

SOLVING A SSCFLP IN A SUPPLY CHAIN WITH ACO

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Abstract

In this article, we have examined a Single-Source Capacitated Facility Location Problem (SSCFLP), in which each customer in the chain can only be supplied from exactly one facility. Given a set of potential locations and the facility capacities, it must be decided where and how many facilities must be opened and which customer must be assigned to each opened facility. The objective of this study is to find the suitable facilities to be opened and therefore the corresponding costs that consist of the cost of establishing facilities, the transportation cost and the uncovered demand cost which will be optimized. We have constructed an algorithm based on the Ant Colony Optimization (ACO) and tested by a simulation test. The results have then been compared with the general algorithm and the optimal solution to identify the efficiency of the algorithm. The simulation test has shown that the proposed algorithm performed very well.

Keywords: Supply chain management, facility location, single-source capacitated facility location problem, ant colony optimization

Introduction

In a NP-problem, one way for solving it to find a good result is to use search approaches such as Genetic Algorithm (GA), Simulated Annealing (SA), Taboo Search (TS) etc. Nevertheless, the search algorithm requires a lot of effort of calculation or in turn a long calculation time is necessary. Therefore, search approaches had been less effective methods in the past. Luckily, current development in computers has made it easy to perform the search algorithm within a very short time. This causes many search approaches to be very effective methods. Among the search techniques, well known methods such as GA, SA and TS have been studied by many

researchers. From the study by Dorigo *et al.*, (2000) about the applications of the ACO, we have found that there have been very few papers related to the Ant Colony Optimization (ACO) especially in the field of Supply Chain Management. Consequently, in this article, we have decided to research an application of ACO in a Single-Source Capacitated Facility Location Problem (SSCFLP).

Dorigo *et al.* (1996); Dorigo and Di Caro (1999); Bonabeau *et al.* (2000) have researched applications of the ACO. The ACO is a cooperative algorithm inspired by the foraging behavior of real ants. Ants lay down in some quantity an

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aromatic substance, known as pheromone, on their way to food and on their way back to the nest. They choose to follow a pheromone trail with a probability proportional to the pheromone trail intensity. Research on real ants has shown that such a trail-following behavior allows the ants to identify the shortest path between a food source and their nest. The applications of the ACO have been studied in many areas such as the Traveling Salesman Problem (TSP), Quadratic Assignment Problem (QAP), Generalized Assignment Problem (GAP), Gutjahr (2000); Maniezzo and Carbonaro (2000), Aguilar (2001), etc. The ACO has never been applied to the SSCFLP; however, the GAP is similar to the SSCFLP and therefore the ACO should have a possibility of being applied as an approach to solve a SSCFLP.

LourenÁo and Serra (2002) have added some additional ideas into the GAP. They have tested a GAP by using numerical examples and then compared the total cost of assigning facilities with other algorithms. The result has indicated that the algorithm that applied the ACO has given a better result.

Therefore, in this paper we have proposed an ACO algorithm into a SSCFLP with, in addition, consideration of the cost of uncovered demand. The experiments have been conducted to find the efficiency of the proposed algorithm in terms of both performance measure and calculation time.

Definition of A Single-Source Capacitated Facility Location Model

The SSCFLP is a special case of the capacitated facility location problem in which each customer can only be supplied from one facility (e.g. a retailer or a distribution center). In this article, the SSCFLP based on the model of Nozick and Turnquist (2001) has been observed and extended. Definitions of the facility location model are shown below.

Notations

The following notations have been identified.

m the number of potential facilities,

n the number of customers,
 d_j demand of customer j ; $j = 1, 2, \dots, n$
 f_i the fixed cost of facility i ; $i = 1, 2, \dots, m$
 c_i the capacity of facility i ,
 l_{ij} the distance from facility i to demand at customer j ,
 u unit cost per unit of distance,
 v truck capacity,
 p unit price,
 α penalty factor when customers cannot be served (in times of price),
 y_i = 1 if facility i is opened, 0 otherwise,
 x_{ij} = 1 if facility i serves customer j , 0 otherwise,
 q_{ij} = 1 if facility i cannot cover demand from customer j , 0 otherwise,
 x_{ij}' = x_{ij} , $\forall x_{ij} \neq 0$, where $i = 1, 2, \dots, m$ and $x_{ij}' = 1$, $\forall x_{ij} = 0$, where $i = 1, 2, \dots, m$
 x_{ij}' is facility i that cannot be assigned to customer j after all facilities have been assigned.

A Single-Source Capacitated Facility Location Model

A SSCFLP treated in this article can be expressed through an example composed of two components that are facilities and customers. Given a set of potential locations and facility capacities, it must be decided where and how many facilities must be opened and which customer must be assigned to each opened facility. Each customer must be assigned to only one facility. And, the customer demand must be fulfilled or on the other hand there will be no backlog. Figure 1 has illustrated the model studied in this article.

Objective Function

Generally, the objective function of a SSCFLP to be minimized consists of the fixed cost and variable costs. The fixed cost is the cost of establishing facilities and the variable cost is the cost of transportation from the opened facility to the assigning customer. A mathematical model to maximize coverage ensures that the proportion of demand that is within a specified "coverage" distance of a facility will be met. Delivery is then guaranteed for the set of customers within a certain radius (e.g., 400 km)

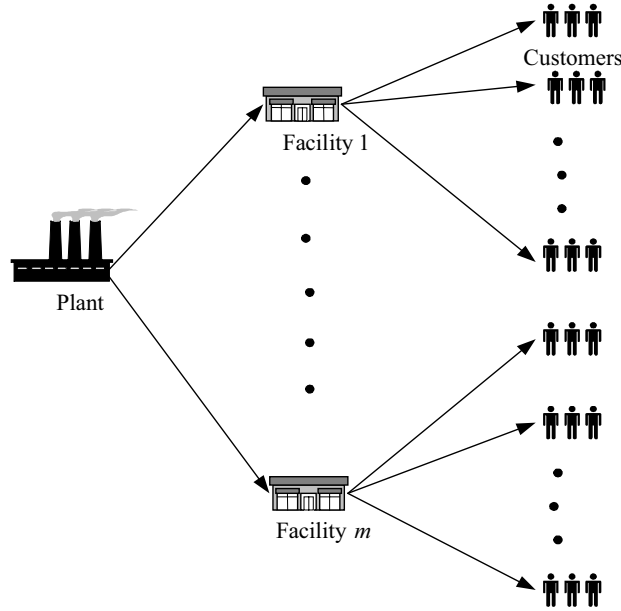


Figure 1. SSCFL model

of the facility. The objective of maximizing the proportion of total demand covered by a set of N facilities was first described by Church and ReVelle (1974). Hillsman (1984) has proposed an equivalent model, which minimizes the uncovered demand. In the research of Nozick and Turnquist (2001), they have not considered the capacity of facilities, so in this research we have considered the capacity of the facility and some variables have been adapted. A mathematical model for the SSCFLP considering the uncovered demand cost has been expressed as follows:

$$\begin{aligned} \text{Minimize } & \sum_{i=1}^m f_i y_i + \sum_{i=1}^m \sum_{j=1}^n u \left(l_{ij} \left[\frac{d_j}{v} \right] x_{ij} \right) \\ & + \sum_{i=1}^m \sum_{j=1}^n \alpha p (q_{ij} d_j x_{ij}^{\odot}) \end{aligned} \quad (1)$$

Subject to

$$\sum_{i=1}^m x_{ij} = 1; j = 1, 2, \dots, n \quad (2)$$

$$\sum_{i=1}^m d_j x_{ij} \leq c_i y_i; i = 1, 2, \dots, m \quad (3)$$

$$x_{ij} \leq y_i; i = 1, 2, \dots, m, j = 1, 2, \dots, n \quad (4)$$

$$y_i \in (0, 1); i = 1, 2, \dots, m \quad (5)$$

$$x_{ij} \in (0, 1); i = 1, 2, \dots, m, j = 1, 2, \dots, n \quad (6)$$

$$q_{ij} \in (0, 1); i = 1, 2, \dots, m, j = 1, 2, \dots, n \quad (7)$$

$$x_{ij} \in (0, 1); i = 1, 2, \dots, m, j = 1, 2, \dots, n \quad (8)$$

The objective function (1) consists of the cost of establishing facilities, the cost of assigning customers to open facilities and the cost of uncovered demand. Constraint (2) is the demand constraint (or the customers constraint), and ensures that each customer is assigned to exactly one facility. Constraint (3) is the capacity constraint (or the facility constraint), and ensures that the customer demand served by a certain facility does not exceed its capacity. Constraint (4) ensures that the assignments are made only to the opened facilities. Constraints (5)~(8) are the integrality constraints.

Facility Location Algorithms

The SSCFLP is a special case of the Facility Location problem and also a combinatorial optimization problem that has been classified as a NP-hard problem. Previously, many techniques have been proposed to solve this problem, such as Lagrangian heuristics, Lagrangian relaxation, Subgradient optimization, Repeated matching etc. However in this article, an Ant Colony Optimization has been proposed to solve a SSCFLP.

Ant Colony Optimization (ACO)

The ACO algorithm has been shown as follows and we have also illustrated the flow charts of the algorithm in Figure 2.

The Algorithm:

- Step 1: At the initialize phase $t = 0$, set the number of cycle $NC = 0$ and assign the set of potential assignment $X = \{x_{ij}\}$, $i = 1, 2, \dots, m$, and $j = 1, 2, \dots, n$.
- Step 2: Calculate the pheromone trails, τ_{ij} for each i, j .

$$\tau_{ij} = \frac{1}{f_i + (ul_{ij} \left[\frac{d_j}{v} \right]) + (\alpha p d_j)}$$

- Step 3: Assign the facility with the highest capacity to be the first priority.
- Step 4: Construct the set of alternatives F_i , the set of already assigned S_i , the set of infeasible assigned U_i , and the set of being assigned L_i .
- Step 5: Randomly select the value, a , of probability in assigning that can occur in the case of best choice, P_0 or in the normal case $(1 - P_0)$. In this article P_0 is set to be 0.10 according to our fine-tuned experiment that stated that 0.10 is the most efficient P_0 in giving the best result.

- 5.1) If the random probability “ a ” belongs to the case of best choice ($a \leq P_0$), facility i is then assigned to customer j with maximum value of τ_{ij} by a probability P_0 .

- 5.2) If the random probability “ a ” belongs to the normal case ($a > P_0$), then a considerable facility i is assigned to customer j according to the following probability:

$$P_{ij} = \begin{cases} \frac{\tau_{ij}}{\sum_{l \in K_{ij}} \tau_{il}} & \text{if } j \in K_{ij} \\ 0 & \text{otherwise} \end{cases}$$

At each considerable facility i , repeat the calculation j times and compute accumulate P_{ij} (where $\sum_j P_{ij} = 1$). After that random the probability value “ b ” to identify the range of accumulate P_{ij} for choosing the being assigned customer.

- Step 6: Check capacity constraint by considering the total demand of the being assigned customer whether it is less than or equal to capacity of facility i .

- 6.1) If total demand of the being assigned customer satisfies the capacity constraint, assigns customer j to facility i and update the pheromone trail:

$$\tau_{ij}^{new} = \rho \tau_{ij}^{old} + \Delta \tau_{ij}$$

where ρ is the persistence of the trails ($0 \leq \rho < 1$), and $1 - \rho$ represents the evaporation rate.

The amount of pheromone deposited by the current local optimal solution is:

$$\Delta \tau_{ij} = \tau_{ij}(\max) * Q$$

where $Q = 0.05$ in the case of the feasible solution.

- 6.2) If total demand of the being assigned customer is over the capacity constraint, put the unassigned customer j at facility i into the infeasible set and update pheromone trail:

$$\tau_{ij}^{new} = \rho \tau_{ij}^{old} + \Delta \tau_{ij}$$

The amount of pheromone deposited by the current local optimal solution is:

$$\Delta\tau_{ij} = \tau_{ij}(\max) * Q$$

where $Q = 0.01$ in the case of the infeasible solution.

Delete the assigned customer j , and repeat the steps until all customers have been considered.

Step 7: Let all variables x_{ij} , q_{ij} , x'_{ij} and y_i have values as assigned below.

$$y_i = \begin{cases} 1 & \text{if a facility is opened} \\ 0 & \text{otherwise} \end{cases}$$

$$x_{ij} = \begin{cases} 1 & \text{if a facility } i \text{ serves customer } j \\ 0 & \text{otherwise} \end{cases}$$

$$q_{ij} = \begin{cases} 1 & \text{if a facility } i \text{ cannot cover demand at customer } j \\ 0 & \text{otherwise} \end{cases}$$

$$x'_{ij} = x_{ij}, \forall x_{ij} \neq 0 \text{ where } i = 1, 2, \dots, m$$

$$x'_{ij} = 1, \forall x_{ij} = 0 \text{ where } i = 1, 2, \dots, m$$

Step 8: Calculate the total cost Z by using the values from step (7).

$$\begin{aligned} \text{Total cost } Z = & \sum_{i=1}^m f_i y_i + \sum_{i=1}^m \sum_{j=1}^n u \left(l_{ij} \left[\frac{d_j}{v} \right] x_{ij} \right) \\ & + \sum_{i=1}^m \sum_{j=1}^n \alpha p(q_{ij} d_j x_{ij}^{\odot}) \end{aligned}$$

Step 9: Keep the minimum value of Z and then replace. Repeat Step 2 to Step 9 until NC reaches NC_{\max} .

Following the idea of Ramalhinho Lourenço and Serra (1998), the proposed method for solving a SSCFLP problem has been presented as above mentioned. For updating pheromone step, the values of parameter Q were set by preliminary tests of Stützle and Hoos (1999), and they proved to be robust. Note that, if the solution is feasible, the pheromone trail has a bigger increment, trying to give larger probability to feasible assignments. Consequently, we have come up with an algorithm for SSCFLP and have been shown in Figure 2. This algorithm is based on the idea proposed by Ramalhinho

Lourenço and Serra (1998) which is extracted from a special Ant Colony Optimization algorithm, MAX-MIN Ant System; Stützle and Hoos (1997, 1999, 2000), and GRASP algorithm; Feo and Resende (1995).

Compared Algorithms

In order to evaluate the efficiency of the proposed algorithm, we have applied three general algorithms as described below:

Algorithm 1: Firstly choose the highest capacity facility and then the lower.

Algorithm 2: Firstly choose the lowest fixed cost facility and then the next higher value of the lowest value.

Algorithm 3: Firstly choose the lowest ratio (total cost of fixed cost of opening the facility plus the cost of transporting goods from the facility to customer j divided by the capacity of that facility) and then the next higher value of the lowest value.

Details of each algorithm have been shown as follows:

Algorithm 1: Highest Capacity Algorithm

This algorithm is best used in order to locate the minimum number of facilities while the effectiveness of the method depends on how large is the transportation cost. Therefore it will produce a practical solution when the transportation cost is low.

Steps:

- 1) Choose highest capacity facility i .
- 2) Assign customer j that has the lowest variable transportation cost between facility and customer. Eliminate the selected customer j .
- 3) Repeat Step 2 until capacity of facility i can no longer satisfy customer demand d_j . Eliminate the selected facility i .
- 4) Repeat steps 1-3 until all customer demand is satisfied.

Algorithm 2: Lowest Fixed-cost Algorithm

This algorithm is practical when the fixed cost of locating a facility is extremely high compared with the variable cost of distribution.

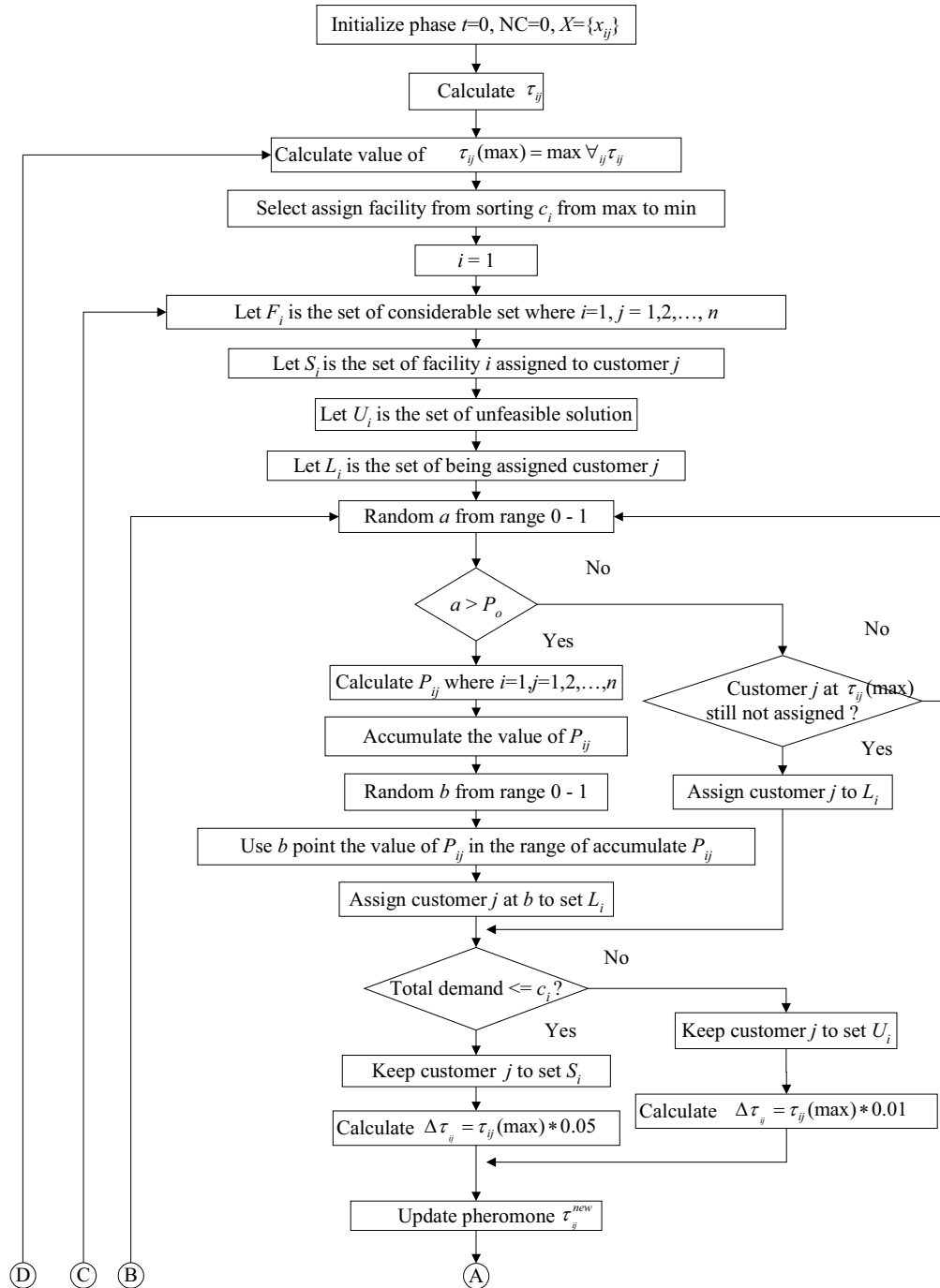


Figure 2. Flow chart of ant colony algorithm

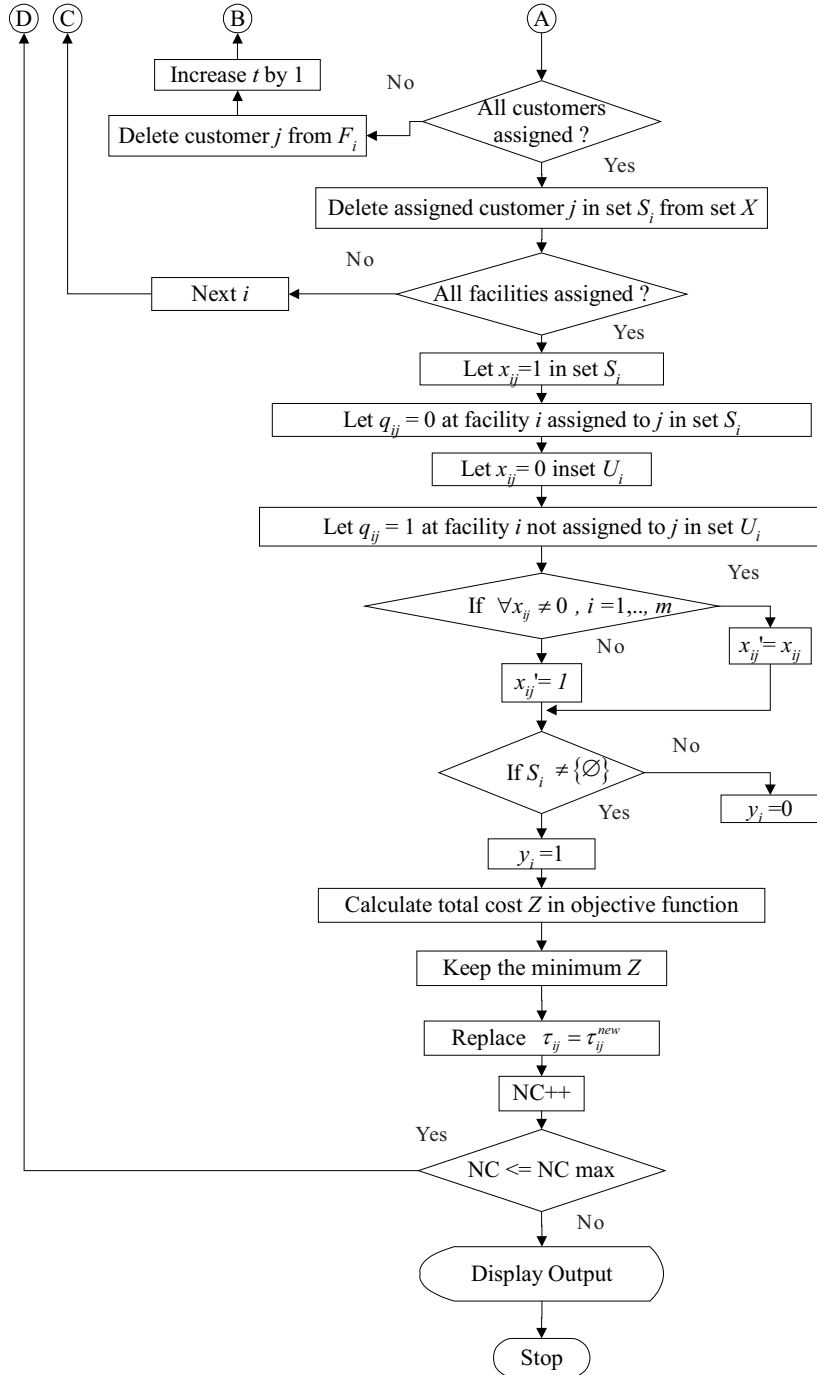


Figure 2. Flow chart of ant colony algorithm (Continued)

Steps:

- 1) Choose the lowest fixed cost of opening a facility i .
- 2) Assign customer j that has the lowest variable transportation cost between facility and customer. Eliminate the selected customer j .
- 3) Repeat step 2 until capacity of facility i can no longer satisfy customer demand d_j . Eliminate the selected facility i .
- 4) Repeat steps 1-3 until all customer demand is satisfied.

- 4) Assign customer j that has the lowest variable transportation cost between facility and customer. Eliminate the selected customer j .
- 5) Repeat step 2 until capacity of facility i can no longer satisfy customer demand d_j . Eliminate the selected facility i .
- 6) Repeat steps 1-5 until all customer demand is satisfied.

Algorithm 3: Lowest Ratio of Total Cost to Capacity Algorithm

This algorithm considers the fixed cost of opening a facility and the cost of transporting goods from the facility to the customer with respect to the capacity of the facility. This algorithm has served well when considering both the fixed cost and the number of warehouses to be opened.

Steps:

- 1) Find the total cost of the fixed cost of opening the facility plus the cost of transporting goods from the facility to customer j .
- 2) Divide the total cost of each facility by the capacity of that facility.
- 3) Chose the lowest ratio to open facility i .

Numerical Examples

Experimental tests have been conducted to seek the efficiency of the proposed algorithm. In this article, we have performed two comparison tests, which are, (1) comparison of the proposed algorithm with simple heuristics and (2) comparison of the proposed algorithm with the optimum value.

Comparison Test with Simple Heuristics

We have performed the simulation test to compare our algorithm with the simple algorithms, which have been explained previously. Details of the test have been illustrated as following:-

Characteristic of Input Data

The input data have been shown in Table 1.

Table 1. Input data for comparison test with simple heuristics

Variable	Description	Data
m	No. of potential facility	5, 10
n	No. of customer	10, 20
u	Unit cost per unit of distance	\$10 per km
p	Unit price	\$ 10 per unit
v	Truck capacity	1,000 units per truck
α	Penalty	1.5
ρ	Persistence of the trail	0.9
NC_{\max}	Maximum number of cycle	80,000
P_o	Probability of the best choice	0.10
c_i	Capacity of facility i	1 - 200,000 unit(s) (random)
f_i	Fixed cost of facility i	1 - 1,000,000 dollar (random)
d_j	Demand of customer j	1 - 100,000 unit(s) (random)
l_{ij}	Distance from facility i to customer j	0 - 1,000 km (random)

There are three parameters have been assigned by the preliminary test to determine the appropriate value, P_o , NC_{\max} and ρ . The way to seek for the appropriate values can be explained as follow.

(1) For seeking the P_{best} , assume P_o then run program at different number of NC_{\max} and plot graph to find P_o that give a constant cost. After that, fixed a NC_{\max} to find P_o which give the minimum cost. Run program in 30 samples by varying P_o . From the test, $P_o = 0.10$ gave the minimum cost among 30 samples.

(2) For NC_{\max} , set $P_o = 0.10$ then vary NC_{\max} from 10,000 – 150,000 cycles, test for 30 samples and then calculate the mean cost of each NC_{\max} . From the test, $NC_{\max} = 80,000$ cycles gave the minimum mean cost among 30 samples.

(3) The persistence of the trial ρ was determined by setting $NC_{\max} = 80,000$ and $P_o = 0.10$ then vary the value of ρ in 30 samples. The average cost among 30 samples is minimized at $\rho = 0.9$.

For other data in Table 1, we have assumed some values to test the algorithm. Some of the input data are random values such as c_i, f_i, d_i and l_{ij} , they are different in each example but the constant values (m, n, u, p, v and α) are fixed for every example. This input data can be change to be any value, only three parameters should not be change for this kind of problem. For this experimental test, we have set the input data for testing algorithm as presented in Table 1 and Table 3.

Results of the Numerical Examples

As stated in the previous section, we have considered four algorithms: the ACO, and algorithms 1, 2 and 3 for evaluating the

effectiveness of the proposed algorithm. The input data have been shown in Table 1. Results are presented by the value of RPD (Relative Percentage Deviation), which can be calculated by applying equation (9).

$$RPD = ((Z_i - Z_{Al}) / Z_{Al}) \times 100\% \quad (9)$$

Where Z_{Al} is the result obtained from the base algorithm and Z_i is result obtained from the algorithm that will be compared. Results of the numerical tests are shown in Table 2.

Once the computation has been completed for Table 2, the ACO algorithm gives the best result in all cases in terms of mean RPD . Values of the mean RPD of algorithms 1 and 3 are quite high and relatively high in algorithm 2.

When the number of customers is increased the mean RPD also increases and the efficiency of the simple algorithms tends to decrease.

In all cases, the ACO performs excellently compared with the simple algorithms.

Comparison Test with the Optimal Solution

In this section, we have evaluated the performance of the proposed ACO method by comparing costs and running times with the optimal solution obtained by the enumeration method. The input data have been shown in Table 3 and results have been shown in Figure 3 and Figure 4.

From Figure 3, the calculation time required to obtain the optimum solution is very long compared with the time required by the ACO. Values are from around 200% up to 400% longer. It means that in order to find the optimal solution, it requires lot of effort compared with the ACO.

Table 2. Results of numerical test

Tested model		Mean RPD			
No. of facility	No. of customers	ACO	Algorithm 1	Algorithm 2	Algorithm 3
5	10	0	98.67	52.36	81.40
	20	0	106.27	69.13	107.98
10	20	0	39.60	9.95	38.71
	40	0	88.82	49.11	89.62

From Figure 4, it can be observed that the ACO algorithm provided satisfactory results. Especially, the ACO provided 20 cases of the optimum from the total of 30 cases observed or about 67% which is fairly good. Moreover, from the value of average *RPD*, it can be observed that the efficiency of the ACO is around 98% of the optimum value or on the other hand its efficiency is only around 2% worse than the optimum value in average.

Conclusions

This research has examined a Single-Source Capacitated Facility Location Problem (SSCFLP) to determine where and how many facilities should be opened and which customers should

be assigned to each opened facility. The objective of this study is that the total cost of establishing facilities, the cost of assigning a customer and the uncovered demand cost should be kept to a minimum, while the demand of a customer should be satisfied without violating the capacity constraints of the facilities. Ant Colony Optimization has been proposed to solve our SSCFLP based on the concept of the ACO previously applied for the Generalized Assignment Problem.

From the numerical experiments, the proposed algorithm (ACO) has performed excellently compared with the other referenced approaches. Moreover, by testing with the optimal solution, the algorithm has performed very well within a reasonably short time.

Table 3. Input data for comparison test with optimal solution

Variable	Description	Data
m	No. of potential facility	3
n	No. of customer	5
u	Unit cost per unit of distance	\$10 per km
p	Unit price	\$ 10 per unit
v	Truck capacity	200 units per truck
α	Penalty	1.5
ρ	Persistence of the trail	0.9
NC_{\max}	Maximum number of cycle	80,000
P_o	Probability of the best choice	0.10
c_i	Capacity of facility i	1 – 2,000 unit(s) (random)
f_i	Fixed cost of facility i	1 – 5,000 dollar (random)
d_j	Demand of customer j	1 – 1,000 unit(s) (random)
l_{ij}	Distance from facility i to customer j	0 – 100 km (random)

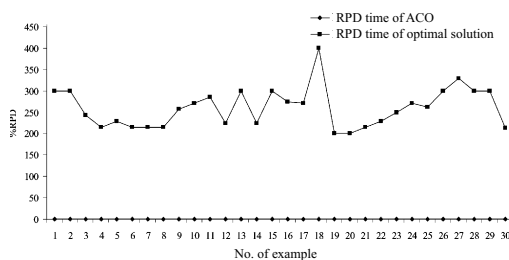


Figure 3. Comparison of % RPD of running time between ACO and the optimal solution

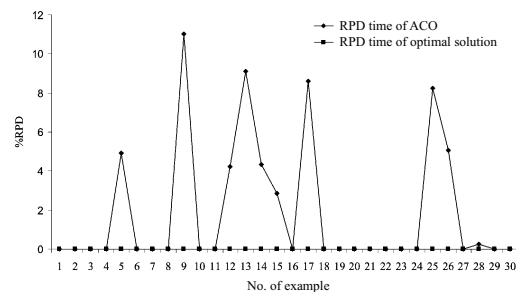


Figure 4. Comparison of % RPD of total cost between ACO and optimal solution

Considering the contribution made by this research, we do hope that our research will be a useful reference for researchers who want to study optimization in the field of Supply Chain Management, especially, the facility location problem and the logistics problem.

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