THE IMPROVEMENT OF MIMO CAPACITY USING SIMPLE TECHNIQUE REALIZED BY BUTLER MATRIX

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Abstract

In the literature, among many techniques improving MIMO capacity (Foschini, 1996; Foschini *et al.*, 1998; Kermoal *et al.*, 2000; Molisch *et al.*, 2002; Stridh *et al.*, 2000; Telatar, 1995; Tsoulos, 2006; Vieira *et al.*, 2006), the concept of eigen-beamforming has been recognized as the best technique to provide an enhanced capacity. However, the expense of this technique is the cost of feedback channel and complexity processing. Therefore, this article aims to present a simple technique based on angle domain processing which does not require a feedback channel and has low complexity. A Butler matrix is chosen for 4 × 4 MIMO systems in order to prove the concept of the proposed system in practice. The simulation and measurement results indicate the enhancement of MIMO capacity when using Butler matrix.

Keywords: MIMO Channel Capacity, Array domain processing, Angle domain processing, Eigen-beamforming, Butler matrix

Introduction

So far in the literature, the MIMO (Multiple Input Multiple your Output) systems provide a promising quality of service including a great channel capacity. Many works have proposed the method of eigen beamforming technique (Bishwarup *et al.*, 2006; Liang Sun *et al.*, 2009; Sirikiat *et al.*, 2006; Xiayu Zheng *et al.*, 2007;) to improve the capacity. This technique utilizes the properties of estimated channels by performing singular value decomposition on channel matrix. Then, eigen-vectors compositing of channel matrix are considered as pre and post coding schemes for MIMO systems. From analysis, the eigen beamforming offers the optimal performance in comparing with other techniques. However, the drawback of this technique is the requirement of feedback channel information which increases the overhead of data transmission and the expense of data processing. In addition, the complexity of pre and post coding is so difficult that it is unattractive to be implemented for real application. Therefore, the search of new technique to replace eigen-beamforming technique is still in focus.

In this article,, the simple technique based on the concept of angle domain processing is introduced. This is because angle domain

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processing does not require any additional complexity like feedback channel for pre and post coding schemes. Instead, their schemes are designed by fixed angles of arrival and departure which operate as blind switched beamforming. Although the performance of angle domain processing can be predicted to be lower than eigen-beamforming but the ease of implementation might be a good tradeoff to attract MIMO designers. Also in this article, the practical realization of the proposed system has been demonstrated by using Butler matrix. A low profile manufacturing is constructed and also tested under real environments. By only inserting Butler matrix next to antenna arrays at both transmitter and receiver, the improvement of MIMO capacity is able to be obtained as reported in simulations and measurements.

MIMO System Model

A. Array domain processing

This section details the array domain representation of MIMO systems (Tse and Viswanath, 2005). Let **x** be a vector of the transmitted signals with N_t transmitted antennas and **y** be a vector of the received signals with N_r received antennas. Then, the relation between transmitted and received signals is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

Where **n** is an ($N_r \times 1$) noise vector and **H** is an ($N_r \times N_t$) channel matrix. With this notation channel output sequence can be written in matrix form as:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{N_r} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1N_t} \\ h_{21} & h_{22} & \cdots & h_{2N_t} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_r1} & h_{N_r2} & \cdots & h_{N_rN_t} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N_t} \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_{N_r} \end{bmatrix} \tag{2}$$

Figure 1 shows the angle domain representation of MIMO systems. There is an arbitrary number of physical paths between the transmitter and receiver; the ith path having attenuation of α_i, makes an angle of ϕ_{ti} ($\Omega_{ti} := \cos\phi_{ti}$) with the transmit antenna array and angle of ϕ_{ri} ($\Omega_{ri} := \cos\phi_{ri}$) with the receive antenna array. The channel matrix H can be written as:

$$\mathbf{H} = \sum_i a_i^b \mathbf{e}_r(\Omega_{ri})\mathbf{e}_t(\Omega_{ti})^* \tag{3}$$

Where

$$a_i^b := a_i\sqrt{N_tN_r}\exp\left(\frac{j2\pi d_i}{\lambda_c}\right) \tag{4}$$

$$\mathbf{e}_t(\Omega) := \frac{1}{\sqrt{N_t}}\begin{bmatrix} 1 \\ \exp[-j(2\pi\Delta_t\Omega)] \\ \vdots \\ \exp[-j(N_t-1)(2\pi\Delta_t\Omega)] \end{bmatrix} \tag{5}$$

And

$$\mathbf{e}_r(\Omega) := \frac{1}{\sqrt{N_r}}\begin{bmatrix} 1 \\ \exp[-j(2\pi\Delta_r\Omega)] \\ \vdots \\ \exp[-j(N_r-1)(2\pi\Delta_r\Omega)] \end{bmatrix} \tag{6}$$

Also, α_i is the distance between transmit and receive antennas along path ith. The vector $\mathbf{e}_t(\Omega)$ and the vector $\mathbf{e}_r(\Omega)$ are, respectively transmitted and received unit spatial signatures along the direction Ω, λ_c is the wavelength of the center frequency in the whole signal bandwidth. Δ_t is the normalized transmit antenna separation and Δ_r is the normalized receive antenna separation. When Channel State Information (CSI) is not available at the transmitter, the capacity of MIMO systems expressed in bits per second per hertz (bps/Hz) can be written as

$$C = \log_2 \det\left(\mathbf{I}_{N_r} + \frac{P_t}{P_N N_t}\mathbf{H}\mathbf{H}^*\right) \tag{7}$$

where $\mathbf{I}_N$, is the identity matrix of size $N_r \times N_r$, **H** is the channel matrix of size $N_r \times N_t$ with **H*** being its transpose conjugate, and P_t gives the average Signal-to-Noise Ratio (SNR) per receiver branch independent of the number of transmitting antennas N_t.

B. Angle domain processing

The concept of angle domain (Li *et al*., 2007-2008) can be represented by the transmitted and received signals. The signal arriving at a directional Ω onto the receive antenna array is along the unit spatial signature $\mathbf{e}_r(\Omega)$ given by (6). Hence, the N_r fixed vector is given by

$$\xi_r := \left\{ \mathbf{e}_r(0), \mathbf{e}_r(\frac{1}{L_r}), \cdots, \mathbf{e}(\frac{N_r - 1}{L_r}) \right\} \tag{8}$$

In (8), it can be noticed that there is a set of orthogonal basis for the received signal space. This basis provides the representation of received signals in the angle domain.

It is similarly defined for the angle domain representation of the transmitted signal. The signal transmitted at direction Ω is along the unit vector $\mathbf{e}_r(\Omega)$, defined in (5). The N_t fixed vector is given by

$$\xi_t := \left\{ \mathbf{e}_t(0), \mathbf{e}_t(\frac{1}{L_t}), \cdots, \mathbf{e}(\frac{N_t - 1}{L_t}) \right\} \tag{9}$$

Where $L_t = N_t\Delta_t$ and $L_r = N_r\Delta_r$ are the normalized antenna array lengths of the transmitter and receiver, respectively. Let $\mathbf{U}_t$ and $\mathbf{U}_r$ be the unitary matrices whose columns are the basis vector in (8) and (9), respectively, can be written as:

$$\mathbf{U}_t = \frac{1}{\sqrt{N_t}} \exp\left(\frac{-j2\pi kl}{N_t} \right) \qquad k, l = 0, 1, \ldots, N_t - 1. \tag{10}$$

And

$$\mathbf{U}_r = \frac{1}{\sqrt{N_r}} \exp\left(\frac{-j2\pi kl}{N_r} \right) \qquad k, l = 0, 1, \ldots, N_r - 1. \tag{11}$$

We can transform the array domain into the angle domain by

$$\mathbf{H}^a = \mathbf{U}_r^* \mathbf{H} \mathbf{U}_t \tag{12}$$

Thus, the capacity of MIMO systems is given by

$$C = \log_2 \det\left(\mathbf{I}_{N_r} + \frac{P_t}{P_N N_t} \mathbf{H}^a \mathbf{H}^{a^*} \right) \tag{13}$$

Where $\mathbf{I}_N$, is the identity matrix of size $N_r \times N_r$, $\mathbf{H}^a$ is the channel matrix of size $N_r \times N_t$.

Figure 2 shows the simulated channel matrices from statistical modeling adopted by Fundamentals of Wireless Communication book. The basis for the statistical modeling of MIMO fading channels is approximated by

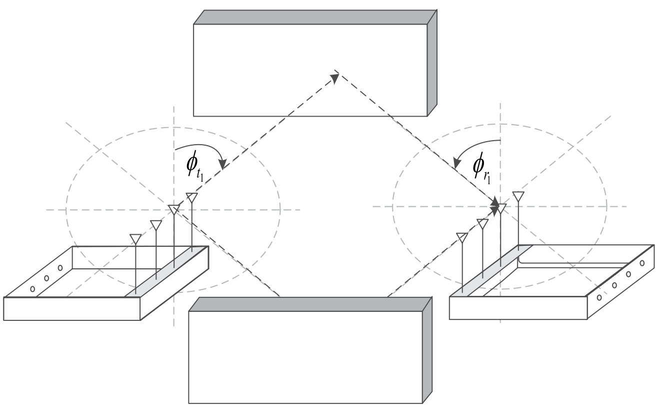

Figure 1. Angle domain representations of 4×4 MIMO channel with four transmit and receive antennas

the physical paths partitioning into angularly resolvable bins and aggregated to form resolvable paths whose channel gains are $\mathbf{H}^a_{kl}$. Assuming that ai of the physical paths is independent. Then, we used equations (3)-(6) to find channel matrix for array domain and (10)-(12) to find channel matrix for angle domain.

C. Eigen beamforming technique

We used the channel matrix **H** from array domain processing. Consider a MIMO channel with $N_r \times N_t$ channel matrix **H** that is known to both the transmitter and the receiver, the singular value can be found by using SVD technique in MATLAB programming. We can obtain its singular value decomposition (SVD) as

$$\mathbf{H} = \mathbf{USA}^* \tag{14}$$

Where $N_r \times N_r$ matrix **U** and the $N_t \times N_t$ matrix **V** are unitary matrices, **S** is an $N_r \times N_t$ diagonal matrix. So, the capacity of MIMO system is given by

$$C = \log_2 \det\left(\mathbf{I}_{N_r} + \frac{P_t}{P_N N_t}\mathbf{SS}^*\right) \tag{15}$$

Practical realization using Butler Matrix

Figure 3 shows a block diagram of Butler matrix (Liberti and Rappaport, 1999) which is applied for the concept of angle domain processing for 4 × 4 MIMO systems. The fixed beamforming matrix is bi-direction, which means that each port corresponds to particular received as well as transmitted signals from the same radiation pattern.

It is clearly shown that the weight vectors corresponding to each port in Table 1 are mutually orthogonal. Therefore, instead of using (10) and (11), the basis vector of applying Butler matrix can be written by the following:

$$\mathbf{B}_r = e^{-j\theta_{kl}} \qquad k,l = 0,1,\cdots,N_r - 1 \tag{16}$$

And

$$\mathbf{B}_t = e^{-j\theta_{kl}} \qquad k,l = 0,1,\cdots,N_t - 1 \tag{17}$$

Figure 4 shows a configuration of manufactured Butler matrix. The dimensions in Butler matrix can be calculated from transmission line theory. The manufactured

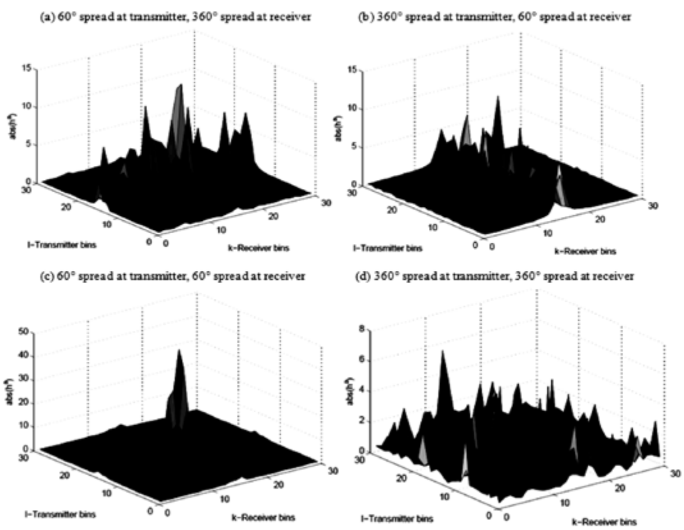

Figure 2. An example of with different angle spreads at the transmitter and receiver

product is also confirmed by measuring inter-element phasing and beam direction which are shown in Table 2. In Table 2, the distributions of all inter-element phasing are similar to conceptual Butler matrix but they are slightly deviated by ± 10 degree. However, the beam direction is deviated by just only 0.6 degree.

Figure 5. illustrates the beam direction of applying Butler matrix to both transmitter and receiver. It is interesting to see that the concept of angle domain processing is successfully achieved by simply adding Butler matrices next to antenna elements. Then, the channel matrix realized by Butler matrix can be written as:

$$\mathbf{H}^b := \mathbf{B}_r^* \mathbf{H} \mathbf{B}_t \tag{18}$$

Where $\mathbf{B}_t$ and $\mathbf{B}_r$ are the unitary matrices whose columns are the basis vector in four direction for transmitter and receiver and $\mathbf{H}$ is channel matrix of size $N_r \times N_t$ to get array domain. Thus, the capacity of MIMO systems when applying Butler matrix is given by

$$C = \log_2 \det\left(\mathbf{I}_{N_r} + \frac{P_t}{P_N N_t}\mathbf{H}^b\mathbf{H}^{b*}\right) \tag{19}$$

Measurement

Figure 6 shows a block diagram of measurement set up for 4 × 4 MIMO system. The network analyzer is used for measurement channel coefficients in magnitude and phase. The power amplifier (PA) is used at transmitter to provide more transmitted power. Low noise

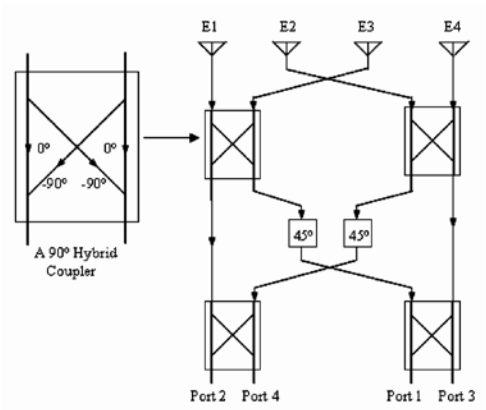

Figure 3. A Block diagram of Butler matrix

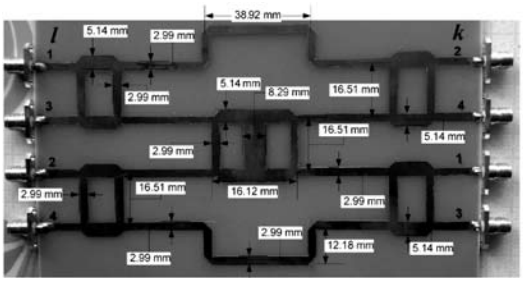

Figure 4. Configuration of manufactured Butler matrix

Table 1. Element phasing, beam direction and inter-element phasing for the Butler matrix shown in Figure 3 (Conceptual)

θ	E1 (l=1)	E2 (l=2)	E3 (l=3)	E4 (l=4)	Beam Direction	Inter-Element Phasing
Port 1 (k =1)	-45°	-180°	45°	-90°	138.6°	-135°
Port 2 (k =2)	0°	-45°	90°	135°	104.5°	-45°
Port 3 (k =3)	-135°	-90°	-45°	0°	75.5°	45°
Port 4 (k =4)	-90°	-45°	-180°	-45°	41.4°	135°

amplifier (LNA) is used at receiver to increase received signal level. The channel measurements are undertaken by five times at each location (Promsuvana and Uthansakul, 2008). In each location two modes of MIMO operation, conventional array and angle domain processings are measured. The Butler matrices are inserted at both transmitter and receiver when measuring MIMO channels with angle domain processing.

Figure 7 shows measurement scenarios. We chose a large room to provide various test conditions. The location of transmitter is fixed as shown in Figure 7 with rectangular symbol. There are five measured locations for receiver shown by circular symbol in Figure 7. It is easy to measure both array domain processing and angle domain processing by using switches presented in Figure 6. The measured results achieved by network analyzer are used as a channel response in MIMO system. Also seen in Fig. 6, apart from Butler matrix, all components of array and angle domain are the same. Therefore, the measured channels can be directly compared to each other as presented in the next section.

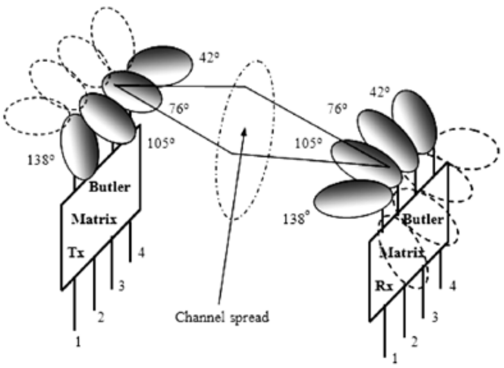

Figure 5. Illustration of applying Butler matrix for 4x4 MIMO systems

Results and Discussions

A. Simulation Results

The simulations are undertaken by MATLAB programming and the capacity results are evaluated by using (7), (15) and (19). For array domain processing approach, the channel matrix $\mathbf{H}$ is found by assumptions in (4), (5) and (6). For optimum eigen-beamforming approach, the channel matrix $\mathbf{H}$ in (3) is utilized. For angle domain processing approach realized by Butler matrix, the channel matrix $\mathbf{H}^b$ is calculated from basis vectors in Table 1 resulting in (16) and (17). The channel fading environments are simulated by changing the conditions of angle spreads at transmitter and receiver. Four cases are

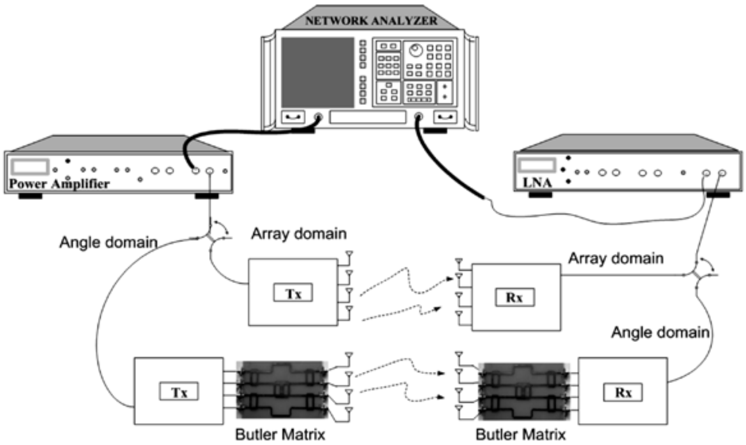

Figure 6. Block diagram of measurement set up

considered as (i) 60° spread at transmitter and 360° spread at receiver, denoted as 60-360 (ii) 360° spread at transmitter and 60° spread at receiver, denoted as 360-60 (iii) 60° spread at transmitter and 60° spread at receiver, denoted as 60-60 (iv) 360° spread at transmitter and 360° spread at receiver, denoted as 360-360. Note that case (iii) is equivalent to line of sight scenario while case (iv) is equivalent to Rayleigh fading channel.

In Figure 8, the capacity comparison between 4 × 4 MIMO systems with array domain processing, angle domain processing and eigen-beamformin technique is presented. The results indicate that to use angle domain processing realized by Butler matrix can improve the channel capacity for any fading conditions. The range of capacity enhancement is from 5 to 10 dB depending on characteristic of fadings. However, the optimum eigen-beamforming technique offers a better performance than angle domain processing.

B. Measurement results

The channel matrix **H** and $\mathbf{H}^b$ is found by measured data from network analyzer. The channel fading environments are measured by changing the locations of receiver. Five locations

Table 2. Element phasing, beam direction and inter-element phasing for the Butler matrix shown in Figure 4 (Manufactured)

θ	E1 (l=1)	E2 (l=2)	E3 (l=3)	E4 (l=4)	Beam Direction	Inter-Element Phasing
Port 1 (k =1)	158°	25°	-112°	118°	138°	-130°
Port 2 (k =2)	-87°	-137°	176°	137°	105°	132°
Port 3 (k =3)	178°	-139°	-98°	76°	-42°	50°
Port 4 (k =4)	136°	-90°	40°	176°	42°	138°

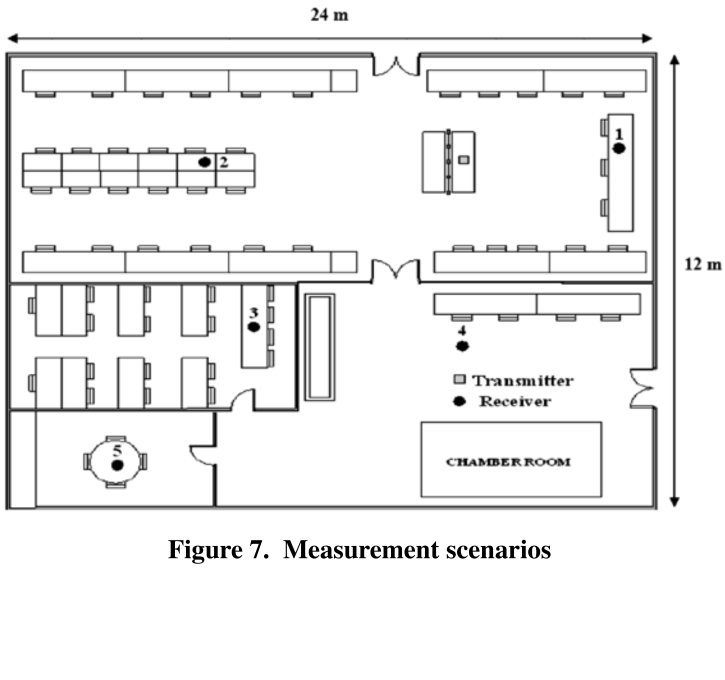

Figure 7. Measurement scenarios

are considered in Figure 7. We also believe that the mismatches among RF circuits in transmit/receive components and mutual coupling effects are included in the measured channel. The simulations are undertaken by utilizing measured data into MATLAB programming and the capacity results are evaluated by using (7), (15) and (19).

Figure 9 shows comparison between array and angle domain channels of 4 x 4 MIMO systems at location 5, where $\mathbf{H}_{ij}$ is referred to the channel coefficient at ith receive antenna and jth transmit antenna. It can be observed that channels of array domain processing and angle domain processing are quite different. The amplitude deviation is about ±5 dB and the phase deviation is about ±100°. These deviations are dominant to the capacity performance of MIMO system. For other locations, the deviations of amplitude and phase are similar to location 5.

In Figure 10, the average capacity by averaging overall locations versus signal to noise ratio (SNR) is presented. The results indicate that to use the angle domain processing realized by Butler matrix offers better performance than array domain processing. However, the best performance is achieved by the optimum eigen-beamforming technique. In order to justify the results, the numeric values of average capacity at SNR = 10 dB are given in Table 3. It is noticed that the benefit of angle domain processing is more pronounced at location 1 and 5. The reason is that these locations are close to wall and there are many surrounding furniture providing more multipath. However, the improvement of MIMO capacity can be observed from all locations with a little expense of inserting Butler matrices at both transmitter and receiver.

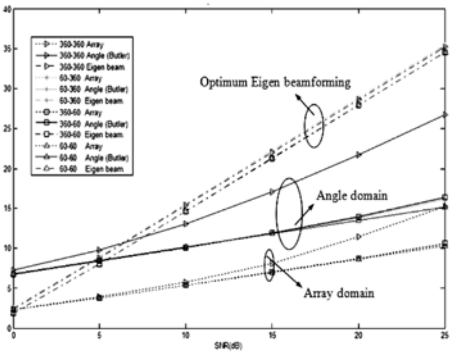

Figure 8. Average capacity (bits/s/Hz) vs. SNR (dB) for 4 conditions of angle spread, Δt = Δr = 0.5

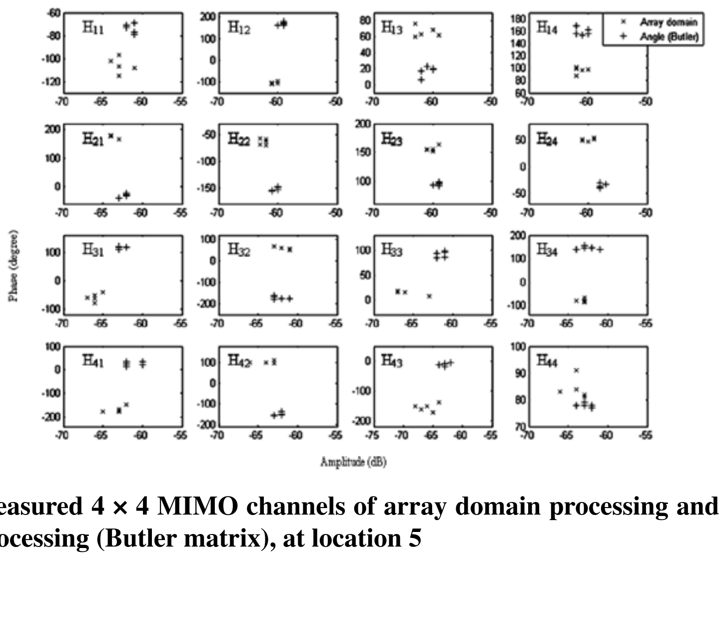

Figure 9. Measured 4 × 4 MIMO channels of array domain processing and angle domain processing (Butler matrix), at location 5

In Table 4, result comparisons between array domain processing, angle domain processing and eigen-beamforming technique. The complexity of eigen-beamforming can be reduced by using the propose system. However, the capacity of propose system is 8.74 bits/s/Hz lower than the eigen-beamforming technique. This is the tradeoff between using both techniques in which the MIMO designers have to realize.

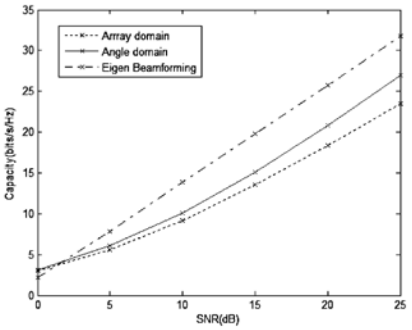

Figure 10. Average capacity (bits/s/Hz) vs. SNR (dB) at each location

Conclusions

This article presents the performance of MIMO systems using angle domain processing realized by Butler matrix. The simulation result reveals that the proposed system outperforms the conventional array domain processing for every fading case. And then, this paper verifies the benefit of using angle domain processing for 4×4 MIMO systems by measured results. The angle domain processing realized by Butler matrix is implemented and compared with array domain processing. The results reveal that the angle domain processing

Table 3. Average capacity at over all locations for SNR=10 dB

Location	Average capacity (bits/s/Hz)		
	Array domain	Angle domain (Butler matrix)	Eigen beamforming
1	8.72	10.12	13.93
2	8.43	8.52	14.72
3	6.46	6.65	15.12
4	6.88	7.37	15.75
5	10.57	11.03	10.62

Table 4. Result comparisons between array domain processing, angle domain processing (Butler matrix) and eigen-beamforming technique

Processing	Array domain	Angle domain (Butler matrix)	Eigen beamforming
Complexity in processing	None	None	Additional SVD technique
Complexity in feedback	None	None	Additional algorithms for feedback channel
Complexity in hardware	None	Additional butler matrix	None
Average capacity (bits/s/Hz) at SNR=10 dB	8.21	8.74	14.03

outperforms the conventional array domain processing for all fading locations. Hence, the proposed system is attractive to be practically implemented on MIMO systems due to its ease and low complexity.

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