HEAT AND MASS NATURAL-CONVECTIVE FLOW OF MICROPOLAR AND VISCOUS FLUIDS THROUGH A POROUS MEDIUM IN A VERTICAL CHANNEL

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Received: Oct 17, 2011; Revised: Oct 18, 2011; Accepted: Nov 30, 2011

Abstract

The problem of fully-developed natural-convective heat and mass transfer through a porous medium in a vertical channel is investigated analytically. One region is filled with a micropolar fluid and the other region with a viscous fluid or both regions are filled with viscous fluids. Using the boundary and interface conditions, the expressions for linear velocity, micro-rotation velocity, temperature, and mass have been obtained. Numerical results are presented graphically for the distribution of velocity, micro-rotation velocity, temperature, and mass fields for various values of physical parameters such as the ratio of the Grashof number to Reynolds number, viscosity ratio, channel width ratio, conductivity ratio, and micropolar fluid material parameter. It is found that the effect of the micropolar fluid material parameter suppresses the velocity whereas it enhances the micro-rotation velocity. The effect of the ratio of the Grashof number to Reynolds number is found to enhance both the linear velocity and the micro-rotation velocity. The effects of the width ratio and the conductivity ratio are found to enhance the temperature distribution.

Keywords: Natural-convection, micropolar fluid, porous medium

Introduction

The research area of micropolar fluids has been of great interest because the Navier-Stokes equations for Newtonian fluids cannot successfully describe the characteristic of fluid with suspended particles. There exist several approaches to study the mechanics of fluids with a substructure. Ericksen (1960 a and b) derived field equations which account for the presence of substructures in the fluid. It has been experimentally demonstrated by Hoyt and Fabula (1964) and Vogel and Patterson (1964) that fluids containing a small amount of polymeric additives display a reduction in skin friction. Eringen (1966) formulated the theory of micropolar fluids which display the effects of local rotary inertia and couple stresses. This theory can be used to explain the flow of colloidal fluids, liquid crystal, animal blood, etc. Eringen (1972) extended the micropolar fluid theory and developed the theory of thermomicropolar fluids. Extensive reviews of

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the theory and application can be found in the review articles by Ariman *et al.* (1974) and the recent books by Lukaszewicz (1999) and Eringen (2001).

Physically, micropolar fluids may be described as non-Newtonian fluids consisting of dumb-bell molecules or a short rigid cylindrical element, polymer fluids, fluid suspension, etc. The presence of dust or smoke, particularly in a gas, may also be modeled using micropolar fluid dynamics. The theory of micropolar fluids first proposed by Eringen (1966) is capable of describing such fluids.

Studies of external convective flows of micropolar fluids have focused mainly on free, forced and mixed convection problems. Applications are found in a variety of engineering problems, such as air conditioning of a room, solar energy collecting devices, material processing, and passive cooling of nuclear reactors. Studies of the flows of heat convection in micropolar fluids have focused mainly on a flat plate Ahmadi (1976); Jena and Mathur (1982); Ÿucel (1989); and Rahman *et al.* (2009) or on regular surfaces Balram and Sastry (1973); Lien *et al.* (1990). Chamkha *et al.* (2002) analyzed numerical and analytical solutions of the developing laminar free convection of a micropolar fluid in a vertical parallel plate channel with asymmetric heating. The subject of 2-fluid flow and heat transfer has been extensively studied due to its importance in the chemical and nuclear industries. The design of a 2-fluid heat transport system for space application requires knowledge of heat and mass transfer processes and fluid mechanics under reduced gravity conditions. Identification of the 2-fluid flow region and determination of the pressure drop, void fraction, quality reaction, and 2-fluid heat transfer coefficient are of great importance for the design of 2-fluid systems. Lohsasbi and Sahai (1988) studied 2-phase magnetohydrodynamic (MHD) flow and heat transfer in a parallel plate channel with the fluid in 1 phase being electrically conducted. Malashetty and Leela (1992) have analyzed the Hartmann flow characteristic of 2 fluids in a horizontal channel. The study of 2-phase flow and heat transfer in an inclined channel has been studied by Malashetty and Umavathi (1997) and Malashetty *et al.* (2001). Fully-developed free-convective flow of micropolar and viscous fluids in a vertical channel was investigated by Kamar *et al.* (2009). Kumar and Gupta (2009) considered the unsteady MHD and heat transfer of 2 viscous immiscible fluids through a porous medium in a horizontal channel.

The aim of this paper is to investigate the fully-developed heat and mass natural-convective flow through a porous medium in a vertical channel with an asymmetric wall temperature distribution.

Formulation of the Problem

The geometry under consideration illustrated in Figure 1, consists of 2 infinite vertical parallel walls maintained at different or equal constant temperatures extending in x^* - and z^* - directions. y^* - direction is taken as normal to the nonconducting walls. The region $-h_1 \le y^* \le 0$ is occupied by micropolar fluid of density ρ_1, viscosity μ_1, vortex viscosity k, thermal conductivity k_1, and thermal expansion coefficient β_1, and the region $0 \le y^* \le h_2$ is occupied by viscous fluid of density ρ_2, viscosity μ_2, thermal conductivity k_2, and thermal expansion coefficient β_2.

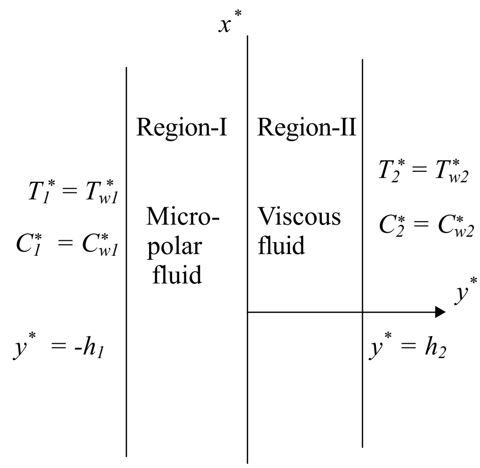

Figure 1. Geometrical configuration

The fluids are assumed to have constant properties except the density in the buoyancy terms $\rho_1 = \rho_0[1-\beta_1(T_1^* - T_0^*)]$ and $\rho_2 = \rho_0[1-\beta_2(T_2^* - T_2^*)]$ in the momentum equation, where T_0^* is the mean temperature. Let us assume that the walls of the channel are isothermal, in particular the temperature and concentration of the boundary at $y^* = -h_1$ is T_{w1}^* and C_{w1}^*, while the temperature at $y^* = h_2$ is T_{w2}^* and C_{w2}^* with $T_{w2}^* \leq T_{w1}^*$ and $C_{w2}^* \leq C_{w1}^*$. A fluid rises in the channel driven by buoyancy forces. The transport properties of both the fluids are assumed to be constant. It should be mentioned here that the micropolar and viscous fluids are immiscible (that is, no mixing between the fluids exists) and the constitutive equations for micropolar and viscous fluids are different. Also, the viscosity of both fluids is different. For instant, Synovial fluid, which is a clear thixotropic lubrication fluid, is a good example of a micropolar fluid and water is a good example of a viscous fluid and it is well known that a Synovial fluid and water cannot be mixed. Since our model is general, one can choose any 2 different fluids which are immiscible.

It is assumed that the only non-zero component of the velocity $\vec{q}$ along x^* direction is. $u_i^* (i = 1,2)$ Thus, as a consequence of the mass balance equation, we obtain

$$\frac{\partial u_i^*}{\partial x^*} = 0 \tag{1}$$

so that $u_i^*(1,2)$ depends only on y^*.

Under these assumptions, the momentum, energy, and mass equations are given by

Region-I

$$(\mu_1 + k)\frac{d^2u_1^*}{dy^{*2}} + k\frac{d\omega^*}{dy^*}\rho_1 g\beta_{1T}(T_1^* - T_0^*) - \frac{\mu_1^*}{K^*}u_1^* + \rho_1 g\beta_{1C}(C_1^* - C_0^*) = 0 \tag{2}$$

$$\text{y}\frac{d^2\omega^*}{dy^{*2}} - k\left(2\omega^* + \frac{du_1^*}{dy^*}\right) = 0 \tag{3}$$

$$\frac{d^2T_1^*}{dy^{*2}} = 0 \tag{4}$$

$$\frac{d^2C_1^*}{dy^{*2}} = 0 \tag{5}$$

Region-II

$$\mu_2\frac{d^2u_2^*}{dy^{*2}} + \rho_2 g\beta_{2T}(T_2^* - T_0^*) - \frac{\mu_2^*}{K^*}u_2^* + \rho_2 g\beta_{2C}(C_2^* - C_0^*) = 0 \tag{6}$$

$$\frac{d^2T_2^*}{dy^{*2}} = 0 \tag{7}$$

$$\frac{d^2C_2^*}{dy^{*2}} = 0 \tag{8}$$

where ω^* is the component of micro-rotation vector normal to the plane $x^* y^*$, g the acceleration due to gravity, σ the coefficient of electrical conductivity, K^* the permeability of porous medium, and γ the spin gradient viscosity. To solve the above set of differential equations from (2) to (6), 6 boundary conditions are required for velocity, 4 boundary conditions for temperature, and 4 boundary conditions for mass. The first 2 boundary conditions are obtained from the fact that there is no slip near the wall. The next condition is obtained by assuming the continuity of velocity and the last 4 conditions are obtained from the equality of stresses at the interface and constant cell rotational velocity at the interface as proposed by Ariman *et al.* (1973). Thus, the appropriate boundary and interface conditions on velocity in the mathematical form are

$$u_1^* = 0 \; at \; y^* = -h_1, \; u_2^* = 0 \; at \; y^* = h_2, \; u_1^*(0) = u_2^*(0)$$

$$(\mu_1 + k)\frac{du_1^*}{dy^*} + k\omega^* = \mu_2\frac{du_1^*}{dy^*} \; at \; y^* = 0 \tag{9}$$

$$\frac{d\omega^*}{dy^*} = 0 \; at \; y^* = 0, \; \omega^* = 0 \; at \; y^* = -h_1$$

For the corresponding temperature and mass boundary conditions, it is assumed that the temperatures and heat fluxes are continuous at the interface

$$T_1^* = T_{w1}^* \; at \; y^* = -h_1, \; T_2^* = T_{w2}^* \; at \; y^* = h_2,$$

$$T_1^*(0) = T_2^*(0), \; \kappa_1\frac{dT_1^*}{dy^*} = \kappa_2\frac{dT_2^*}{dy^*} \; at \; y^* = 0 \tag{10}$$

$$C_1^* = C_{w1}^* \; at \; y^* = -h_1, \; C_2^* = C_{w2}^* \; at \; y^* = h_2,$$

$$C_1^*(0) = C_2^*(0), \; \kappa_1\frac{dC_1^*}{dy^*} = \kappa_2\frac{dC_2^*}{dy^*} \; at \; y^* = 0$$

Also we assume that

$$\gamma=\left(\mu_1+\frac{k}{2}\right)j=\left(\mu_1+\frac{k_1}{2}\right)j \tag{11}$$

where j is the micro-inertia density and $k_1 = k/\mu_1$ is the micropolar fluid material parameter of Region-I. We notice that $k_1 = 0$ describes the case of a viscous or Newtonian fluid. Relation (11) expresses the fact that the micropolar fluid field can predict the correct behavior in the limiting case when the micro-structure effects become negligible and total spin reduces to the angular flow velocity or flow vorticity. Relation (9) was established by Ahmadi (1976) and Kline (1977) and it has been used by many researchers, for example Rees and Bassom (1996), Gorla (1988), and Rees and Pop (1998).

Method of Solution

We introduce the following non-dimensional quantities

$$y_i=\frac{y^*}{h_i},\ u_i=\frac{u_i^*}{U_0},\ \theta_i=\frac{T_i^*-T_0^*}{\Delta T},$$
$$\phi_i=\frac{C_i^*-C_0^*}{\Delta C},\ \Omega=\frac{h_1}{U_0}\omega^*,\ \mathrm{K}=\frac{K^*}{h_I^2},$$
$$Gr_T=\frac{g\beta_{1T}\Delta T h_I^3}{v_1^2},\ Gr_C=\frac{g\beta_{1C}\Delta C h_I^3}{v_1^2}, \tag{12}$$
$$\mathrm{Re}=\frac{U_0h_1}{v_1},\ GR_T=\frac{Gr_T}{\mathrm{Re}},$$
$$GR_C=\frac{Gr_C}{\mathrm{Re}},\ k_1=\frac{k}{\mu_1}$$

where Gr is the Grashof number, Re the Reynolds number, GR the mixed convection parameter, $j = h_I^2$ the characteristic length, and ΔT and ΔC the characteristic temperature and concentration which are defined as $\Delta T = T_{w1}^* - T_{w2}^*\ \ if\ T_{w1}^* > T_{w2}^*$ and $\Delta C = C_{w1}^* - C_{w2}^*\ \ if\ C_{w1}^* > C_{w2}^*$ respectively.

Using (12), Equations (2) to (6) in non-dimensional form become

Region-I

$$\frac{d^2u_1}{dy^2}-\frac{1}{K(1+k_1)}u_1+\frac{k_1}{1+k_1}\frac{d\Omega}{dy}=-\frac{GR_T}{1+k_1}\theta_1-\frac{GR_C}{1+k_1}\phi_1 \tag{13}$$

$$\frac{d^2\Omega}{dy^2}-\frac{2k_1}{2+k_1}\left(2\Omega+\frac{du_1}{dy}\right)=0 \tag{14}$$

$$\frac{d^2\theta_1}{dy^2}=0 \tag{15}$$

$$\frac{d^2\phi_1}{dy^2}=0 \tag{16}$$

Region-II

$$\frac{d^2u_2}{dy^2}-\frac{h^2}{K}u_2=-mb\rho h^2GR_T\theta_2-mb\rho h^2GR_C\phi_2 \tag{17}$$

$$\frac{d^2\theta_2}{dy^2}=0 \tag{18}$$

$$\frac{d^2\phi_2}{dy^2}=0 \tag{19}$$

with the boundary conditions

$$u_1(-1)=0,\ u_2(1)=0,\ u_1(0)=u_2(0),$$
$$\frac{du_1(0)}{dy}+\frac{k_1}{1+k_1}\omega(0)=\frac{1}{mh(1+k_1)}\frac{du_2(0)}{dy},$$
$$\frac{d\omega(0)}{dy}=0,\ \omega(-1)=0,$$
$$\theta_2(-1)=\frac{T_{w1}^*-T_0^*}{\Delta T}=m_1,\ \theta_2(1)=\frac{T_{w2}^*-T_0^*}{\Delta T}=m_2, \tag{20}$$
$$\theta_1(0)=\theta_2(0),\ \frac{d\theta_1}{dy}=\frac{1}{h\kappa}\frac{d\theta_2}{dy}$$
$$\phi_1(-1)=\frac{C_{w1}^*-C_0^*}{\Delta C}=n_1,\ \phi_2(1)=\frac{C_{w2}^*-C_0^*}{\Delta C}=n_2,$$
$$\phi_1(0)=\phi_2(0),\ \frac{d\phi_1}{dy}=\frac{1}{h\kappa}\frac{d\phi_2}{dy}$$

where

$$h=\frac{h_1}{h_2},\ \mathrm{m}=\frac{\mu_1}{\mu_{21}},\ \kappa=\frac{\kappa_1}{\kappa_2},\ \rho=\frac{\rho_1}{\rho_2},\ and\ b=\frac{\beta_1}{\beta_2}$$

are the channel width ratio, viscosity ratio, thermal conductivity ratio, density ratio, and thermal expansion ratio, respectively.

Solution

On solving coupled linear differential Equations from (13) to (19) under boundary and interface conditions (20), we have the solutions

$$\theta_1 = c_1 y + c_2 \quad (21)$$

$$\phi_1 = c_3 y + c_4 \quad (22)$$

$$\theta_2 = c_5 y + c_6 \quad (23)$$

$$\phi_2 = c_7 y + c_8 \quad (24)$$

$$c_1 = -\frac{1}{1+h\kappa},\ c_2 = \frac{h\kappa}{1+h\kappa},$$

$$c_5 = -\frac{h\kappa}{1+h\kappa},\ c_6 = \frac{h\kappa}{1+h\kappa}$$

Region I

$$\Omega(y) = -\frac{1}{2}c_1 KGR_T - \frac{1}{2}c_3 KGR_C + c_{11}e^{-\sqrt{\frac{L_1-\sqrt{R}}{2L_2}}y} + c_{12}e^{\sqrt{\frac{L_1-\sqrt{R}}{2L_2}}y} + c_{13}e^{-\sqrt{\frac{L_1+\sqrt{R}}{2L_2}}y} + c_{14}e^{\sqrt{\frac{L_1+\sqrt{R}}{2L_2}}y} \quad (25)$$

$$u_1(y) = \frac{1}{8L_2k_1}\left(L_4 y + L_5 + L_6 e^{\sqrt{\frac{L_1-\sqrt{R}}{2L_2}}y} + L_7 e^{-\sqrt{\frac{L_1-\sqrt{R}}{2L_2}}y}\right)$$

Region II

$$u_2(y) = c_9 e^{\frac{h}{\sqrt{K}}y} + c_{10} e^{\frac{h}{\sqrt{K}}y} + mb\rho KGR_C (c_7 y + c_8) + mb\rho KGR_T (c_5 y + c_6) \quad (26)$$

$$R = 4 + 4k_1 - 16Kk_1 - 32Kk_1^2 + k_1^2 - 12Kk_1^3 + 16K^2k_1^2 + 16K^2k_1^3 + 4K^2k_1^4$$

where $L_1 = 2 + k_1 + 4Kk_1 + 2Kk_1^2$

$L_2 = K(2 + k_1)(1 + k_1)$

$L_3 = 8 + 8k_1 - 32Kk_1 - 64Kk_1^2 + 2k_1^2 - 24Kk_1^3 + 32K^2k_1^2 + 32K^2k_1^3 + 8K^2k_1^4$

$L_4 = 24k_1^2GR_CK^2c_3 + 8k_1^3GR_CK^2c_3 + 24k_1^2GR_TK^2c_1 + 8k_1^3GR_TK^2c_1 + 16k_1GR_TK^2c_1 + 16k_1GR_CK^2c_3$

$L_5 = 24k_1^2GR_TK^2c_2 + 8k_1^3GR_TK^2c_2 + 24k_1^2GR_CK^2c_4 + 8k_1^3GR_CK^2c_4 + 16k_1GR_TK^2c_2 + 16k_1GR_CK^2c_4$

$$L_6 = -4c_{17}Kk_1\sqrt{2L_2(L_1+\sqrt{R})} + c_{16}k_1\sqrt{2L_2(L_1-\sqrt{R})} - 2c_{17}Kk_1^2\sqrt{2L_2(L_1+\sqrt{R})} - 4c_{16}Kk_1\sqrt{2L_2(L_1-\sqrt{R})} + c_{17}\sqrt{L_2L_3(L_1+\sqrt{R})} + 2c_{16}\sqrt{2L_2(L_1+\sqrt{R})} + 2c_{17}\sqrt{2L_2(L_1+\sqrt{R})} + c_{16}\sqrt{L_2L_3(L_1-\sqrt{R})} + c_{17}k_1\sqrt{2L_2(L_1+\sqrt{R})} - 2c_{16}k_1^2K\sqrt{2L_2(L_1-\sqrt{R})}$$

$$L_7 = c_{15}\sqrt{L_2L_3(L_1-\sqrt{R})} - 2c_{18}\sqrt{2L_2(L_1+\sqrt{R})} + 4c_18Kk_1\sqrt{2L_2(L_1+\sqrt{R})} - c_{18}\sqrt{L_2L_3(L_1+\sqrt{R})} + k_1c_{15}\sqrt{2L_2(L_1-\sqrt{R}-)} - 2c_{15}\sqrt{L_2(L_1-\sqrt{R})} - c_{18}k_1\sqrt{2L_2(L_1+\sqrt{R})} + 4c_{15}Kk_1\sqrt{2L_2(L_1-\sqrt{R})} + 2c_{15}k_1^2K\sqrt{2L_2(L_1-\sqrt{R})} + 2c_{18}k_1^2K\sqrt{2L_2(L_1+\sqrt{R})}$$

where c_i constants are constants of integration, not included here for the sake of brevity.

Limiting Case

For a Newtonian fluid $k_1 = 0$, the solution of Equations (13) and (19) using boundary and interface conditions (20) are

$$u_2(y) = c_9' e^{\frac{h}{\sqrt{k}}y} + c_{10}' e^{\frac{h}{\sqrt{k}}y} + mb\rho KGR_C (c_7 y + c_8) + mb\rho KGR_T(c_5 y + c_6) \quad (27)$$

$$u_1(y) = c_{15}' e^{\frac{1}{\sqrt{k}}y} + c_{16}' e^{-\frac{1}{\sqrt{k}}y} + KGR_C (c_3 y + c_4) + KGR_T(c_1 y + c_2) \quad (28)$$

where c_i' constants are constants of integration, not included here for the sake of brevity.

Results and Discussion

An analytical solution for the problem of heat and mass fully-developed natural-convective flow of micropolar and viscous fluids through a porous medium in a vertical channel is investigated. The analytical solutions are evaluated numerically for different values of governing parameters and the results are presented through a graph by assuming that, at the second wall, the temperature is alike to the mean temperature, i.e. $T_{w2}^* \approx T_0^*$, so that $m_1, n_1 \to 1$ and $m_2, n_2 \to 0$.

The effect of the mixed convection parameter or the Grashof to Reynolds numbers ratio GR_T and GR_C on the linear velocity and micro-rotation velocity are shown in Figures 2 and 5, respectively. An increase in the mixed convection parameter means an increase of the buoyancy force which supports the motion. It is also observed from Figure 2 that if the micropolar fluid is replaced by the clear viscous fluid, the effect of the mixed convection parameter GR_T and GR_C is still retained. But the magnitude of promotion is large for a viscous–viscous fluids system compared with a micropolar–viscous fluids system. Figures 3 and 5 show the effect of the mixed convection parameter on micro-rotation velocity. It is observed from the figure that an increase of buoyancy force reduces the magnitude of micro-rotation velocity.

Figures 6 and 7 display the effect of the viscosity ratio $m = \mu_1/\mu_2$ on the linear velocity and micro-rotation velocity, respectively. As the viscosity ratio m increases, the linear velocity increases, but the magnitude of promotion is large for $k_1 = 0$ (Newtonian fluid) compared with $k_1 = 1$ (micro-polar fluid). The effect of the viscosity ratio m is found to reduce the micro-rotational velocity.

The effect of the channel width ratio h on the linear velocity and micro-rotation velocity is shown in Figures 8 and 9, respectively. As the width ratio h increases, both the linear velocity and the micro-rotation velocity increase for viscous–viscous $k_1 = 0$ and micropolar–viscous $k_1 = 1$ fluids systems. The effect of the width ratio h is also found to promote the temperature and mass fields as seen in Figures 15 and 17, respectively.

The effect of the permeability parameter K on the linear velocity and micro-rotational velocity is presented through Figures 10 and 11, respectively. It is clear from Figure 10 that an increase in the permeability parameter promotes the linear velocity. It is also observed that if the micropolar fluid is replaced by the clear viscous fluid, the effect of the permeability parameter is still maintained, but the magnitude of promotion is large for the viscous-viscous fluids system compared with the micropolar-viscous fluids system. Figure 11 shows the effect of the permeability parameter on micro-rotation velocity. It is observed from the figure that an increase of the permeability parameter reduces the magnitude of micro-rotation velocity.

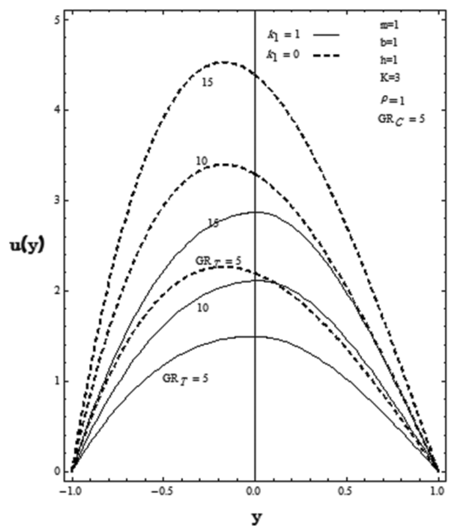

Figure 2. Velocity distribution for different values of GR_T

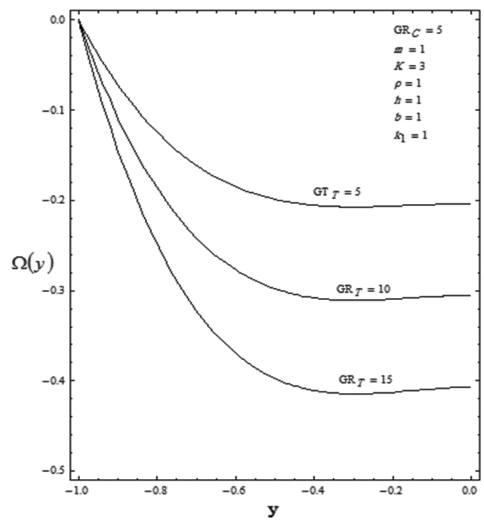

Figure 3. Micro-rotational velocity distribution for different values of GR_T

The effect of the conductivity ratio κ on the linear velocity and micro-rotational velocity are presented through Figures 12 and 13, respectively. The effect of the conductivity ratio is predicted to increase the linear velocity for the micropolar-viscous and viscous-viscous (Figure 12) fluids systems, but the effect of the conductivity ratio κ is found to reduce the micro-rotational velocity as seen in Figure 13.

The effects of the conductivity ratio κ on the temperature and mass fields are shown in Figures 14 and 16, respectively. The effect of the conductivity ratio κ is predicted to increase both the temperature and mass fields, i.e. the larger the conductivity of the micropolar fluid compared with the viscous fluid, the larger the flow nature.

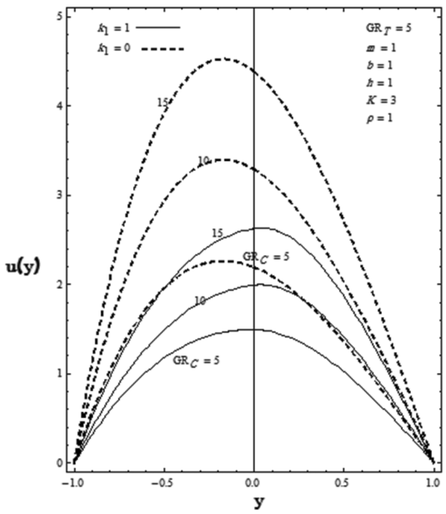

Figure 4. Velocity distribution for different values of GR_C

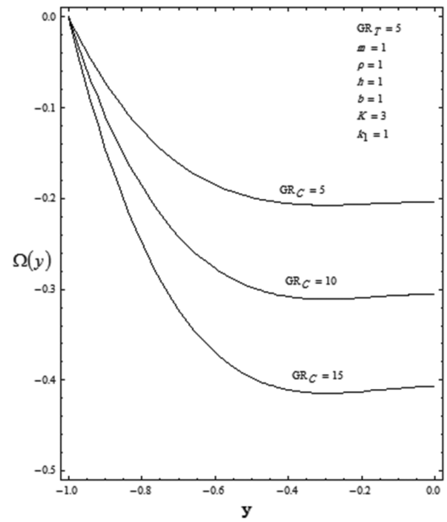

Figure 5. Micro-rotational velocity distribution for different values of GR_C

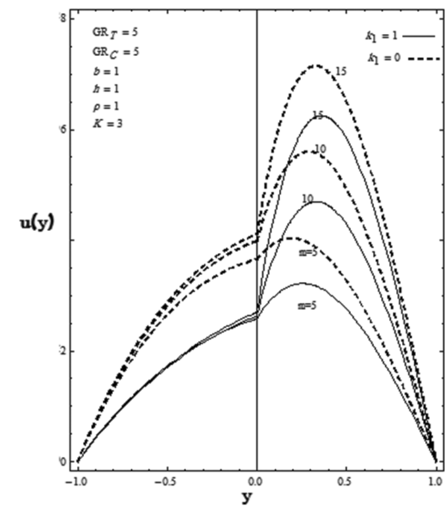

Figure 6. Velocity distribution for different values of viscosity ratio m

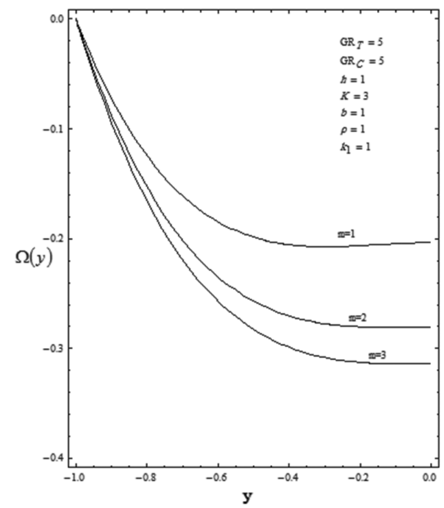

Figure 7. Micro-rotation velocity distribution for different values of viscosity ratio m

Conclusions

There was considered the fully-developed laminar natural-convective flow through a porous medium in a vertical channel in which 1 region is filled with a micropolar fluid and the other region with a viscous fluid. It is found that the effects of the Grashof to Reynolds number ratio, channel width ratio, conductivity ratio, and permeability parameter are to promote the linear velocity, whereas the micropolar fluid material parameter suppressed the velocity. Further, the Grashof to Reynolds number ratio, channel width ratio, conductivity ratio, micropolar fluid material parameter, and permeability parameter repressed the micro-rotational velocity. The effect of the width and conductivity ratio parameters promotes the temperature and mass fields.

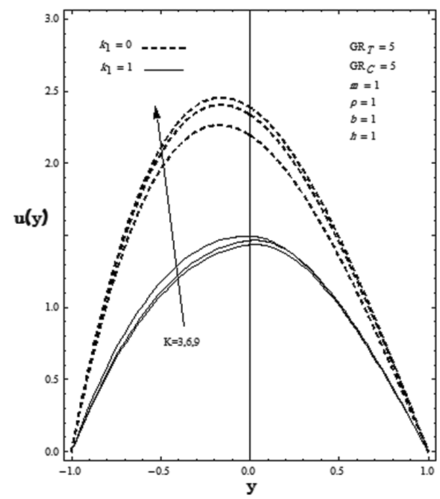

Figure 8. Velocity distribution for different values of width ratio h

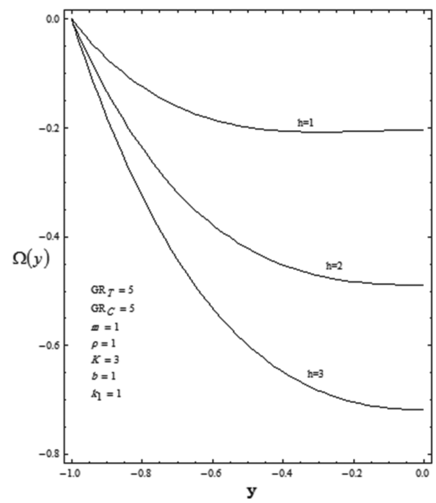

Figure 9. Micro-rotational velocity distribution for different values of width ratio h

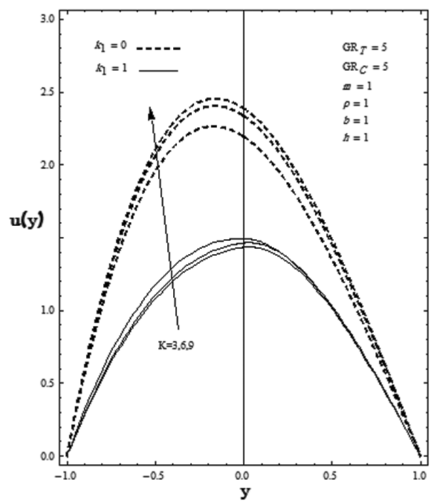

Figure 10. Velocity distribution for different values of permeability parameter K

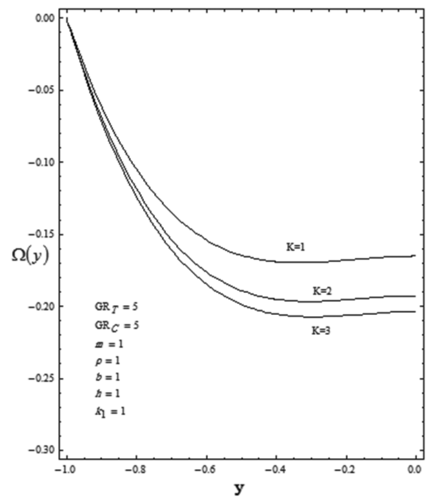

Figure 11. Micro-rotational velocity distribution for different values of permeability parameter K

Nomenclature

b	thermal expansion coefficient ratio, β_2/β_1
C	concentration
g	acceleration due to gravity
Gr_T	Grashof number for heat transfer
Gr_c	Grashof number for mass trasnfer
GR_T	Grashof to Reynolds numbers ratio for heat transfer, Gr_T/R_e
GR_C	Grashof to Reynolds numbers ratio for mass transfer, Gr_C/R_e
h	channel width ratio, h_1 / h_2
h_1	Height of Region-I
h_2	Height of Region-II
j	micro-inertia density
κ	ratio of thermal conductivities, κ_1/κ_2
κ_1	thermal conductivity of the fluid in Region-I
κ_2	thermal conductivity of the fluid in Region-II

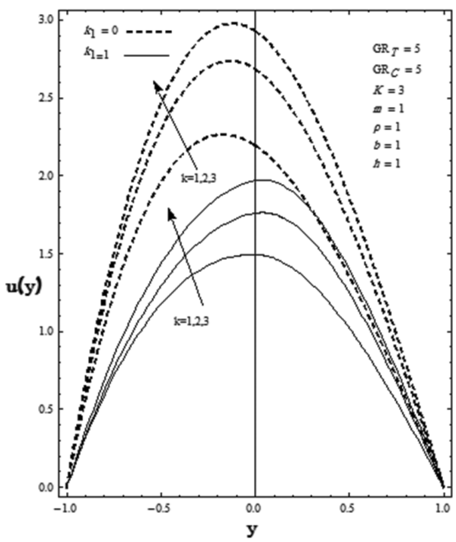

Figure 12. Velocity profiles for different values of conductivity ratio κ

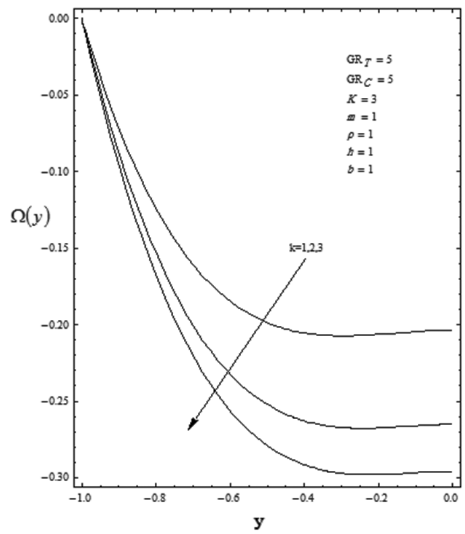

Figure 13. Micro-rotational velocity profiles for different values of conductivity ratio κ

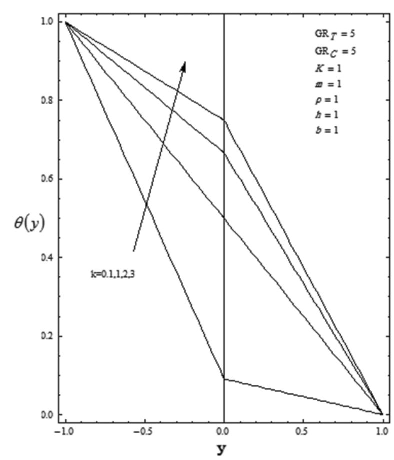

Figure 14. Temperature profiles for different values of conductivity ratio κ

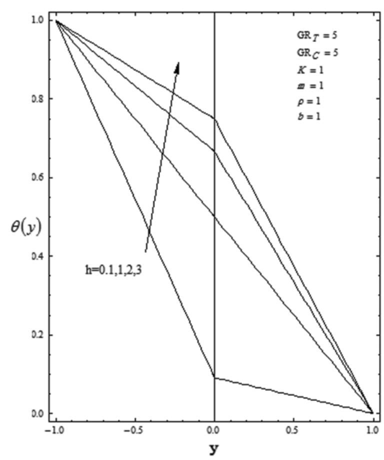

Figure 15. Temperature profiles for different values of width ratio h

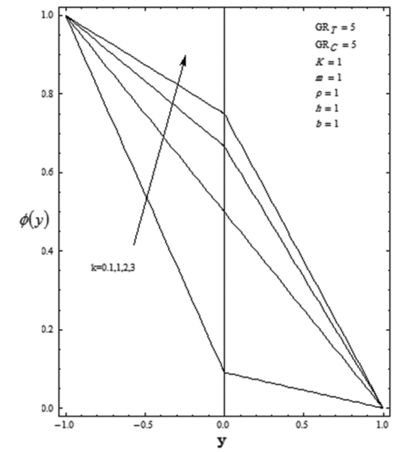

Figure 16. Mass profiles for different values of conductivity ratio κ

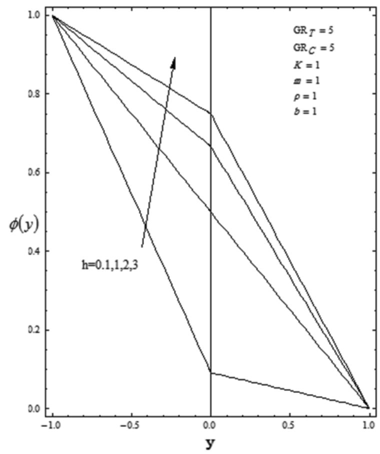

Figure 17. Mass profiles for different values of width ratio h

K permeability parameter
k_1 micropolar fluid material parameter
m ratio of viscosities, μ_1/μ_2
Re Reynolds number
T_0 average temperature
T temperature
T_1, T_2 temperature of the boundaries
U_0 average velocity
U velocity
x^*, y^* space coordinates

Greek letters

β coefficient of thermal expansion
γ spin gradient viscosity
Ω micro-rotational velocity
μ viscosity
ρ_1 density of Region-I
ρ_2 density of Region-II
ρ ratio of densities, ρ_1/ρ_2
ΔT difference in temperature
θ_t dimensionless temperature
ϕ_t dimensionless mass

Subscript

1,2 reference quantities for Region-I and Region-II, respectively
w condition at the wall

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