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# SOLITON AND BISOLITON MODEL FOR PAIRING MECHANISM OF HIGH - TEMPERATURE SUPERCONDUCTIVITY

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## Abstract

Soliton is a nonlinear solitary wave moving without energy loss and without changing its form and velocity. It has particlelike properties. The extraordinary stability of solitons is due to the mutual compensation of two phenomena, dispersion and nonlinearity. Solitons can be paired in a singlet state called bisoliton due to the interaction with local chain deformation created by them. Bisolitons are Bose particles and when their concentration is higher or lower than some critical values they can move without resistance. The bisoliton model can be applied for a pairing mechanism in cuprate superconductors due to their layered structure and the relatively small density of charge carriers. Cooper pairs breaking is a result of a paramagnetic effect and the Landau diamagnetic effect. The influence of magnetic impurities and the Meissner effect on cuprate superconductor based on the concepts of the bisoliton model are discussed.

**Keywords:** Soliton, bisoliton, pairing mechanism, cuprate superconductors, paramagnetic effect, diamagnetic effect, Meissner effect

## Introduction

The fascinating nonlinear wave structures known as solitary waves or solitons have become a subject of deeply interested for physicists in recent years. A soliton is a wave packet in which the wave field is localized in a limited (generally propagating) spatial region and is absent outside this region. Soliton, however, differs fundamentally from the classical wave packet which, being a linear formation, is known to spread out rapidly owing to the variation of the group velocity in the whole wavelength range of the packet. Soliton is essentially nonlinear; the (linear) dispersion of the group velocity in it

is exactly compensated by the reverse phenomenon, namely, nonlinear self-compression of the wave packet, and therefore the soliton propagates without spreading out and it conserves its shape. Thus, soliton is a nonspreading, nonlinear wave packet in which the phases and amplitudes of the waves are appropriately self-consistent.

It would seem natural to regard exact mutual compensation of dispersion and nonlinearity as being hardly probable and soliton as being a product of the theorists' imagination, rather than a real phenomenon. But, strange as it may seem, numerous theoretical studies and

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calculations and some experiments definitely demonstrate the existence and wide occurrence of solitons. Moreover, physicists are increasingly coming to the belief that solitons may play an essential role in such different fields of physics and even biology.

The history of solitons started about 170 years ago with the famous Scott-Russell's observation in an English canal in 1834. Korteweg and de Vries were the first to introduce solitons into theory in 1895. However, only in 1965 solitary waves were fully understood and later on it was realized that solitary waves on the water surface, nerve pulses, tornados, blood-pressure pulses, and so on all belong to the same category, i.e. they are all solitons. The most striking property of solitons is that they behave like particles and their solitary waves that are not deformed after collision with other solitons. They move without changing their shape and velocity, and without energy loss.

Solitons may be classified into two types, **nontopological** and **topological solitons**. The nontopological nature of the soliton is that it returns to its initial state after the passage of the wave while it is different from its initial state for the topological soliton. Another way to classify solitons is in accordance with nonlinear equations which describe their evolution. They are;

Korteweg de Vries (KdV) equation:

$$u_t = 6uu_x - u_{xxx}$$

Sine-Gordon equation:  $u_{tt} = u_{xx} - \sin u$

Nonlinear Schrodinger (NLS) equation:

$$iu_t = -u_{xx} \pm |u|^2 u$$

where  $u = u(x, t)$  denotes the height of the wave above the equilibrium position,  $t$  is the time and  $x$  is the coordinate along propagation of the wave, and the following notations:

$$u_x \equiv \frac{\partial u}{\partial x}, \quad u_{xx} \equiv \frac{\partial^2 u}{\partial x^2}, \quad u_{xxx} \equiv \frac{\partial^3 u}{\partial x^3}, \quad u_t \equiv \frac{\partial u}{\partial t},$$

$$\text{and } u_{tt} \equiv \frac{\partial^2 u}{\partial t^2}$$

The KdV soliton is nontopological and has a  $\text{sech}^2$ - shape. The topological soliton is described by the Sine-Gordon equation and has a tanh-like shape. The envelope soliton is described by the NLS equation and has a sech-shape which is slightly wider than the  $\text{sech}^2$ -shape of KdV soliton.

Solitons represent localized states and in exchange of interaction with a medium, an excitation or a particle locally deforms the medium in such a way that it is attracted by the deformation, and hence there exists a self-trapped state. In a self-trapped state, both the particle wave function and lattice deformation are localized. In one-dimensional systems, a self-trapped quasiparticle is called the **Davydov soliton or electrosoliton**. A self-trapped state can be described by two coupled differential equations for a field that determines the position of a quasiparticle, and a field that characterizes local deformation of the medium. The NLS equation is used to describe the Davydov soliton.

The concept of soliton is often confused with that of a polaron. Solitons arise when quasiparticles, neutral or charged, interact with local deformations described by virtual **acoustic phonons** with large dispersion. They can be at rest or move with a velocity which is less than the velocity of a longitudinal sound wave and their great stability is due to the compensation of dispersion and nonlinearity. Polarons, however, are self-localized states arising when a charged quasiparticle interacts with virtual **optical phonons** with a very small dispersion and hence the polarons are practically at rest.

## Forming into Bisoliton

The quasiparticles can be paired into a **singlet state** due to the interaction with local deformation of the lattice created by them. The bound pair of a quasiparticle and a local deformation is called a **bisoliton**. A bisoliton can be formed either from two electrosolitons or directly from two quasiparticles. Bisoliton has a double electric charge and a zero spin. The simplest theory of

quasiparticle pairing in quasi-one dimensional systems was developed by Brizhik and Davydov in 1984. Bisolitons do not interact with acoustic phonons since the interaction is completely taken into account in the coupling of quasiparticles with a local deformation and hence they do not radiate phonons. At low temperature, bisolitons are stable if the binding energy gained when they are coupling exceeds the screened Coulomb repulsion between their charges.

The quasiparticles joined by local deformation in singlet spin state form a single Bose particle or **boson**. The energy of a bisoliton at rest lies under the conduction band edge of two quasiparticles. The effective mass of a bisoliton is larger than the sum of two masses of solitons because its motion is connected with the motion of a deeper local deformation. The energy gap in the quasiparticle spectrum resulting from a pairing is half of the energy of formation of a bisoliton. Since bisolitons are formed when two quasiparticles are bounded with the local deformation, the coherence velocity  $v$  of bisolitons condensate motion is therefore limited by a longitudinal sound velocity  $v_0$  whose order of magnitude  $\sim 10^5$  cm/s. Since the velocity  $v$  of the bisoliton condensate is always less than  $v_0$  of the sound in the medium, the bisoliton does not lose energy to produce the acoustic phonons. Unlike the ideal Bose gas condensate that expresses the state of boson with zero energy and velocity, the bisolitons condensate moving with velocity  $v$  much less than transport energy velocity  $v_0$ . This energy is the sum of the energy of quasiparticles trapped by the local deformation and the energy of deformation.

The properties of a bisoliton condensate are dependent on the exchange interaction integral  $J$  and the dimensionless coupling parameter  $g$  of the electron-phonon interaction, and on the ratio  $L = N_0 / N$  of the bisoliton number  $N$  to the general number  $N_0$  of unit cells in the chain. At rather small values of the density of the quasiparticles or  $gL \gg 1$ , with increasing concentration of bisolitons the value  $gL$  decreases, the pairing energy increases and the effective mass decreases. Such changes are

due to the mutual indirect influence of bisolitons in a condensate. At rather large density of quasiparticles or  $gL < 5$ , the magnitude of the pairing gap decreases as the density of quasiparticles increases. The Coulomb interaction also reduces the magnitude of the energy gap at a large density of bisolitons.

### Bisoliton Model for Pairing Mechanism

The bisoliton model proposed by Brizhik and Davydov was first utilized to explain superconductivity in quasi - one dimensional organic conductors, which were discovered in 1979. In this conductor, the bonds between plane molecules in stacks arise from weak van de Waals forces. Hence, due to the deformation interaction, a quasiparticle (electron or hole) causes a local deformation of a stack of molecules, which also induces intramolecular atomic displacements in molecules. The deformation interaction results in the nonlinear equations as mentioned earlier. The bisoliton model of high temperature superconductivity in cuprates was proposed by Davydov in 1990. From only three experimental facts: the isotope effect in optimally doped cuprates is nearly absent, the coherence length in hole-doped cuprates is very short, and cuprates become superconductive only when they are slightly doped, he concluded that the superconductivity in cuprates is caused by soliton-like excitations.

According to Davydov, the sharp decrease of the coherence length in cuprates in comparison with conventional superconductors indicates a rather strong interaction of quasiparticles with the acoustic branch of phonons inherent in cuprates. Since the  $CuO_2$  planes in cuprates consist of quasi-infinite parallel chains of alternating ions of copper and oxygen, he assumed that each  $Cu-O$  chain in the  $CuO_2$  plane can be considered as a quasi-one dimensional system. The doping level in superconducting cuprates is small implies that  $gL \gg 1$  for small density of doped charge carriers in cuprates. The energy of pairing does not depend on the mass of an elementary cell since this mass appears in the kinetic energy of the bisolitons. Therefore

the isotope effect is very small in spite of the fact that the basis of pairing is the electron-phonon interaction.

Even though the bisoliton model can describe some pairing characteristics, it is only in first approximation. This is probably because in the bisoliton model the Coulomb repulsion between quasiparticles in a bisoliton is not taken into account. However, the main idea of the bisolitons model seems to be correct, that is, the moderately strong and nonlinear electron-phonon interaction mediates the pairing in cuprates. The main result of the model is that in the presence of strong electron-phonon interaction the isotope effect will be absent or rather small. In spite of the fact that Cooper pairs in cuprates are not Davydov's bisolitons in a direct sense, the Cooper pairs consist of two quasi-one dimensional soliton-like excitations, therefore the name bisolitons is probably the best choice. For example, the name bipolarons does not reflect the presence of one dimensionality in the system.

In 1994, Alexandrov and Mott pointed out that the internal wave function of a Cooper pair and the order parameter of the Bose-Einstein condensation may have different symmetries in cuprates. Emery and Kilvelson, in 1995, proposed that superconductivity requires pairing and long-rang **phase coherence**. In conventional superconductors, the two mechanisms occur simultaneously at  $T_c$  since the carrier density is relatively high so that the average distance between the Cooper pairs is much smaller than the coherence length (the size of a Cooper pair). In other words, the phase stiffness which measures the ability of the superconducting state to carry a supercurrent, is much larger than the energy gap which reflects the strength of the binding of the electrons into Cooper pairs. In cuprates, depending on the doping level, the size of a Cooper pair is smaller than or equal to the distance between the Cooper pairs. Under such conditions, the pairing may occur above  $T_c$  without the phase coherence which is established at  $T_c$ .

In the framework of the bisoliton model, the mechanism for establishing phase coherence among bisolitons is not specified. As emphasized

by Davydov, phonons cannot mediate the phase coherence since bisolitons do not interact with acoustic phonons. This interaction is completely taken into account in the coupling of quasiparticles with a local lattice deformation. He assumed that the phase coherence among bisolitons is established due to the overlap of their wave functions as in the BCS theory for conventional superconductors. In the bisoliton model, the wave function coupling cannot mediate the phase coherence among bisolitons because the wave function of any self-trapped state is by definition very localized.

Davydov defined the critical temperature  $T_c$  as the temperature at which the energy gap vanishes. With such a definition for  $T_c$ , the wavefunction coupling can formally be considered as the mediator of the phase coherence. In reality, however, the wavefunction coupling cannot be responsible for mediating the phase coherence among the bisolitons. This was the reason why he avoided discussing the bell-like shape of the  $T_c(p)$  dependence in cuprates, where  $p$  is the hole concentration in  $CuO_2$  planes. In other words, he could not explain it in the framework of the model. The bisoliton theory predicts that by increasing the hole (electron) concentration, the magnitude of the pairing gap decreases and hence  $T_c$  will monotonically decrease which is contrary to experiment. Therefore, the phase coherence among bisoliton in cuprate should be established due to a non-phonon mechanism which is different from the wavefunction coupling. The bisoliton model is the theory of quasiparticle pairing, but it lacks the mechanism for establishing phase coherence.

### Breaking of Cooper Pairs in Magnetic Field

Cuprates are the type II - superconductors where their total superconductivity vanishes when the external magnetic field exceeds the second critical magnetic field  $B_{c2}$ . Experimental data for lanthanum superconductor shows that  $B_{c2}$  has the value 50 - 140T, while it is 80 - 140T for yttrium superconductor. Such a value of  $B_{c2}$  exceeds appreciably the observed value in conventional superconductor by several tesla.

The constant magnetic field destroys superconductivity due to the Cooper pairs breaking as a result of two effects, **the paramagnetic effect** and **the Landau diamagnetic effect**. The paramagnetic effect is caused by the reorientation of electron magnetic moments along the magnetic field which makes the singlet spin state transform to a triplet one. The Landau diamagnetic effect is caused by the helical trajectories of electrons affected by a magnetic field and the quantized intensive circular motion is accompanied by electron depairing. According to the bisoliton model, the paramagnetic effect, and not the diamagnetic one as in the BCS theory, is proved to be crucial in suppressing the superconductivity by the magnetic field.

Due to the paramagnetic effect, some bisolitons can decay from the singlet spin state with wave numbers  $k_1 = 2k + k_F$  and  $k_2 = -k_F$  into free (unpaired) quasiparticles in the triplet spin state with wave numbers  $k_1 = k + k_F$  and  $k_2 = k - k_F$ . The presence of a magnetic field in the system will affect indirectly the bisolitons due to the bisoliton deformation potential well is decreased. The Cooper pairs are broken in the field  $B$  that exceeds the critical value  $B_{c2}$  at which the energy  $2\mu_B B$  of quasiparticle interaction with a magnetic field is equal to the pairing energy  $2\Delta_0$ , i.e., when the equality  $\mu_B B = \Delta_0$  is fulfilled. Therefore,

$$B_{c2}^{(p)} = \frac{\Delta_0}{\mu_B} = \frac{2cm_z\Delta_0}{e\hbar}$$

where  $\mu_B = e\hbar/2m_zc$  is the Bohr magneton and  $m_z$  is an effective mass of a quasiparticle along the field direction. In ceramic superconductors with  $T_c \sim 90 K$  that corresponds to pairing energy  $2\Delta_0 \sim 10meV$ , the critical value  $B_{c2} \sim 100T$  is close to the observed one in  $YBa_2Cu_3O_7$ . Also the critical velocity of a bisoliton condensate is determined by the equality  $v_c = (2\Delta_0 - \mu_B B)/2\hbar k_F$  which decreases

linearly with increasing  $B$  and vanishes at  $B_{c2}$ . There is no superconductivity current in magnetic field  $B$  greater than  $B_{c2}$ .

To study the diamagnetic effect, it is necessary to use the fact that electron motion in a superconductor in a constant magnetic field is of helical character. Due to the quantum theory developed by Landau, the projection of the helical trajectory onto the plane perpendicular to the magnetic field is a circle with **Larmor frequency**  $\omega_L$ , proportional to the magnetic field by  $\omega_L = eB/Mc$  where  $M$  is the cyclotron mass. Anisotropy of cuprate superconductors causes the different effective masses in different directions and if the magnetic field is in the  $z$  axis then  $M = \sqrt{m_x m_y} \gg m_z$ . This rotational motion is quantized and the minimum energy is determined by  $E_{rot} = (1/2)\hbar\omega_L = \hbar eB/2Mc$  which is proportional to the magnetic field strength  $B$ . The rotational motion reduces the pairing energy by the equation  $2\Delta(B) = 2\Delta_0 - \hbar eB/2Mc$  which is zero when the critical field is  $B_{c2}^{(d)} = 4Mc\Delta_0/e\hbar$ . Comparing this value to the case of paramagnetic effect, it is seen that  $B_{c2}^{(d)} = (2M/m_z)B_{c2}^{(p)} \gg B_{c2}^{(p)}$ . Therefore, the pairing energy and the critical temperature at first decreases slowly with increasing magnetic field due to the diamagnetic effect and then falls sharply to zero due to the paramagnetic effect when  $B = B_{c2}^{(p)}$ .

The influence of **magnetic impurities** on superconductivity in cuprate superconductors described by the bisoliton model differs essentially from the result by the BCS theory. According to the BCS theory, as the impurity concentration  $n$  increases, the critical temperature decreases monotonically as  $T_c(n) = T_c(0) - \pi Jn/4k_B E_F$  and anishes at the critical value  $n_c = 4k_B T_c E_F / \pi J$ . Here,  $J$  is the exchange integral between magnetic impurity and the spin of a conduction electron averaged by the atomic volume. In the framework of the bisoliton model, the magnetic impurities



with spin  $S_i$  generate the effective magnetic field  $B_{eff} = nA$  which is linear in the impurity concentration

$n$ . Here  $A \equiv J \left\langle \sum_i S_i \right\rangle / \mu_B$  and  $J$  is the mean

value of the exchange integral between magnetic impurity and unpaired quasiparticle in triplet spin states along the conduction chains in  $z$  axis. Using the effective magnetic field  $B_{eff}$ , we reduce the problem of the influence of magnetic impurities on superconductivity to the problem of the external magnetic field effect as mentioned above. Replacing the external magnetic field strength  $B$  by  $B_{eff}$ , we get the equation that determines the binding energy, and hence  $T_c$ , depend on the impurity concentration  $n$  due to the diamagnetic effect as  $2\Delta(n) = 2\Delta(0) - (\hbar e A / 2Mc)n$ . With increasing concentration  $n$ , the Cooper pairs decay due to the paramagnetic effect at the critical value  $(B_{eff})_c = n_c A = \Delta(n)cm_z / 2e\hbar \approx \Delta_0 cm_z / 2e\hbar$ .

Therefore when the magnetic impurity concentration  $n < n_c$ , with increasing  $n$ , the pairing energy  $\Delta(n)$  and hence  $T_c$  depend weakly on  $n$  which is confirmed by experimental data. At concentration  $n > n_c$ , the superconductivity vanishes stepwise. The bisoliton model describes qualitatively the  $T_c$  dependence on magnetic impurity concentration observed experimentally.

We know that the quantum motion of quasiparticle in a constant magnetic field is of helical character and the projection onto the plane perpendicular to the magnetic field is a circle with Larmor frequency  $\omega_L = eB / Mc$ . The motion of quasiparticle is characterized by two integrals of motion  $E_z$  and  $L_z$ . The transverse energy  $E_z$  and the angular momentum  $L_z$  projected onto the  $z$  axis are determined by the quantum numbers  $n$  and  $m$  by the equalities  $E_z = \hbar\omega_L(n + 1/2)$  and  $L_z = \hbar m$  where  $n = 0, 1, \dots$  and  $m = 0, \pm 1, \dots$ . For fixed value of  $n$  the radius of a circle is given by

$R_n = [\hbar(2n + 1) / M\omega_L]^{1/2}$  and  $R_0 = \sqrt{\hbar / M\omega_L}$  corresponds to  $n = 0$ . Therefore, the area described by two quasiparticles in the plane perpendicular to the magnetic field is determined by  $S = \pi R_0^2 = \pi\hbar / M\omega_L$ . The magnetic flux permeating the closed orbit is, therefore,  $\Phi_0 = BS = \pi\hbar c / e$  which coincides with the universal unit of the magnetic flux  $\Phi_0 \approx 2.07 \times 10^{-7} \text{ Oe.cm}^2$ . The bisoliton model also describes the **magnetic flux quantization** in layered superconductors.

Brizhik applied the bisoliton model to the **Meissner effect** in cuprate superconductor in 1989. To study the Meissner effect, it is necessary to take into account the helical character of the quasiparticle motion in a constant magnetic field. A bisoliton moves in a crystal along the helical orbits perpendicular to the constant external magnetic field less than the critical field. At low temperatures, the bisoliton motion creates an undamped circular current which in its turn creates a magnetic moment in a sample that compensates the external field. This is the main process of the Meissner effect and the bisolitons are not broken by the magnetic field if its intensity does not exceed the critical value  $B_{c2}^{(P)}$ . Due to strong crystal anisotropy, the longitudinal and transverse motion of the quasiparticles is characterized by different effective masses which satisfy the inequality  $m_z \gg m_x, m_y$ . The anisotropy of the magnetic field **penetration depth**  $\lambda_c$  is determined by the quasiparticle effective mass anisotropy as  $\lambda_c^2 / \pi_{ab}^2 = m_c / m_{ab}$  where  $\lambda_{ab}^2$  and  $m_{ab}$  are mean values of the square of the magnetic field penetration depth and of the effective mass in the  $ab$ -plane, respectively. Experimental values for  $YBa_2Cu_3O_7$  are  $\lambda_c \approx 1,740 - 2,700 \text{ \AA}$ ,  $\lambda_{ab} \approx 335 - 525 \text{ \AA}$  and for  $La_{2-x}Sr_xCuO_{4-y}$  ( $x = 0.15$ ) are  $\lambda_c \approx 2 \times 10^4 \text{ \AA}$ ,  $\lambda_{ab} \approx 1,400 \text{ \AA}$ . These values are in agreement qualitatively with the estimation of quasiparticle effective mass anisotropy which attains the value 10-15.

## Conclusion

In summary, the bisoliton model uses the experimental data on their layered structure and the relatively small density of charge carriers to explain superconductivity in cuprate superconductors. Due to the layered structure and relatively large electron-phonon interaction, the superconducting properties of such a system are described by a system of nonlinear equations. The exact solution of nonlinear equations is characterized by a single wavefunction for the whole crystal that describes a condensate of bisoliton. The spinless bisolitons are pairs of quasiparticles surrounded by local deformations and move as a single unit with constant velocity. Bisolitons are stable if the pairing energy exceeds the screened Coulomb repulsion between their charges. The stability of bisolitons determines the critical velocity and critical current resulting in the breaking of pairs. The bisoliton model is the theory for pairing mechanism while it lacks the mechanism for

establishing phase coherence. Several phenomena in cuprate superconductors can be explained without involving nonphonon pairing mechanisms.

## References

- Davydov, A.S., and Ermakov, V.N. (1989). One-particle excitation in a bisoliton model of superconductivity of ceramic oxides. Preprint ITP-89-68E, Inst. Theor. Physics, Kiev, p. 68.
- Davydov, A.S. (1990). Theoretical investigation of high-temperature superconductivity. *Physics Reports*, 190(4&5):91.
- Mourachkine, A. (2002). High-temperature superconductivity in cuprates: the nonlinear mechanism and tunneling measurements 1<sup>st</sup> ed. Kluwer Academic Publishers, The Netherlands, p. 101-135.
- Mourachkine, A. (2004). Room-temperature Superconductivity. 1<sup>st</sup> ed. Cambridge International Science Publishing, Cambridge, UK, p. 96-108.



