electrons are described within the RPA and can be written in the standard form (Bose and Longe, 1992)

$$V(q,\omega) = V_0(q)\left[1 + \int_{-\pi/L}^{\pi/L} dq_z \frac{2\omega_p(q,q_z)|M(q,q_z)|^2}{\omega^2 - \omega_p^2(q,q_z)}\right] \quad (1)$$

where $V_0(q) = 2\pi e^2/\varepsilon_M q$ is the bare 2D Coulomb interaction, $\omega_p(q, q_z)$ is the plasmon frequency and $|M(q, q_z)|^2$ is the square of the electron-plasmon matrix element. The plasmon frequency in the present model is shown to be (Hompanya and Phatisena, 2005)

$$\omega_p^2(q,q_z) = \sigma q\left(1 + \frac{3}{4}\frac{\hbar^2 q}{m^* e^2}\right)R(q,q_z) \quad (2)$$

$$2\omega_p(q,q_z)|M(q,q_z)|^2 = \sigma q\left(1 + \frac{3}{4}\frac{\hbar^2 q}{m^* e^2}\right)|R(q,q_z)|^2 \quad (3)$$

where $$\sigma = 2\pi e^2 n / \varepsilon_M m^* \tag{4}$$

and $$R(q, q_z) = \frac{\sinh(qL)}{\cosh(qL) - \cos(q_z L)} \tag{5}$$

n is 2D electron density, m^* is the electron effective mass, and R (q, q_z) is the layer form factor which reflects the layered nature of the system. Note that for $L \rightarrow \infty$, the effective interaction V (q, ω) becomes $V_0(q)\left[1 + \sigma q/(\omega^2 - \sigma q)\right]$ which is the effective electron-electron interaction in a single 2DEG.

Also to be noted is the appearance of the second term in the parenthesis of Eqn. (2). This term reflects the finite-wave vector effect on plasmon dispersion relation, which is always missing in the otherís calculation. This paper shows the significance of this term to T_c of the layered superconductors.

Indeed, it has been shown (Allen and Dynes, 1975) that if the effective interaction between electrons in a superconductor can be written as given by Eqn. (1), then the coupling strength λ due to the attractive part of the effective interaction and the average value $\langle\omega\rangle$ of the plasmon frequency can be obtained from

$$\lambda = N(0)\left\langle \frac{2\left|M(q, q_z)\right|^2}{\omega_p(q, q_z)} \right\rangle_{FS} \tag{6}$$

and $$\lambda\left\langle\omega^2\right\rangle = N(0)\left\langle 2\left|M(q, q_z)\right|^2 \omega_p(q, q_z)\right\rangle_{FS} \tag{7}$$

where N(0) is the density of states of the electrons at the Fermi surface and $\langle ... \rangle_{FS}$ indicates that an average of the expression is taken over the Fermi surface. Eqns. (6) and (7) can be shown (Longe and Bose, 1992) to be

$$\lambda = \frac{2N(0)}{\pi}\int_{q_m}^{2k_F} dq \frac{V_0(q)}{\left(4k_F^2 - q^2\right)^{1/2}} \tag{8}$$

and $$\lambda\left\langle\omega^2\right\rangle = \frac{2N(0)}{\pi}\sigma\int_{q_m}^{2k_F} dq \frac{q\left(1 + 3\hbar^2 q / 4m^* e^2\right)V_0(q)\coth(Lq)}{\left(4k_F^2 - q^2\right)^{1/2}} \tag{9}$$

It is interesting to note that λ, as given by Eqn. (8), does not depend on the interlayer distance L. This is due to the analytic properties of the RPA potential given by Eqn. (1). Another important point is that the integrals (8) and (9) diverge for small momentum transfer q. The technique to avoid this difficulty is to introduce a cutoff q_m to obtain finite results. Physically one would expect that the effective range of should q_m be of the order of inverse coherence length ξ since charge carriers at distances larger than ξ do not contribute significantly to Cooper pairing. Therefore, one can write $q_m = 1/\xi$ and thus T_c must obviously depend on the value of ξ.

It is also interesting to note that even though integrals (8) and (9) diverge for small q_m, but their ratio, i.e. $\langle\omega^2\rangle$, however does not. For small q_m, $\langle\omega^2\rangle$ tends rapidly to the lower limit of σ/L which is the 3D electron density. On the other hand, for q_m large orq $2k_F$, the average $\langle\omega^2\rangle$ tends rather slowly to the upper limit $2k_F\,\sigma \coth(2k_F L) \approx 2k_F\sigma$. Hence the range of variation of $\langle\omega^2\rangle$ as a function of q_m is not very extended. This is not the case for λ which diverges linearly for small q_m.

It is simpler to scale the parameter, $y = q / k_F$. Eqn. (8) and (9) then become

$$\lambda = N(0)\frac{2\pi e^2}{\varepsilon_M}\frac{1}{\pi k_F}\int_{1/k_F\xi}^{2} \frac{dy}{y\left(1 - y^2/4\right)^{1/2}} \tag{10}$$

$$\lambda\left\langle\omega^2\right\rangle = N(0)\frac{2\pi e^2}{\varepsilon_M}\frac{\sigma}{\pi}\int_{1/k_F\xi}^{2} dy \frac{\left(1 + 3\hbar^2 k_F y / 4m^* e^2\right)\coth(k_F L y)}{\left(1 - y^2/4\right)^{1/2}} \tag{11}$$

where $N(0) = \dfrac{m^*}{2\pi}$ and $k_F^2 = 2\pi n$. The average plasmon frequency, $\langle\omega^2\rangle$, is given by the square root of the ratio of (10) and (11).

It is seen from Eqns. (10) and (11) that the two parameters obviously depend on the dielectric constant ε_M, the effective mass m^*, the surface density n of the electron gas (or equivalently the Fermi wave-vector k_F and hence the Fermi energy ε_F), and the coherence length ξ.

Critical Temperature of $La_{1.85}\,Sr_{0.15}\,CuO_4$

In this section we will focus on the $La_{1.85}\,Sr_{0.15}\,CuO_4$ for which most parameters have been determined and it deserves special attention because of the simplicity of its structure. This system plays a role similar to the hydrogen atom in atomic physics. It is the best test system for understanding the basic principles of high-temperature superconductivity.

Following are the normal state parameters (Bill *et al.,* 2003):

the interlayer distance $L = 6.5$Å

the Fermi wave-vector $k_F = 3.5 \times 10^7 cm^{-1}$

the dielectric constant $\varepsilon_M \approx 5$ - 10
the effective mass $m^* = 1.7m_e$
the coherence length $\xi = 35$Å
and the Coulomb pseudopotential is taken to be $\mu^* = 0.1$ (here, m_e being the mass of the bare electron).

Two equations, McMillan's equation and Kresin's equation, both of which were modified from the Eliashberg theory for strong-coupling superconductors will be used for the calculation of T_c of this material. The McMillan's equation for the plasmon exchange model has the form

$$T_c^{pl} = \frac{\langle\omega\rangle}{1.45}\exp\left[-\frac{1.04(1+\lambda)}{\lambda-\mu^*(1+0.62\lambda)}\right] \quad (12)$$

where the Debye frequency θ_D is replaced by the average frequency of plasmon, $\langle\omega\rangle$, the exchange of which is responsible for superconductivity.

Kresin's equation for the plasmon exchange model to calculate the value of T_c is given by

$$T_c^{pl} = \frac{0.25\langle\omega\rangle}{\left[e^{2/\lambda_{eff}}-1\right]^{1/2}} \quad (13)$$

where the effective interaction strength λ_{eff} is given by

$$\lambda_{eff} = \frac{\lambda-\mu^*}{1+2\mu^*+\lambda\mu^* t(\lambda)} \quad (14)$$

and the analytical expression for the function t (λ) is given (Longe and Bose, 1992) by

$$t(\lambda) = 0.75+0.8/(1+\lambda)-0.12(\lambda-0.5) \quad (15)$$

The results of T_c obtained by these two equations will be compared with the recent work by Bill *et al.* (2003). We will start with the calculation of λ and ⟨ω⟩ given by Eqns. (10) and (11) respectively. It can be seen from the given parameters that the value of dielectric constant ε_M is in the range 5 - 10, and λ and ⟨ω⟩ are obviously sensitive to this choice of ε_M. We, therefore, calculate the value of λ, ⟨ω⟩ and then T_c^{pl} by using different values of ε_M. The result is shown in Table 1. The finite-wave vector (higher order in q) effect of the plasmon dispersion relation given by Eqn. (2), which is the term $(3/4)\hbar^2 q/me^2$, on T_c^{pl} is also shown in the Table. It is seen that the values of T_c^{pl} obtained by using Kresin's equation are higher than those by McMillan's equation and the finite-wave vector effect enhances the values of T_c^{pl} significantly.

As reported by Bill *et al.* (2003), the experimental value of T_c of $La_{1.85}$ $Sr_{0.15}$ CuO_4 is $T_c^{exp} \approx 38K$. Their numerical result is $T_c = 36.5$ K whereas in the absence of acoustic plasmons it is $T_c^{ph} = 30$ K. Therefore, the value of T_c due to acoustic plasmons is $T_c^{pl} \approx 8K$. It is seen from Table 1 that the expected results correspond to the dielectric constant $\varepsilon_M = 8$ (8.38K and 8.97K by Kresin's equation, 6.96K and 7.45K by McMillan's equation). These results are quite different from the values that correspond to $\varepsilon_M = 7$ and $\varepsilon_M = 9$. It is, therefore, necessary to obtain T_c^{pl} that corresponds to the dielectric constant around $\varepsilon_M = 8$. The result is shown in Table 2. It is seen from Table 2 that the appropriate value of the dielectric constant is $\varepsilon_M \approx 8.0$ for $\mu^* = 0.1$.

Table 1. The calculated values for T_c^{pl} (in K) as obtained from Kresin's equation (T_{c1} and T_{c2}) and from McMillan's equation (T_{c3} and T_{c4}). T_{c1} and T_{c3} are the values without the finite-wave vector effect whereas T_{c2} and T_{c4} are the values including that effect

ε_M	By Kresin's equation		By McMillan's equation	
	T_{c1}	T_{c2}	T_{c3}	T_{c4}
5	136.821	146.540	130.213	139.463
6	57.943	62.059	53.459	57.256
7	22.978	24.610	20.279	21.720
8	8.377	8.973	6.960	7.454
9	2.753	2.948	2.109	2.259
10	0.797	0.854	0.548	0.587

Finally, to find the proper value of the effective repulsive strength μ^* (rather than 0.1) that fit the expected result of $T_c^{pl} \approx 8K$, we use it here as a second parameter varying from 0.07 to 0.13 in steps of 0.01. The critical temperature T_c^{pl} as a function of the dielectric constant ε_M for 7 values of μ^* is shown in Table 3 and Figure 1. It is seen from the Figure that the proper value of ε_M and μ^* that fit the expected result of $T_c^{pl} \approx 8K$ are $\varepsilon_M \approx 8.0$ and $\mu^* = 0.1$.

Discussion and Conclusions

Using the plasmon exchange model in the framework of the Eliashberg theory for strong coupling superconductors, the plasmon contribution to the critical temperature T_c^{pl} could be obtained. In this model the plasmons are assumed to be attractive bosons in the pairing effect. The effective interactions between the electrons are described within the RPA. The electrons interact with each other within the same layer as well as from layer to layer via an effective interaction involving plasmon exchanges among all layers. Eliashberg's equation for the calculation of T_c has been modified into McMillan's equation and Kresin's equation. These two equations contain two basic parameters to be evaluated, λ and $\langle\omega\rangle$. The quantity λ represents the attractive strength between electrons, which in this model is essentially mediated by plasmons. The quantity $\langle\omega\rangle$ is the average value of the frequency of the plasmons, the exchange of which is responsible for superconductivity. Both λ and $\langle\omega\rangle$ obviously depend on the dielectric constant ε_M, the effective massm*, the Fermi wave-vector k_F, the interlayer dist ance L, and the coherence length ξ (to specify the lower limit of integration for λ and $\langle\omega\rangle$). The third parameter entered in the two equations for T_c^{pl} is the Coulomb repulsion strength μ^*. This parameter is generally not well known, but one knows that it is limited by the condition $0 < \mu^* < 0.5$. Many other investigators take its numerical value to be 0.1. In this work, μ^* is kept as an undefined parameter around 0.1.

Table 2. The calculated values for T_c^{pl} (in K) as obtained from the same process as Table 1 with dielectric constant around $\varepsilon_M = 8$

ε_M	By Kresin's equation		By McMillan's equation	
	T_{c1}	T_{c2}	T_{c3}	T_{c4}
7.5	14.0379	15.0351	12.0473	12.9031
7.75	10.8781	11.6508	9.19075	9.8436
8	8.3779	8.9730	6.96018	7.4545
8.25	6.4107	6.8661	5.23028	5.6018
8.5	4.8721	5.2182	3.89835	4.1752

Table 3. The calculated values for T_c^{pl} (in K) as a function of ε_M by using Kresin's equation for various values of effective repulsive strength around $\mu^* = 0.1$. Kresin's equation without finite-wave vector effect has been used

μ^*	ε_M				
	7.5	7.75	8.0	8.25	8.5
0.07	32.286	26.591	21.845	17.899	14.625
0.08	24.980	20.211	16.296	13.091	10.477
0.09	18.940	15.018	11.853	9.310	7.274
0.10	14.038	10.878	8.378	6.411	4.872
0.11	10.141	7.654	5.731	4.255	3.131
0.12	7.116	5.210	3.776	2.706	1.917
0.13	4.830	3.414	2.381	1.637	1.109

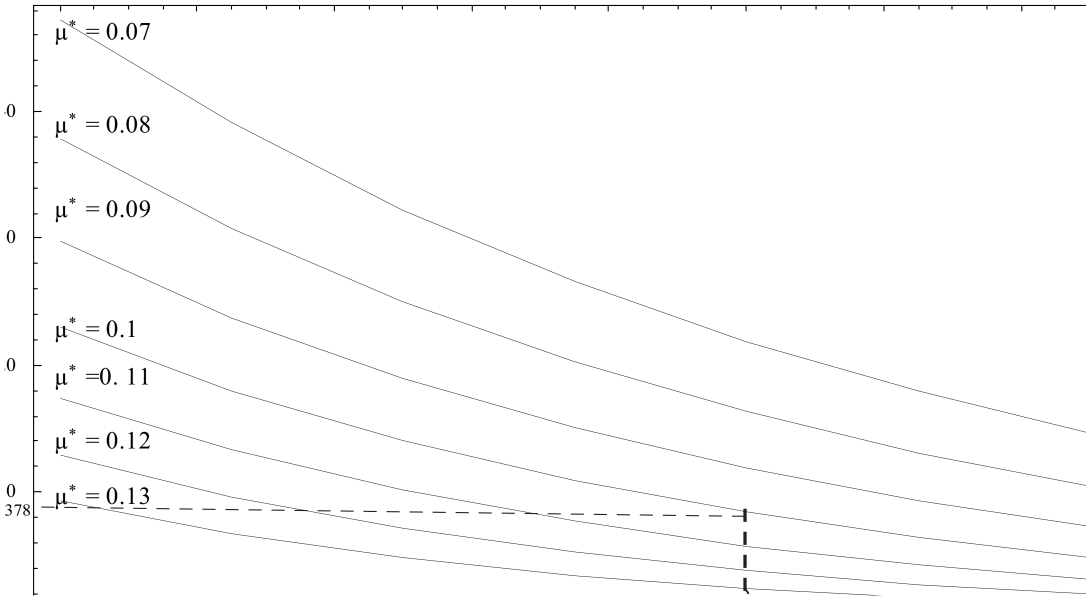

Figure 1. The critical temperrature as a function of constant ε_M for seven values of electron-electron repulsive strength μ^*, varying from 0.07 to 0.13 in steps of 0.01

A specific cuprate superconductor, $La_{1.85}$ $Sr_{0.15}$ CuO_4, for which most parameters have been determined, is selected for the calculation of T_c^{pl}. Since the experimental value of T_c of this material is $T_c^{exp} \approx 38K$ and the phonon contribution to the T_c is shown to be T_c^{ph} = 30 k, hence the plasmon contribution should be $T_c^{pl} \approx 8K$. Indeed the critical temperature is sensitively dependent on parameters mentioned above. However, only ε_M is not known precisely and the value of μ^* should be tested around the value of 0.1. Variation of ε_M and μ^* shows that their proper values for $T_c^{pl} \approx 8K$ are $\varepsilon_M \approx 8$ and $\mu^* \approx 0.1$.

The plasmon exchange model is very simple since the microstructure of the superconductors is completely neglected. The model is characterized by four parameters only. For reasonable values of these parameters the calculated value of T_c^{pl} is found to be in reasonable agreement with the experimental values of the materials. In the case of high-temperature oxides, the contribution of low-energy plasmons to the critical temperature is significant but not dominant. The phonon contribution is still largest in this model. In some classes of layered superconductors, the acoustic plasmon contribution is shown to be dominant or of the same order as the phonon contribution.

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