RESEARCH ARTICLE
doi: 10.2306/scienceasia1513-1874.2009.35.196

Robust motion estimation methods using gradient orientation information

Toshiaki Kondo*, **Waree Kongprawechnon**

Sirindhorn International Institute of Technology, Thammasat University, Pathumthani 12000, Thailand

*Corresponding author, e-mail: tkondo@siit.tu.ac.th

Received 7 Nov 2007
Accepted 23 Mar 2009

ABSTRACT: Changing lighting conditions cause temporal variations of image intensities and make most existing motion estimation techniques ineffective. As a solution to this problem, we use gradient orientation information, which depends very little on changes of image intensities, in place of commonly used image features such as intensities and gradients. By employing gradient orientation, two conventional motion estimation techniques (the spatio-temporal gradient method and the gradient structure tensor method) can be transformed into methods that are far more robust to changing image intensities. Simulation results on both synthetic and real image sequences show that the proposed methods perform motion estimation remarkably well irrespective of time-varying image intensities. In addition, the proposed methods also cope better with the aperture problem in which only unidirectional gradients are available and large erroneous motion estimates are produced.

KEYWORDS: motion vectors, optical flow, spatio-temporal gradient method, gradient structure tensor

INTRODUCTION

Motion estimation is an important task in the fields of image sequence processing and computer vision. It has various applications, including object-based video coding (e.g. MPEG-4), object detection and tracking, scene change detection for video editing, image stabilization for camcorders, and dynamic 3-d scene analysis for autonomous navigation. Motion estimation techniques in the spatial domain may be classified as being either gradient-based or correlation-based methods (also commonly referred to as block matching or template matching)[1]. Gradient-based techniques can be further divided into spatio-temporal gradient methods (often simply referred to as gradient methods)[1–3] and gradient structure tensor methods (also referred to as gradient square tensor methods or 3-d structure tensor methods)[1,4–7]. Gradient-based methods are in general used to obtain a dense optical flow field or motion vectors. These techniques are effective especially when the displacement between images over time is small, typically a few pixels. On the other hand, correlation-based methods may be the most intuitive approach as they search for similar patterns between two images[1,8–10]. Since they can handle larger displacements, they are used not only for motion estimation, but also for establishing correspondence between images captured at different viewpoints such as stereo vision. They are, however, not suitable for computing dense motion vectors owing to their high computational cost.

This paper is concerned with gradient-based methods. They are based on the optical flow constraint equation which assumes that image intensities are constant along motion trajectories. This assumption, however, is often violated by changes of lighting conditions that is a common occurrence in outdoor environments. To circumvent this varying illumination problem, it is reasonable to employ a feature that is less dependent on image intensities. Gradient orientation (or direction) is an attractive feature because it is not sensitive to variations in illumination[11–14]. In this paper, we propose the use of gradient orientation information, instead of the commonly used image features such as intensities and gradients, to achieve a robust performance for motion estimation.

OPTICAL FLOW CONSTRAINT EQUATION

An optical flow field, showing dense motion vectors within an image, is generally estimated under the assumption that the image intensities of an object are constant over time. By assuming that the image intensities I at points (x, y) at time t are constant over time $t + \mathrm{d}t$, we have

$$I(x, y, t) = I(x + \mathrm{d}x, y + \mathrm{d}y, t + \mathrm{d}t), \qquad (1)$$

where $\mathrm{d}x$ and $\mathrm{d}y$ denote the displacements in the x and y directions between two images recorded at times t and $t + \mathrm{d}t$. By taking the first-order Taylor series expansion of the right-hand side of (1), we obtain

$$\begin{aligned} &I(x+\mathrm{d}x, y+\mathrm{d}y, t+\mathrm{d}t) \\ &\qquad \approx I(x,y,t) + I_x \mathrm{d}x + I_y \mathrm{d}y + I_t \mathrm{d}t, \end{aligned} \quad (2)$$

where the subscripts denote partial derivatives. From (1) and (2), one can derive the well-known optical flow constraint equation (OFCE),

$$I_x u + I_y v + I_t = 0, \quad (3)$$

in which $(u, v) \equiv (\mathrm{d}x/\mathrm{d}t, \mathrm{d}y/\mathrm{d}t)$ are motion vectors. Eq. (3) is apparently underdetermined because it contains two unknowns, u and v. There are two popular approaches to solve (3) for (u, v), leading to two motion estimation techniques, namely, the spatio-temporal gradient method and the gradient structure tensor method.

THE SPATIO-TEMPORAL GRADIENT METHOD

The spatio-temporal gradient method, which we will refer to as the gradient method (GM), is a traditional optical flow estimation method[1–3]. It is based on regression analysis where the two unknowns (u, v) are computed by the least-squares method assuming that I_t is a dependent (or target) variable and I_x and I_y are independent variables. To solve the OFCE for (u, v) we may introduce either a global or local smoothness constraint. The former constraint assumes that the optical flow changes smoothly over the entire image[2], while the latter assumes that the optical flow in a small region is constant[3]. To avoid the possibly time-consuming iterative procedure involved in the former approach, we employed the latter. Under the local smoothness constraint, motion vectors (u, v) can be straightforwardly determined by minimizing the quadratic cost-function,

$$F = \sum (I_x u + I_y v + I_t)^2, \quad (4)$$

where the summation is performed over a small region or block.

The procedure of the GM is equivalent to determining of the best-fitting plane to a given set of points (I_x, I_y, I_t) by minimizing the sum of the squares of the distances between the points and the plane along the I_t axis. The normal vector of the plane gives the motion vector (u, v) of the small region. The least-squares solution of (4), which we denote by $(\tilde{u}, \tilde{v})$, is given by

$$\begin{aligned} \tilde{u}_{\mathrm{GM}} &= \frac{(\sum I_x I_y)(\sum I_y I_t) - (\sum I_y^2)(\sum I_x I_t)}{(\sum I_x^2)(\sum I_y^2) - (\sum I_x I_y)^2}, \\ \tilde{v}_{\mathrm{GM}} &= \frac{(\sum I_x I_y)(\sum I_x I_t) - (\sum I_x^2)(\sum I_y I_t)}{(\sum I_x^2)(\sum I_y^2) - (\sum I_x I_y)^2}. \end{aligned} \quad (5)$$

The implementation of (5) is easy and fast, and is often used for obtaining dense motion vectors.

THE GRADIENT STRUCTURE TENSOR METHOD

The gradient structure tensor method (GSTM) is a newer approach[1,4–7]. With the rapid advance of computer technology, the GSTM has been employed for real-time motion estimation in recent years[15,16]. The GSTM is based on orthogonal regression (or total least-squares fitting) using principal component analysis (PCA). The gradient structure tensor T of the image intensities I is defined as

$$T = \begin{pmatrix} \sum I_x^2 & \sum I_x I_y & \sum I_x I_t \\ \sum I_y I_x & \sum I_y^2 & \sum I_y I_t \\ \sum I_t I_x & \sum I_t I_y & \sum I_t^2 \end{pmatrix}, \quad (6)$$

where $\sum$ indicates the summation within a local 3-d region such as a cube. T can be viewed as a covariance matrix of the spatio-temporal image gradients in PCA. PCA can be used to find the best-fitting plane to a given set of points (I_x, I_y, I_t) in the 3-d space spanned by the I_x, I_y, and I_t axes. The GSTM determines the best-fitting plane by minimizing the sum of the squares of the orthogonal offsets of the points from the plane. In contrast, in the GM, errors are defined as the sum of the squared vertical offsets from the data points to the fitted plane. In other words, errors are measured along the I_t axis because I_t is considered as a variable dependent on the other two variables, I_x and I_y. Such fitting model of the GM is often criticized for its unequal treatment of spatial and temporal derivatives[1]. When there is no clear relationship among variables, whether they are independent or dependent variables, it makes more sense to measure errors by treating all three variables equally and minimize the orthogonal offsets of the points to the plane. Our previous work also revealed that the GSTM performs motion estimation much better than the GM largely owing to this difference in fitting models[17].

Another interpretation of the GSTM is that when a small object P is moving at a constant speed in a certain direction in a series of time-sequential images, the trajectory of the moving object P forms a tubular structure in 3-d spatio-temporal space. By applying PCA to (I_x, I_y, I_t), we obtain three eigenvectors $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ and their corresponding eigenvalues $(\lambda_1, \lambda_2, \lambda_3)$ where $\lambda_1 \geqslant \lambda_2 \geqslant \lambda_3 \geqslant 0$. The first two eigenvectors will point in directions that are perpendicular to the trajectory of P, while the third eigenvector $\mathbf{e}_3 = [x_3\, y_3\, t_3]^{\mathrm{T}}$, associated with the smallest eigenvalue λ_3, will indicate the direction along the trajectory. Hence, motion vectors may be estimated from

$$\tilde{u}_{\mathrm{GSTM}} = x_3/t_3, \qquad \tilde{v}_{\mathrm{GSTM}} = y_3/t_3. \tag{7}$$

It is relevant to note that we may evaluate the confidence of estimated motion vectors by analysing the relationship between the three eigenvalues of the tensor[1,4,6,7,15]. For instance, if $\lambda_1 \approx \lambda_2 \gg \lambda_3 \geqslant 0$, the trajectory forms a linear structure in the spatio-temporal space and the confidence is high. When $\lambda_1 \gg \lambda_2 \approx \lambda_3 \geqslant 0$, the motion trajectory is plane-like, which indicates that there is an aperture problem where we can estimate motion only along the gradient vector available. When $\lambda_1 \approx \lambda_2 \approx \lambda_3 > 0$, the corresponding local region may have impulsive noise or motion discontinuities, and the resultant motion vectors may be inaccurate. Finally, if $\lambda_1 \approx \lambda_2 \approx \lambda_3 \approx 0$, there is no gradient information and motion estimation is impossible.

GRADIENT ORIENTATION INFORMATION

As a robust image feature, we propose using gradient orientation information (GOI) rather than the conventional image features such as intensities and gradients. We now describe how to extract GOI and how to use it in a computationally efficient manner.

Extraction of gradient orientation information

Let $I(x, y)$ be the image intensities at pixel coordinates (x, y). The gradient vectors are then (I_x, I_y). By convention, the upper left corner of the image is the origin, the x-axis is directed downwards, and the y-axis is horizontal. The unit gradient vectors (n_x, n_y) are obtained by dividing (I_x, I_y) by their magnitudes where we assign zeros to $n_x(x, y)$ and $n_y(x, y)$ if the magnitude is zero. Notice that unit gradient vectors carry rich spatial information even in relatively low-contrast areas (Fig. 1d). As the unit gradient vectors (n_x, n_y) are represented by two scalars, n_x and n_y, ranging from -1 to 1, they can be treated as two separate intensity patterns. We call them gradient orientation patterns and make use of them as a robust image feature for motion estimation. It should be emphasised that the use of unit gradient vectors is

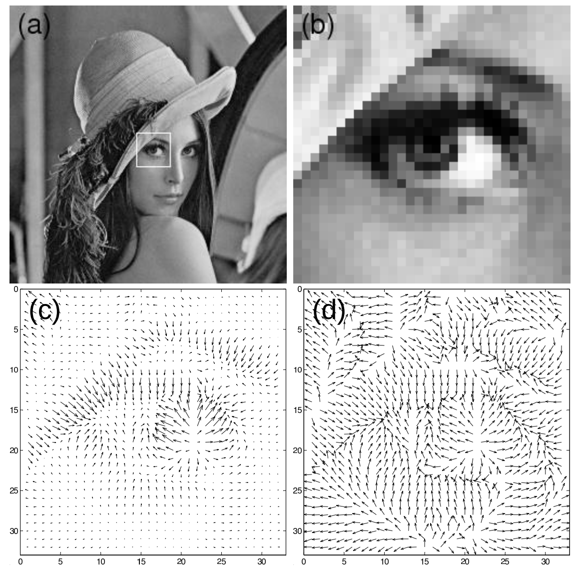

Fig. 1 (a) An 8-bit grey-scale 256×256 pixel image – the Lena image (b) an enlarged (32×32 pixel) subimage (c) subimage gradient vectors (d) subimage unit gradient vectors.

preferable to using angular values θ directly because angular values require modulo calculations (e.g., the difference between two angles cannot exceed π)[14].

Intensity-invariance of gradient orientation

One way to model the intensity variations is to assume $I' = aI + b$ where I and I' are image intensities before and after a lighting condition is changed, while a and b are scalar constants[18]. Gradient-based, including second order derivative-based[19], methods can work irrespective of additive variations of intensities ($b \neq 0$). They are, however, susceptible to multiplicative variations ($a \neq 0$). Fig. 2a shows the same subimage as in Fig. 1b, except that the intensities of the upper half of it are reduced by 50% (i.e., $a = 0.5$, $b = 0$). Fig. 2b shows the gradient vectors of the subimage. They differ considerably from those of Fig. 1c because of the multiplicative changes of image intensities. On the other hand, Fig. 2c shows unit gradient vectors that are identical to those of Fig. 1d, except at the boundary between the halves. Since the boundary acts as a step edge, it disturbs its neighbouring gradient orientations. Therefore, except at such boundaries, we may state that unit gradient vectors are insensitive to both additive and multiplicative changes of image intensities and thus can maintain gradient orientation patterns well regardless of varying illumination.

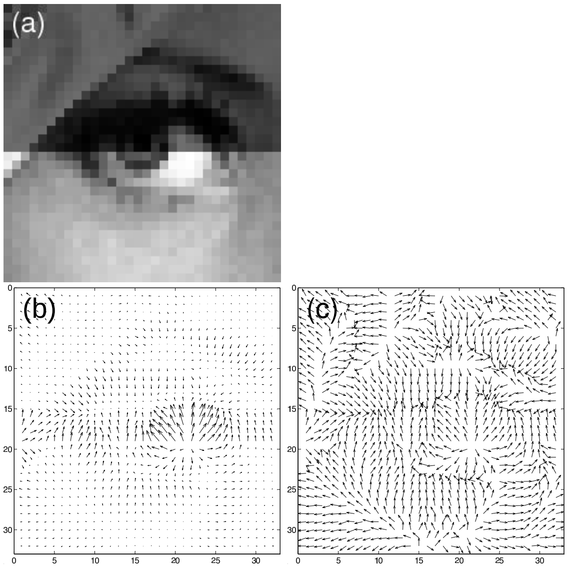

Fig. 2 (a) Partially shaded subimage (b) gradient vectors (c) unit gradient vectors.

APPLICATION OF GRADIENT ORIENTATION INFORMATION TO MOTION ESTIMATION

Gradient orientation based gradient method

In the first proposed approach, we use the unit gradient vectors (n_x, n_y), instead of image intensities I, and assume that (n_x, n_y) are constant over time:

$$\begin{aligned} n_x(x, y, t) &= n_x(x + \mathrm{d}x, y + \mathrm{d}y, t + \mathrm{d}t), \\ n_y(x, y, t) &= n_y(x + \mathrm{d}x, y + \mathrm{d}y, t + \mathrm{d}t). \end{aligned} \tag{8}$$

From (8), we obtain two modified OFCEs:

$$\begin{aligned} X_x u_1 + X_y v_1 + X_t = 0, \\ Y_x u_2 + Y_y v_2 + Y_t = 0, \end{aligned} \tag{9}$$

where $X \equiv n_x$ and $Y \equiv n_y$. We have used the Sobel operators for computing the partial derivatives because they yield good approximations of the derivatives with minimal computation[11]. In place of (4) we have the following two cost functions

$$\begin{aligned} F_1 &= \sum (X_x u_1 + X_y v_1 + X_t)^2, \\ F_2 &= \sum (Y_x u_2 + Y_y v_2 + Y_t)^2. \end{aligned} \tag{10}$$

From (10), we then obtain two motion estimates, $(\tilde{u}_{\mathrm{GOGM}x}, \tilde{v}_{\mathrm{GOGM}x})$ and $(\tilde{u}_{\mathrm{GOGM}y}, \tilde{v}_{\mathrm{GOGM}y})$ using the GM.

Because there should be only one motion vector per block, we need to unify those two motion estimates into one. For this we introduce some weighting

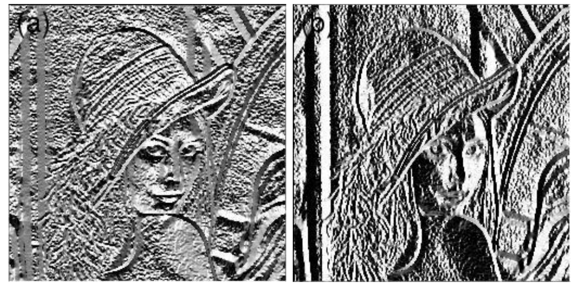

Fig. 3 Gradient orientation patterns of the Lena image: (a) n_x (b) n_y.

factors that are dependent on the gradient orientation patterns. Figs. 3a and 3b show gradient orientation patterns n_x and n_y of the Lena image in Fig. 1a, scaled between 0 and 255 for visualization purposes. Vertical gradients (i.e., horizontal lines) are rich in n_x, whereas horizontal gradients (vertical lines) are more dominant in n_y. This observation indicates that n_x is more suitable for computing vertical motion u and n_y is better for horizontal motion v.

The reliability of each motion estimate may be judged from the diversity of gradient orientations (directions) in its local area. The diversity of gradient orientations can be measured by the two eigenvalues of the covariance matrix $\mathbf{C}$ between n_x and n_y,

$$\mathbf{C} = \begin{bmatrix} \sum n_x^2 & \sum n_x n_y \\ \sum n_x n_y & \sum n_y^2 \end{bmatrix}, \tag{11}$$

where $\lambda_1 \geqslant \lambda_2 \geqslant 0$ are the eigenvalues of $\mathbf{C}$. This approach resembles the analysis of eigenvalues from gradient covariance[1,4,6,7,15], but we can focus on the analysis of gradient orientations regardless of variations of image intensities by employing unit gradient vectors. If $\lambda_1 \gg \lambda_2 \approx 0$, the gradients are unidirectional and we can only estimate motion along the direction parallel to the gradients. This is the so-called aperture problem. In this case, vertical and horizontal motion estimation should rely heavily on the pattern of n_x and n_y, respectively. Meanwhile, if $\lambda_1 \approx \lambda_2 > 0$, there are omni-directional gradients and similar weights can be used for motion estimates from n_x and n_y as they are considered equally reliable. In the proposed method this weight control is achieved by using the following unified motion estimates

$$\begin{aligned} \tilde{u}_{\mathrm{GOGM}} &= \frac{\lambda_1}{\lambda_1 + \lambda_2} \tilde{u}_{\mathrm{GOGM}x} + \frac{\lambda_2}{\lambda_1 + \lambda_2} \tilde{u}_{\mathrm{GOGM}y}, \\ \tilde{v}_{\mathrm{GOGM}} &= \frac{\lambda_2}{\lambda_1 + \lambda_2} \tilde{v}_{\mathrm{GOGM}x} + \frac{\lambda_1}{\lambda_1 + \lambda_2} \tilde{v}_{\mathrm{GOGM}y}. \end{aligned} \tag{12}$$

We call this approach the gradient orientation based gradient method (GOGM).

Gradient orientation structure tensor method

In the second proposed method we use (n_x, n_y) instead of the image intensities I. Thus, as in (6), we have two local tensors T_X and T_Y

$$T_X = \begin{pmatrix} \sum X_x^2 & \sum X_x X_y & \sum X_x X_t \\ \sum X_y X_x & \sum X_y^2 & \sum X_y X_t \\ \sum X_t X_x & \sum X_t X_y & \sum X_t^2 \end{pmatrix}, \quad T_Y = \begin{pmatrix} \sum Y_x^2 & \sum Y_x Y_y & \sum Y_x Y_t \\ \sum Y_y Y_x & \sum Y_y^2 & \sum Y_y Y_t \\ \sum Y_t Y_x & \sum Y_t Y_y & \sum Y_t^2 \end{pmatrix}. \tag{13}$$

We use the 3-d Sobel operators for computing the partial derivatives[20]. By applying PCA to T_X and T_Y separately, we obtain two estimates of image motions $(\tilde{u}_{\text{GOSTM}x}, \tilde{v}_{\text{GOSTM}x})$ and $(\tilde{u}_{\text{GOSTM}y}, \tilde{v}_{\text{GOSTM}y})$. Following the same procedure described above, we integrate these two motion estimates depending on the diversity of local gradient orientations by using

$$\tilde{u}_{\text{GOSTM}} = \frac{\lambda_1}{\lambda_1+\lambda_2}\tilde{u}_{\text{GOSTM}x} + \frac{\lambda_2}{\lambda_1+\lambda_2}\tilde{u}_{\text{GOSTM}y}, \quad \tilde{v}_{\text{GOSTM}} = \frac{\lambda_2}{\lambda_1+\lambda_2}\tilde{v}_{\text{GOSTM}x} + \frac{\lambda_1}{\lambda_1+\lambda_2}\tilde{v}_{\text{GOSTM}y}. \tag{14}$$

We call this approach the gradient orientation structure tensor method (GOSTM).

RESULTS AND DISCUSSION

We evaluate the motion estimates by GOGM and GOSTM by comparing their performances with those of conventional approaches on four standard test images (Fig. 1a, Fig. 4) converted to 8-bit grey-scale 256×256 pixel images.

Synthetic time-sequential images were produced by translating the four test images by 2 pixels in both the vertical and horizontal directions. Gaussian noise was added to all images (signal-to-noise ratio $\sim$ 40 dB). For evaluation, we assume that the estimated motion vectors are correct when they satisfy the condition,

$$\max\max(|\Delta x|, |\Delta y|) \leqslant 0.5, \tag{15}$$

where Δx and Δy denote the deviations (in pixels) between true and estimated motions. Hence, if rounded motion vectors are equal to the true motion, those motion estimates are considered successful. Note that the motion estimates are not integers, but real numbers.

Fig. 4 Standard test images: (a) girl (b) cameraman (c) house.

Gradient orientation based gradient method

Prior to motion estimation it is always necessary to apply a certain low-pass filter for gradient-based methods because the computation of gradients is sensitive to high frequency components in an image. A Gaussian low-pass filter of size 13×13 pixels whose standard deviation is half of the filtering mask (σ=13/2) was applied to all the four image sequences[17]. We then computed 14×14 motion vectors for each sequence with the blocks of size 16×16 pixels.

We first tested the GM and GOGM under mildly noisy (40 dB) and constant lighting conditions. It is apparent that the GOGM achieves much higher success rates than the GM for all the four image sequences (Table 1). Next, the intensities of the second frame were uniformly reduced by 10% to simulate a time-varying lighting condition. Note that this is a robustness test to temporal variation of image intensities that is different from a spatial variation of image brightness shown in Fig. 2a. The GM completely breaks down whereas the GOGM performs motion estimation regardless of the variation of image intensities (Table 1).

Gradient orientation structure tensor method

Next we compare the GOSTM with the GSTM in the same settings as above. Both methods work equally well under constant lighting conditions (Table 1). For varying lighting conditions, the stark contrast between

Table 1 Percentage success rates of 196 motion estimates of the true motion (2,2) by the GM, GOGM, GSTM, and GOSTM under constant (Sim 1) and time-varying (Sim 2) lighting conditions.

Image name	GM		GOGM		GSTM		GOSTM	
Sim:	1	2	1	2	1	2	1	2
Lena	38.3	0.5	62.8	64.3	97.4	0.0	98.0	98.0
Girl	56.1	4.1	77.6	77.0	99.0	3.1	96.4	94.4
Cameraman	42.9	5.1	60.7	57.7	90.8	6.1	86.2	81.6
House	27.6	0.0	48.0	49.0	77.6	0.5	77.0	73.5

the GSTM and GOSTM clearly shows that even a slight change of image intensities makes the GSTM completely ineffective, while the GOSTM performs motion estimation irrespective of varying image intensities. Notice that the successful motion estimate rates in the cameraman and house images tend to be low for every approach. This is because these images contain large extremely low-contrast areas, such as sky and wall, where no gradient information is available.

Improved performance to the aperture problem

A close observation of the motion estimates by the four methods also reveals that the proposed techniques tend to work better where only one-directional gradients are available in a local area (the aperture problem). For this comparison, the block size of the GSTM and GOSTM has been reduced to 8×8 pixels to highlight the differences between them. This modification is necessary because both methods produce equally high success rates when a block size of 16×16 pixels is used (Table 1). Under the new setting, the success rates of the GSTM and GOSTM are respectively 67.5% and 77.6%. In Fig. 5a, most of the motion vectors along the vertical edges on the left-hand side are incorrect. Similarly, Fig. 5c shows several large erroneous motion vectors along the curved lines located on the right-hand side. In both cases the incorrect motion vectors tend to have large motion estimation errors in the direction tangent to the edges. They are typical faulty motion vectors resulting from the aperture problem. Many of these unsuccessful motion estimations, however, have been improved or corrected by the GOGM and GOSTM (Fig. 5b,d). The improved performance may be attributed to the weighting factors based on the diversity of local gradient orientations. To summarize, large erroneous motion estimates are well suppressed by the weighted sum of two motion estimates in (12) and (14).

Motion estimation on a real-image sequence

Finally, we demonstrate the feasibility of the GOGM and GOSTM on a real image sequence of size 240×320 pixels that contain local motions. The sequence shows a walking woman with stationary background. We have reduced the image intensities uniformly by 10% in one frame in the sequence. The size of the block is set at 8×8 pixels. Since there is no ground truth data for this sequence, the proposed methods GOGM and GOSTM are compared with the GM and GSTM based on visual inspection. It is obvious that the conventional approaches GM (Fig. 6a) and GSTM (Fig. 6c) cannot produce reliable motion estimates at all under the time-varying lighting condition. They produce numerous faulty motion estimates in the background where there is no motion. On the other hand, both GOGM (Fig. 6b)

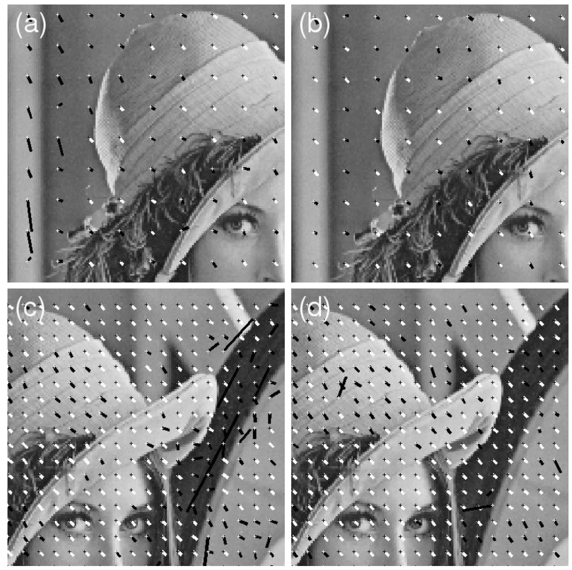

Fig. 5 Parts of the motion vectors estimated under mildly noisy constant lighting conditions using (a) GM (b) GOGM (c) GSTM (d) GOSTM. White vectors indicate correctly estimated motions, i.e., displacements of (2,2) pixels after rounding. Black vectors show incorrect estimations.

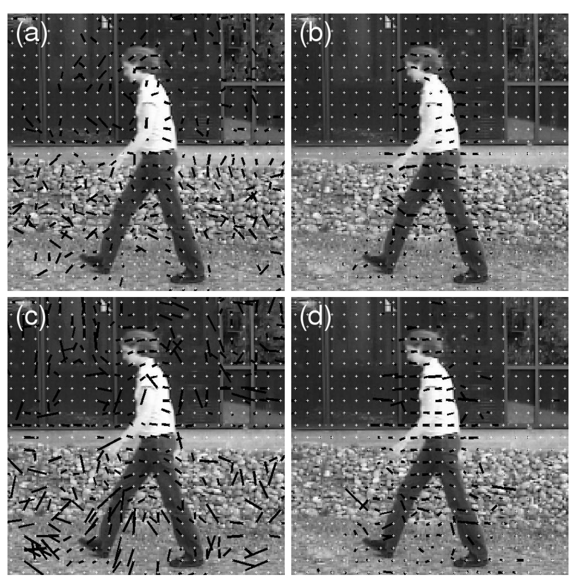

Fig. 6 Parts of the motion estimation results on a real image sequence under time-varying lighting conditions using (a) GM (b) GOGM (c) GSTM (d) GOSTM.

and GOSTM (Fig. 6d) work reasonably well under the same condition, and there are far less faulty responses to the stationary background. Therefore, it is confirmed that the proposed techniques outperform their conventional counterparts, especially under time-varying illumination conditions.

CONCLUSIONS

We have presented two motion estimation techniques, the gradient orientation based gradient method and the gradient orientation structure tensor method that are based, respectively, on the spatio-temporal gradient method and the gradient structure tensor method. Unlike the conventional approaches utilizing image gradients, we make use of gradient orientation information (GOI) by means of unit gradient vectors. Since GOI is insensitive to changes of image intensities, the proposed methods have achieved a significant robustness to time-varying lighting conditions. They also perform better than the previous methods when encountering the aperture problem. The implementation of the proposed methods is straightforward because image gradients are commonly computed at an early stage in image sequence processing and computer vision applications and are readily available. At present, we are testing a correlation-based method using GOI. We plan to evaluate the performances of these GOI-based approaches further on more real image sequences.

Acknowledgements: This research work was supported by research grants from the Thammasat University Research Fund and Thailand Research Fund MRG5180364. The authors are also deeply grateful for the constructive comments from an anonymous reviewer.

REFERENCES

1. Reed TR (2005) *Digital Image Sequence Processing, Compression, and Analysis*, CRC Press.
2. Horn BKP, Schunck BG (1981) Determining optical flow. *Artif Intell* **17**, 185–203.
3. Kearney JK, Thompson WB, Boley DL (1987) Optical flow estimation: an error analysis of gradient-based methods with local optimization. *IEEE Trans Pattern Anal Mach Intell* **9**, 229–44.
4. Westin CF, Knutsson H (1994) Estimation of motion vector fields using tensor field filtering. In: Proceedings of the IEEE International Conference on Image Processing, vol 2, pp 237–41.
5. Zhang J, Gao J, Liu W (2001) Image sequence segmentation using 3-D structure tensor and curve evolution. *IEEE Trans Circ Syst Video Tech* **11**, 629–41.
6. Spies H, Scharr H (2001) Accurate optical flow in noisy image sequences. In: Proceedings of the IEEE International Conference on Computer Vision, pp 587–92.
7. Liu H, Chellappa R, Rosenfeld A (2003) Accurate dense optical flow estimation using adaptive structure tensors and a parametric model. *IEEE Trans Image Process* **12**, 1170–80.
8. Giachetti A (2000) Matching techniques to compute image motion. *Image Vis Comput* **18**, 247–60.
9. Ye M, Haralick RM, Shairo LG (2003) Estimating piecewise-smooth optical flow with global matching and graduated optimization. *IEEE Trans Pattern Anal Mach Intell* **25**, 1625–30.
10. Alkaabi S, Deravi F (2003) Gradient direction similarity measure. *Electron Lett* **39**, 1643–4.
11. Gregson PH (1993) Using angular dispersion of gradient direction for detecting edge ribbons. *IEEE Trans Pattern Anal Mach Intell* **15**, 682–96.
12. Kondo T, Yan H (1999) Automatic human face detection and recognition under non-uniform illumination. *Pattern Recogn* **32**, 1707–18.
13. Chen HF, Belhumeur PN, Jacobs DE (2000) In search of illumination invariants. In: Proceedings of the IEEE International Conference on Computer Vision and Pattern Recognition, vol 1, pp 254–61.
14. Burgi PY (2004) Motion estimation based on the direction of intensity gradient. *Image Vis Comput* **22**, 637–53.
15. Strzodka R, Garbe C (2004) Real-time motion estimation and visualization on graphics cards. In: IEEE Visualization, pp 545–52.
16. Pless R, Wright J (2005) Analysis of persistent motion patterns using the 3D structure tensor. In: Proceedings of the IEEE Workshop on Motion and Video Computing, vol 2, pp 14–9.
17. Boonsieng P, Kondo T (2007) Comparative study of motion estimation techniques: the gradient method and structure tensor method. In: Proceedings of the International Workshop on Advanced Image Technology, Bangkok, Thailand.
18. Kim YO, Martínez AM, Kak AC (2005) Robust motion estimation under varying illumination. *Image Vis Comput* **23**, 365–75.
19. Girosi F, Verri A, Torre V (1989) Constraints for the computation of optical flow. In: Proceedings of the Workshop on Visual Motion, pp 116–24.
20. Nikolaidis N, Pitas I (2001) *3-D Image Processing Algorithms*, Wiley.