MODIFIED FUZZY ANALYTICAL HIERARCHY PROCESS FOR MULTIPLE CRITERIA INVENTORY CLASSIFICATION

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Abstract

A systematic approach to inventory control and classification may have a significant influence on company competitiveness. In order to efficiently control the inventory items and to determine suitable ordering policies for them, a multi-criteria inventory classification is used. In this paper, a Modified Fuzzy Analytic Hierarchy Process (FAHP) is proposed to determine the relative weights of the criteria using Chang's extent analysis and to classify inventories into different categories. To accredit the proposed model, it is implemented for 351 raw materials of the switchgear section of Energypac Engineering Limited (EEL), a large power engineering company in Bangladesh. The results of the study show that 22 items are identified as being in class A or the very important group, 47 items as being in class B or the important group, and the remaining 282 items as being in class C or the relatively unimportant group, and these are used as a basis for the control scheme.

Keywords: Multi-criteria inventory classification, fuzzy analytic hierarchy process, triangular fuzzy number

Introduction

An inventory has been looked at as a major cost and a source of uncertainty due to both the volatility within commodity markets and the demand for a value-added product. Manufacturing companies hold inventories for a number of reasons, such as to allow for flexible production schedules and to take advantage of economies of scale when ordering stock (Nahmias, 2004). Classification of an inventory is a crucial element in the operation of any production company. Because of the huge number of inventory items in many companies, great attention is directed to the division of an inventory into different classes, which consequently requires the application of different management tools and policies (Chase *et al*., 2006). ABC inventory management deals with the classification of the items in an inventory into a decreasing order of annual dollar volume. The ABC classification process is an analysis of a range of items, such as finished products or customers, into 3 categories: A - outstandingly important; B - of average importance; C - relatively unimportant, as a basis for a control scheme. Each category can, and sometimes should, be handled in a different way,

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with more attention being devoted to category A, less to B, and even less to C (Nahmias, 2004).

Sometimes, only 1 criterion is not a very efficient measure for decision making. Therefore, multiple-criteria decision making methods are used (Flores and Whybark, 1986, 1987). Other criteria like lead time of supply, part criticality, availability, stock out penalty costs, ordering costs, scarcity, durability, substitutability, and reparability have been taken into consideration (Flores and Whybark, 1986, 1987; Zhou and Fan, 2007). More studies have been done on multi-criteria inventory classification in the past 20 years, so many different methods for classifying an inventory and taking into consideration multiple-criteria have been used and developed.

Flores and Whybark (1986, 1987) proposed the bi-criteria matrix approach, wherein annual dollar usage by a joint-criteria matrix is combined with another criterion. Flores *et al*. (1992) have proposed the use of a joint-criteria matrix for average unit cost and annual dollar usage and applied the Analytic Hierarchy Process (AHP) developed by Saaty (1980). The advantages of the AHP is that it can incorporate many criteria and its ease of use on a massive accounting and measurement system, but its shortcoming is that a significant amount of subjectivity is involved in pairwise comparisons of criteria. The AHP has been used to reduce multiple criteria to a univariate and consistent measure.

Partovi and Burton (1993) applied the AHP to inventory classification in order to include both quantitative and qualitative evaluation criteria. Braglia *et al*. (2004) integrated a decision diagram with a set of AHP models to solve various multi-attribute decision sub-problems at the different levels/nodes of the decision tree. Guvenir and Erel (1998) applied the genetic algorithm technique to the problem of multiple-criteria inventory classification. Partovi and Anandarajan (2002) proposed an artificial neural network approach for inventory classification. Ramanathan (2006) proposed a weighted linear optimization model for multiple-criteria ABC inventory classification, where the performance score of each item is obtained using a data envelopment analysis (DEA)-like model. Liu and Huang (2006) presented a modified DEA model to address ABC inventory classification.

Bhattacharya *et al*. (2007) developed a distance-based multiple-criteria consensus framework utilizing the technique for order preference by similarity to ideal solution (TOPSIS) for ABC analysis. Chen *et al*. (2008) proposed a case-based distance model for multiple-criteria ABC analysis. Jamshidi and Jain (2008) addressed multi-criteria ABC inventory classification by standardizing each criterion and weighting them for classification. Šimunović *et al*. (2009) investigated the application of neural networks in multiple-criteria inventory classification. Hadi-Vencheh (2010) proposed a simple nonlinear programming model which determines a common set of weights for all the items. Yu (2011) compared artificial-intelligence-based classification techniques with the traditional multiple discriminant analysis. Therefore, the main objective of this research is to develop an improved multiple-criteria inventory classification model using the Modified Fuzzy Analytic Hierarchy Process (FAHP) approach.

The remainder of this paper is organized as follows: an overview of the Modified FAHP is presented in the next section; in the following section is the background information for the case study problem and the justification of the proposed model; the discussion that summarizes the empirical results is given in next section; and finally, the last section presents the conclusions, limitations, and scope for future work.

Modified Fuzzy Analytic Hierarchy Process

Fuzzy Analytic Hierarchy Process

In the conventional AHP, the pairwise comparisons for each level with respect to the goal of the best alternative selection are conducted using a 9-point scale. So, the application of Saaty's AHP has some shortcomings as follows (Saaty, 1998): (1) The AHP method is mainly used in nearly crisp decision applications; (2) The AHP method creates and deals with a very unbalanced scale of judgment; (3) The AHP method does not take into account the uncertainty associated with the mapping of one's judgment to a number; (4) The ranking of the AHP method is rather imprecise; and (5) The subjective judgment, selection, and preference of decision makers have great influence on the AHP results. In addition, a decision maker's requirements on evaluating alternatives always contain ambiguity and a multiplicity of meaning. Furthermore, it is also recognized that human assessment on qualitative attributes is always subjective and thus imprecise. Therefore, the conventional AHP seems inadequate to capture a decision maker's requirements explicitly (Kabir and Hasin, 2011).

In order to model this kind of uncertainty in human preference, fuzzy sets could be incorporated with the pairwise comparison as an extension of the AHP. A variant of the AHP, called FAHP, is implemented in order to overcome the compensatory approach and the inability of the AHP in handling linguistic variables. The FAHP approach allows a more accurate description of the decision making process.

One of the important issues of multi-criteria decision making is prioritization of the criteria. Determining the importance of the weights made by managers, especially in terms of the issue of multiple-criteria ABC classification, is always subjective in such a way that inventory managers usually select some important criteria and then prioritize them. There are several methods to determine of the criteria weights, including the AHP, entropy analysis, eigenvector method, weighted least square method, and linear programming for multidimensions of analysis preference. In this model, the FAHP is applied.

Generally, it is impossible to reflect the decision makers' uncertain preferences through crisp values. Therefore, the FAHP is proposed to relieve the uncertainty of the AHP method, where the fuzzy comparisons ratios are used.

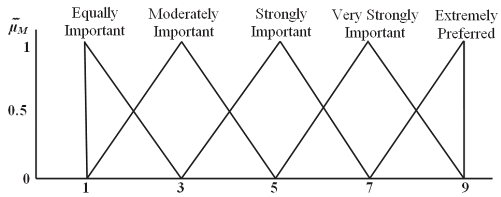

Figure 1. Linguistic variables for the importance of weight of each criterion

Table 1. Linguistic variables describing weights of the criteria and values of ratings

Linguistic scale for importance	**Fuzzy numbers**	**Membership function**	**Domain**	**Triangular fuzzy scale *(cl, m, u)***
Just equal				(1, 1, 1)
Equally important	$\tilde{1}$	$\mu_M(x) = (3-x) / (3-1)$	$1 \leq x \leq 3$	(1, 1, 3)
Weakly important	$\tilde{3}$	$\mu_M(x) = (x-1) / (3-1)$ $\mu_M(x) = (5-x) / (5-3)$	$1 \leq x \leq 3$ $3 \leq x \leq 5$	(1, 3, 5)
Essential or Strongly important	$\tilde{5}$	$\mu_M(x) = (x-3) / (5-3)$ $\mu_M(x) = (7-x) / (7-5)$	$3 \leq x \leq 5$ $5 \leq x \leq 7$	(3, 5, 7)
Very strongly important	$\tilde{7}$	$\mu_M(x) = (x-5) / (7-5)$ $\mu_M(x) = (9-x) / (9-7)$	$5 \leq x \leq 7$ $7 \leq x \leq 9$	(5, 7, 9)
Extremely Preferred	$\tilde{9}$	$\mu_M(x) = (x-7) / (9-7)$	$7 \leq x \leq 9$	(7, 9, 9)
If factor i has one of the above numbers assigned to it when compared to factor j, then j has the reciprocal value when compare to i			Reciprocals of above $\tilde{M}_1^{-1} = (1/u_1, 1/m_1, 1/l_1)$	

Source: Bozbura and Beskese (2007)

There are several procedures to attain the priorities in the FAHP. The fuzzy least square method (Xu, 2000), the method based on the fuzzy modification of the logarithmic least square method (Boender *et al*., 1989), the geometric mean method (Buckley, 1985), the direct fuzzification of the method of Csutora and Buckley (2001), The synthetic extent analysis (Chang, 1996), Mikhailov's fuzzy preference programming (Mikhailov, 2003), and the 2-stage logarithmic programming (Wang *et al*., 2005) are some of these methods. Chang's extent analysis is utilized in this research to evaluate the focusing problem.

Chang (1992) introduces a new approach for handling a pair-wise comparison scale based on triangular fuzzy numbers (TFNs), followed by the use of the extent analysis method for the synthetic extent value of the pairwise comparison (Chang, 1996). The first step in this method is to use TFNs for pairwise comparison by means of the FAHP scale, and the next step is to use the extent analysis method to obtain priority weights by using synthetic extent values. Fuzzification or the fuzzy evaluation matrix of the criteria was constructed through the pairwise comparison of different attributes relevant to the overall objective using the linguistic variables and TFNs (Figure 1 and Table 1).

The following section outlines Chang's extent analysis method on the FAHP. Let $X = \{x_1, x_2,\dots, x_n\}$ be an object set and $U = \{u_1,u_2 ,\dots,\mu_m\}$ be a goal set. As per Chang (1992, 1996) each object is taken and an analysis for each goal, g_i, is performed, respectively. Therefore, m extent analysis values for each object can be obtained, as under:

$$M_{gi}^{1} ,M_{gi}^{2} ,\dots\dots, M_{gi}^{m} \quad i = 1, 2, 3,\dots\dots,n$$

where all the M_{gi}^{m} $(j = 1, 2,\dots,m)$ are TFNs whose parameters are depicting the least, most, and largest possible values, respectively and are represented as (a, b, c).

The steps of Chang's extent analysis (Chang, 1992) can be detailed as follows (Kahraman *et al*., 2004; Bozbura *et al*., 2007; Kabir and Hasin, 2011):

Step 1: The value of the fuzzy synthetic extent with respect to I, the object, is defined as

$$S_i = \sum_{j=1}^{m} M_{gi}^{\;j} \otimes \left[\sum_{j=1}^{n} \sum_{j=1}^{m} M_{gi}^{\;j}\right]^{-1} \tag{1}$$

To obtain $\sum_{j=1}^{m} M_{gi}^{\;j}$ perform the fuzzy addition operation of m extent analysis values for a particular matrix such that

$$\sum_{j=1}^{m} M_{gi}^{j} = \left(\sum_{j=1}^{m} a_j ,\ \sum_{j=1}^{m} b_j ,\ \sum_{j=1}^{m} c_j\right) \tag{2}$$

And to obtain $\left[\sum_{j=1}^{n} \sum_{j=1}^{m} M_{gi}^{j}\right]^{-1}$ perform the fuzzy addition operation of M_{gi}^{m} $(j = 1, 2,\dots.,$ m) values such that

$$\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^{j} = \left(\sum_{i=1}^{n} a_i ,\ \sum_{i=1}^{n} b_i ,\ \sum_{i=1}^{n} c_i\right) \tag{3}$$

And then compute the inverse of the vector in Equation (11) such that

$$\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^{j}\right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} c_i},\ \frac{1}{\sum_{i=1}^{n} b_i},\ \frac{1}{\sum_{i=1}^{n} a_i}\right) \tag{4}$$

Step 2: The degree of possibility of $M_2 = (a_2, b_2, c_2) \geq M_1 = (a_1, b_1, c_1)$ is defined as

$$V(M_2 \geq M_1) = \sup\left[\min\left(\mu_{M_1}(x) ,\ \mu_{M_2}(x)\right)\right] \tag{5}$$

and can be equivalently expressed as follows:

$$V(\widetilde{M}_2 \geq \widetilde{M}_1) = hgt\,(\widetilde{M}_1 \cap \widetilde{M}_2)$$

$$= \begin{cases} 1, & if\ b_2 \geq b_1 \\ 0, & if\ a_1 \geq c_2 \\ \dfrac{a_1 - c_2}{(b_2 - c_2) - (b_1 - a_1)}, & \text{otherwise} \end{cases} \tag{6}$$

where d is the ordinate of the highest intersection point D between μ_{M_1} and μ_{M_2} as shown in Figure 2.

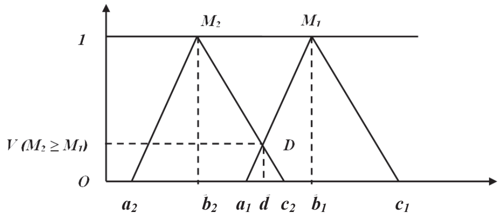

Figure 2. The intersection between M_1 and M_2

To compare M_1 and M_2, both the values of $V\ (M_1 \geq M_2)$ and $V\ (M_2 \geq M_1)$.

Step 3: The degree of possibility for a convex fuzzy number to be greater than k convex fuzzy numbers M_i (i = 1,2,….., k) can be defined by

$$V(M \geq M_1, M_2, ...M_k) = V\ [(M \geq M_1) \text{ and } (M \geq M_2) \text{ and } ...(M \geq M_k)] = \min V\ (M \geq M_i),\ (i = 1,2,3,...,k). \quad (7)$$

Assuming that

$$d'\ (A_i) = \min V\ (S_i \geq S_k) \quad (8)$$

for $k = 1, 2, 3, ..., n;\ k \neq i$, then the weight vector is given by

$$W' = (\ d'\ (A_1),\ d'\ (A_2), ..., d'\ (A_n))^T \quad (9)$$

where $A_i = (i = 1,2,3,...n)$ are n elements

Step 4: By normalizing, the normalized weight vectors are

$$W = (\ d\ (A_1),\ d\ (A_2), ..., d\ (A_n))^T \quad (10)$$

where W is a non-fuzzy number.

Fuzzy pairwise comparisons can be combined by use of the following algorithm (Büyüközkam and Feyzioğlu, 2004; Chang *et al.*, 2009):

$$l_{ij} = \min(l_{ijk}),\ m_{ij} = \left(\prod_{k=1}^{K} m_{ijk}\right)^{1/k},\ u_{ij} = \max(u_{ijk}) \quad (11)$$

where (l_{ijk}, m_{ijk}, u_{ijk}) is the fuzzy evaluation of sample members k (k = 1,2, …., K).

After comparison is made, it is necessary to check the consistencỹ ratio of the comparison. To do so, the graded mean integration approach is utilized for defuzzifying the matrix. According to the graded mean integration approach, a fuzzy number $\tilde{L}_x = (l_1, l_2, l_3)$ can be transformed into a crisp number by employing the equation given below:

$$P\ (\tilde{L}) = L = \frac{l_1 + 4l_2 + l_3}{6} \quad (12)$$

After the defuzification of each value in the matrix, the 'consistency ratio' (CR) of the matrix can easily be calculated and checked to see whether CR is smaller than 0.10 or not.

Modified Fuzzy Analytic Hierarchy Process

Minimum and maximum operations are not appropriate if the sample has a wide range of upper and lower bandwidths, in other words, if the evaluations are inhomogeneous. It has to be considered that if only 1 or a few decision makers deliver extreme l_{ijk} and/or u_{ijk} the whole span of fuzzy numbers (l_{ij}, m_{ij}, u_{ij}) gets huge. Due to the required number of multiplication and addition operations, the aggregated fuzzy weights can even exceed the 0-1 borders or become irrational (Mikhailov, 2003), which is of course, unsatisfactory. Therefore, the geometric mean is used also for l_{ij} and u_{ij} which delivers satisfying fuzzy group weightings (Meixner, 2009). Geometric mean operations are commonly used within the application of the AHP for aggregating group decisions (Davies, 1994):

$$l_{ij} = \left(\prod_{k=1}^{K} l_{ijk}\right)^{1/k},\ m_{ij} = \left(\prod_{k=1}^{K} m_{ijk}\right)^{1/k}, \quad (13)$$

$$u_{ij} = \left(\prod_{k=1}^{K} u_{ijk}\right)^{1/k}$$

This is a rather new approach, which is of course connected to a certain information loss. However, this loss should be tolerable, especially if the main advantage of this approach is taken into account. Isolated runaway values will not affect the results dramatically, unlike minimum amd maximum operations.

Application of the Model

To accredit the proposed model, it is implemented for 351 raw materials of the switchgear section of Energypac Engineering Limited (EEL), one of the leading power engineering companies in Bangladesh. EEL is the manufacturer of transformers (power transformers, distribution transformers, and instrumental transformers) and switchgear (outdoor vacuum circuit breakers, indoor vacuum circuit breakers, controls, metering and relay panels, low tension and power factor improvement panels, indoor-type load break switches, outdoor offload disconnectors, and by-pass switches). FAHP is used to determine the relative weights of the attributes or criteria and to classify inventories into different categories through training the data set.

Determination of Criteria

Based on the extensive literature review, experts participating in the implementation of this model have regarded 5 important criteria for classification of the inventory. Those are: unit price, annual demand, criticality, last use date, and durability.

Determination of the Weights of Criteria Using FAHP

For a multi-criteria inventory classification, a questionnaire was designed to elicit judgments about the relative importance of each of the selected criteria. The questionnaire was completed by14 experts, 3 academicians and 11 professionals including the raw material and inventory manager of EEL.

Table 2 shows the aggregated fuzzy pairwise comparisons (l_{ij}, m_{ij}, u_{ij}) using the algorithm given in Equation (11). Inconsistency of the TFN used can be checked and the CR has to be calculated using Equation (12). The results obtained are: largest eigen value of matrix, λ_{max} = 5.323; Consistency Index (CI.) = 0.08075; Randomly Generated Consistency Index (RI.) = 1.12; and CR = 0.0721. As the CR is < 0.1, the level of inconsistency present in the information stored in the comparison matrix is satisfactory. The aggregated decision matrix as shown in Table 2 is constructed to measure the relative degree of importance for each criterion, based on Chang's extent analysis. The decision matrix consist 5x5 elements:

$$S_U = (1.62, 6.51, 27) \otimes (1/123, 1/27.68, 1/9.88) = (0.013, 0.235, 2.73)$$
$$S_A = (3.34, 8.06, 29) \otimes (1/123, 1/27.68, 1/9.88) = (0.027, 0.291, 2.93)$$
$$S_C = (1.56, 4.37, 21) \otimes (1/123, 1/27.68, 1/9.88) = (0.127, 0.158, 2.13)$$
$$S_L = (1.62, 4.51, 27) \otimes (1/123, 1/27.68, 1/9.88) = (0.013, 0.163, 2.73)$$
$$S_D = (1.74, 4.23, 19) \otimes (1/123, 1/27.68, 1/9.88) = (0.014, 0.153, 1.92)$$

The degrees of possibility of superiority of S_U can be calculated by Equations (7) and (8) and is denoted by $V(S_U \geq S_A)$. Therefore, for the degree of possibility of superiority for the first requirement, the values are calculated as

$$V(S_U \geq S_A) = (0.027 - 2.73) / (0.235 - 2.73) - (0.291 - 0.027) = 0.98,$$
$$V(S_U \geq S_C) = 1, \quad V(S_U \geq S_L) = 1,$$
$$V(S_U \geq S_D) = 1,$$

For the second requirement, the values are calculated as

$$V(S_A \geq S_U) = 1, \quad V(S_A \geq S_C) = 1,$$
$$V(S_A \geq S_L) = 1, \quad V(S_A \geq S_D) = 1,$$

For the third requirement, the values are calculated as

$$V(S_C \geq S_U) = 0.98, \quad V(S_C \geq S_A) = 0.94,$$
$$V(S_C \geq S_L) = 0.99, \quad V(S_C \geq S_D) = 1,$$

For the fourth requirement, the values are calculated as

$$V(S_L \geq S_U) = 0.97, \quad V(S_L \geq S_A) = 0.95,$$
$$V(S_L \geq S_C) = 1, \quad V(S_L \geq S_D) = 1,$$

Table 2. Aggregated fuzzy comparison matrix of the attributes with respect to the overall objective

Attributes	Unit price	Annual demand	Criticality	Last use date	Durability
Unit Price	1,1,1	0.14,1.6,7	0.14,1.07,7	0.14,1.47,7	0.2,1.37,5
Annual Demand	0.14,0.62,7	1,1,1	1,3.08,9	0.2,1,5	1,2.36,7
Criticality	0.14,0.93,7	0.11,0.34,1	1,1,1	0.11,1.11,7	0.2,1,5
Last Use Date	0.14,0.68,7	0.2,1,5	0.14,0.90,9	1,1,1	0.14,0.93,5
Durability	0.2,0.73,5	0.14,0.42,1	0.2,1,5	0.2,1.08,7	1,1,1

For the fifth requirement, the values are calculated as

$V(S_D \geq S_U) = 0.96$, $V(S_D \geq S_A) = 0.93$,
$V(S_D \geq S_C) = 0.99$, $V(S_D \geq S_L) = 0.99$,

With the help of Equations (9) and (10), the minimum degree of possibility of superiority of each criterion over another is obtained. This further decides the weight vectors of the criteria.

Therefore, the weight vector is given as

W' = (0.98, 1, 0.94, 0.95, 0.93)

The normalized value of this vector decides the priority weights of each criterion overanother. The normalized weight vectors are calculated as

W = (0.204, 0.208, 0.196, 0.198, 0.194)

The normalized weight of each attribute is depicted in Figure 3. Figure 3 shows that the annual demand has a higher priority than the other criteria but the normalized weights of each attribute are very close to each other. For this the aggregated fuzzy pairwise comparisons (l_{ij}, m_{ij}, u_{ij}) using Equation (13) are given in Table 3.

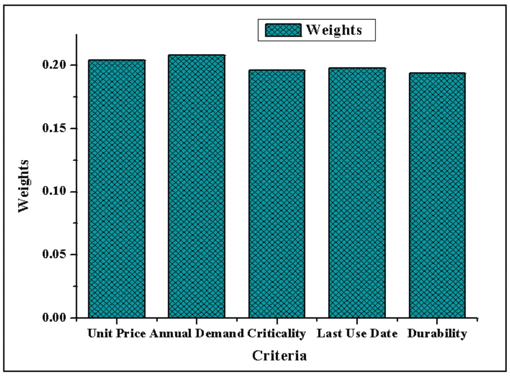

Figure 3. Normalized weights of criteria for multiple criteria inventory classification

The largest eigen value of matrix, λ_{max} = 5.418; the CI = 0.1045; the RI = 1.12; and the CR = 0.0933. As the CR is < 0.1 the level of inconsistency present in the information stored in comparison matrix is satisfactory.

$$S_U = (4.16, 6.51, 11.08) \otimes (1/42.14, 1/27.68, 1/19.13) = (0.09, 0.235, 0.58)$$
$$S_A = (5.43, 8.06, 12.76) \otimes (1/42.14, 1/27.68, 1/19.13) = (0.13, 0.291, 0.67)$$
$$S_C = (3.23, 4.38, 6.41) \otimes (1/42.14, 1/27.68, 1/19.13) = (0.077, 0.158, 0.34)$$
$$S_L = (3.4, 4.51, 6.19) \otimes (1/42.14, 1/27.68, 1/19.13) = (0.08, 0.163, 0.32)$$
$$S_D = (2.91, 4.23, 5.7) \otimes (1/42.14, 1/27.68, 1/19.13) = (0.07, 0.153, 0.30)$$

The degree of possibility of superiority of S_U is calculated and is denoted by $V(S_U \geq S_A)$. Therefore, for the degree of possibility of superiority for the first requirement, the values are calculated as

$V(S_U \geq S_A) = 0.9$, $V(S_U \geq S_C) = 1$,
$V(S_U \geq S_L) = 1$, $V(S_U \geq S_D) = 1$,

For the second requirement, the values are calculated as

$V(S_A \geq S_U) = 1$, $V(S_A \geq S_C) = 1$,
$V(S_A \geq S_L) = 1$, $V(S_A \geq S_D) = 1$,

For the third requirement, the values are calculated as

$V(S_C \geq S_U) = 0.75$, $V(S_C \geq S_A) = 0.61$,
$V(S_C \geq S_L) = 0.98$, $V(S_C \geq S_D) = 1$,

For the fourth requirement, the values are calculated as

$V(S_L \geq S_U) = 0.75$, $V(S_L \geq S_A) = 0.60$,
$V(S_L \geq S_C) = 1$, $V(S_L \geq S_D) = 1$,

Table 3. Aggregated fuzzy comparison matrix of the attributes with respect to the overall objective

Attributes	Unit price	Annual demand	Criticality	Last use date	Durability
Unit price	1,1,1	0.89,1.6,2.25	0.65,1.07,1.88	0.82,1.47,2.76	0.8,1.37,3.19
Annual demand	0.44,0.62,1.12	1,1,1	2.02,3.08,4.64	0.80,1,1.47	1.17,2.36,4.53
Criticality	0.53,0.93,1.53	0.22,0.34,0.50	1,1,1	0.68,1.11,1.66	0.80,1,1.72
Last use date	0.36,0.68,1.21	0.68,1,1.26	0.60,0.90,1.47	1,1,1	0.76,0.93,1.25
Durability	0.31,0.73,1.26	0.22,0.42,0.86	0.58,1,1.26	0.80,1.08,1.32	1,1,1

For the fifth requirement, the values are calculated as

$V\ (S_D \geq S_U) = 0.70,\quad V\ (S_D \geq S_A) = 0.55,$
$V\ (S_D \geq S_C) = 0.98,\quad V\ (S_D \geq S_L) = 0.96,$

The minimum degree of possibility of superiority of each criterion over another is obtained. This further decides the weight vectors of the criteria. Therefore, the weight vector is given as

W' = (0.9, 1, 0.61, 0.60, 0.55)

The normalized value of this vector decides the priority weights of each criterion over another. The normalized weight vectors are calculated as

W = (0.246, 0.273, 0.167, 0.164, 0.15)

The normalized weight of each success factor is depicted in Figure 4. Figure 4 shows that the annual demand has a higher priority than the other criteria. The weights of the criteria represent the ratio of how much more important 1 criterion is than another, with respect to the goal or criterion at a higher level.

Table 4. Transformation of last use date

Range	Value
Used within a day	10
Used within a week	8
Used within a month	6
Used within 6 month	4
Used within a year	2
Used more than a year	1

Table 5. Transformation of criticality

Range	Value
Extremely Critical	5
Moderate Critical	3
Non Critical	1

Table 6. Transformation of durability

Mean Time Between Failure	Value
> 1 Week	10
> 1 Month	8
> 6 Month	5
> 1 year	3
< 1 year	1

Data Collection

The unit price, last year consumption or annual demand, last use date, criticality, and durability of 351 materials of the switchgear section have been collected. The range and value for the transformation of last use date, criticality, and durability are shown in Tables 4-6.

Determination of Composite Priority Weights

In FAHP methodology, for a very large number of alternatives (351), making pairwise comparisons of alternatives, with respect to each criterion, can be time consuming and confusing, because the total number of comparisons will also be very big. Therefore, multiple-criteria inventory classification is carried out by using the modified FAHP methodology, which includes pairwise comparisons of criteria, but not pairwise comparisons of alternatives. Because of the large number of alternatives (351), pairwise comparisons of the alternatives are not performed.

Finally, the composite priority weights of each alternative can be calculated by multiplying the weights of each alternative by the data of the corresponding criteria. The composite priority weight of the alternatives gives an idea about the appropriate class of the alternatives or items. Items are ranked according to overall composite priority weights in descending order. The limits for the classes are derived on the following basis: Class A involves 70% of the total composite priority weights; Class B involves 20% of the total composite priority weights; while

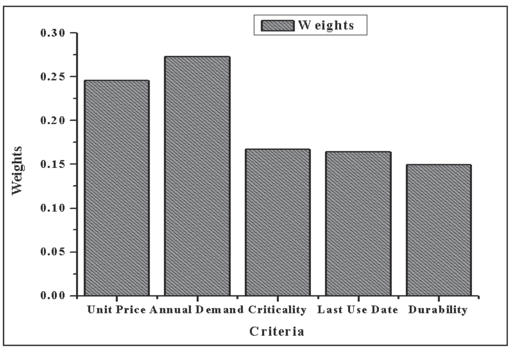

Figure 4. Normalized weights of criteria for multiple criteria inventory classification

10% of the total composite priority weights belong to class C. The results of the study show that, among the 351 items, 22 items are identified as class A or the very important group or are outstandingly important, 47 items as class B or the important group or are of average importance, and the remaining 282 items as class C or the unimportant group or are relatively unimportant as a basis for a control scheme using the Modified FAHP.

Discussions

Fuzzy linguistic terms have been employed for facilitating the comparisons between the subject criteria, since the decision makers feel more comfortable with using linguistic terms rather than providing exact crisp judgments. Using Chang's extent analysis, the normalized weight of each attribute is depicted as shown in Figures 3 and 4 using the conventional FAHP and Modified FAHP, respectively. Both Figures show that the annual demand has a higher priority (0.208 and 0.273 respectively) than the other criteria. The composite priority weight of each alternative has been calculated using the conventional FAHP and Modified FAHP methodology. The composite priority weight of the alternatives gives an idea about the appropriate class of the alternatives or items. Class A involves 70% of the total composite priority weights, Class B involves 20% of the total composite priority weights, while 10% of the total composite priority weights belong to class C.

The results of the comparative analysis of the conventional FAHP and Modified FAHP are given in Table 7. Using the conventional FAHP and Modified FAHP models, the control scheme or appropriate class of 351 raw materials of the switchgear section have been determined. It can be noted that different items belong to different classes or control scheme using the different models. Although the division of the number of items into different classes is almost equal, due to the conventional FAHP different items belong to the wrong classes or control scheme.

Conclusions

Multi-class classification utilizing multiple-criteria requires techniques capable of providing accurate classification and processing of a large number of inventory items. In this research, a new multi-criteria inventory classification model has been proposed using the Modified FAHP approach. The Modified FAHP technique was used to synthesize the opinions of the decision makers to identify the weight of each criterion. The Modified FAHP approach proved to be a convenient method in tackling practical multi-criteria decision making problems. It demonstrated the advantage of being able to capture the vagueness of human thinking and to aid in solving the research problem through a structured manner and a simple process.

The classification system is very flexible in the sense that the user:

a) can incorporate some other criteria or remove any criteria for his/her specific implementation;

b) can conduct different classification analyses for different inventory records;

c) can employ an application-specific linguistic variable set;

d) can substitute the crisp comparison values a_{ij} for the fuzzy comparison values a_{ij} in the optimization program, whenever the fuzzy

Table 7. Comparative analysis of conventional FAHP and Modified FAHP for multi-criteria inventory classification

Model	Class A Very important or outstandingly important	Class B Important or average important	Class C Unimportant or relatively unimportant	Total items
Conventional FAHP	21	45	285	351
Modified FAHP	22	47	282	

comparisons are not available.

Further development of the FAHP application could be for improvement in the determination of the weights of each component and for handling the uncertainty level of the decision environment by using hybrid neuro-fuzzy models, like the quick fuzzy backpropagation algorithm.

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