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Fractal studies on the spatial patterns of trees: A case study of Khao Yai National Park, Thailand

Pornkamon Nalakarn[a], **I-Ming Tang**[b], **Wannapong Triampo**[b,c,*]

[a] Physics Department, Faculty of Science & Technology, Thammasat University, Bangkok 12121, Thailand
[b] R&D Group of Biological and Environmental Physics, Department of Physics, Faculty of Science, Mahidol University, Bangkok 10400, Thailand
[c] Institute for Science & Technology for Research & Development, Mahidol University, Nakhon Pathom 73170, Thailand

[*] Corresponding author, e-mail: scwtr@mahidol.ac.th

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ABSTRACT: The distribution of the trees in a 30 hectare plot in Khao Yai National Park, Thailand containing 16 375 trees, divided into 63 families and 182 species was studied. The allometric scaling exponent in the relation between number and diameter of all the trees taken as one group was approximately -2. The spatial heterogeneity of the forest shown by the variation of this scaling exponent determined for each hectare in the area of study. The box counting method was used to determine the fractal dimensions (d_f) of the spatial patterns of the trees. The spatial pattern for all the trees taken as a whole had a d_f of 1.81. The fractal dimensions for the patterns of the six most abundant species ranged from 1.73 to 1.75. On the other hand, values of d_f as measured from each hectare were less than 1.5, suggesting that the pattern is not self-similar over a significant range of length scales.

KEYWORDS: tropical forest, allometric scaling exponent, scaling laws, fractal dimension

INTRODUCTION

An ecosystem is a complex system. One recent tool used to study the universal principles that govern the structure and dynamics of ecosystems comes from the concept of fractals[1,2]. A fractal is a nontrivial geometrical structure which, in at least a statistical sense, is invariant under a scale transformation[3]. The mathematics of fractals has developed to a point where it has become a promising way to quantify patterns exhibiting complex geometries. The need for a new geometry stems from the fact that classical Euclidean geometry is not suitable for analysing some aspects of the irregular shapes that appear in nature. The non-Euclidean concept of fractional dimensions allows for a quantitative description of rather abstract quantities such as heterogeneity, irregularity and complexity[4].

Recently, long-range power-law correlations have been discovered in a wide range of scenarios in nature[5]. These features have been interpreted as the result of scale invariance[6–8]. A power-law dependence of dependent variable Y on independent variable X takes the form $Y = Y_0 X^n$ where Y_0 is a constant and n is the exponent. A power-law dependence with an exponent which is a multiple of 1/3 indicates that the phenomenon exhibits a Euclidean geometric scaling. The simplest examples of this are the relationships between the length (L), area (A), and volume (V). Self-similar objects of this class scale as $A \propto L^2$, $V \propto L^3$, and $A \propto V^{2/3}$. Biological allometries, however, exhibit exponents that are multiples of a quarter[8–15]. Examples of biological allometries are the $M^{3/4}$ scaling of the whole body metabolic rate, the $M^{1/4}$ scaling of the development time, lifespan, and other biological times, and the $M^{-1/4}$ scaling of the heart rate and the maximum rate of population growth, where M is the mass of the organism.

Most landscapes exhibit patterns intermediate between a complete spatial independence ($d_\mathrm{f} = 1$) and a complete spatial dependence ($d_\mathrm{f} = 2$). The fractal dimension provides a measure of the degree of correlation between 'points' or patches over space or time. It provides a new way to understand and to analyse rough and fragmented spatial phenomena in terms of self-similarity or self-affinity. Again, the concept of fractal geometry can provide new insight into the spatial variability of tree patterns. Recently, there have been several reviews on the applications of fractals to ecological research and landscapes[16,17]. Because a fractal dimension is scale invariant, it provides us with a new index to measure 'point' patterns and diversity. Information on the fractal dimensions can be used in quantifying the landscape habitat diversity[18] and non-geometric ecological properties such as permeability[19].

Habitat complexity and spatial heterogeneity are important factors in community structure, contributing to community diversity and to species co-existence in an ecosystem. Habitat complexity has been shown to alter the size-density scaling of species in an ecosystem. Li[20] has proposed a theoretical explanation for species-area scaling based on the generalized MacArthur-Wilson model with a fractal interpretation of habitats by relating mechanisms of species migration with habitat heterogeneity. Organizing principles are needed to link the organism, the community, and the ecosystem properties across spatial and temporal scales. A central goal of plant ecology is to understand the mechanisms controlling the structure and the dynamics of plant communities[21,22]. The mechanistic principles connecting the essential characteristics of different ecological communities across diverse ecosystems have gradually started to emerge. West et al[8,14,15] have hypothesized that the unique 1/4 power exponent is due to the fractal designs of the distribution networks and the exchange surfaces. The fractal nature of the geometry endows these systems with a 'fourth spatial dimension.'

The spatial patterns of plants can also be characterized by a fractal analysis. The fractal dimension of the structural components of the ecosystem could reflect the emerging pattern being produced as a result of the spatiotemporal interactions between the components of the ecosystem. Fractal structures have two important properties: (a) spatial heterogeneity and (b) invariability through a range of scales over which the plant community is self-similar, i.e., displays similar behaviour, appears to be structurally similar, and is affected by the same processes[23]. Fractal dimension can be used as a criterion to evaluate and diagnose successional vegetation changes. Changes in the fractal dimension might indicate substantial change in the processes that generate spatial patterns of plants[2]. Different species may perceive environmental variability in different ways and thus exhibit different patterns. The distribution pattern of plants in the landscape can be characterized by spectral analysis of intra-specific distribution ($1/f$ noise), which measures the level of autocorrelation among the spatially distributed objects. Spectral analysis has generally been applied to temporal fluctuations, but can also be applied equally to characterize spatial correlations[24].

The purpose of the present study is to investigate the fractal properties of the distribution of different species of trees in a forest in Thailand. Brown et al[9] mentions some work done on 0.1 ha plots in two forests in Colombia; one in Tutunento having 590 trees of 121 species and the other in Baja Calima, having 556 trees of 263 species. They found that the scaling between the numbers of stems and the diameters of the trees in the two forests went as -2.08 and -2.12, respectively.

Our analysis of the data is based on the box counting method. The box counting method provides a simple way to determine the fractal dimensional geometry which here is defined as how much area is filled with trees (of all species, a given species or a given family). We wish to see if the spatial pattern formation of a tropical forest can be explained in terms of fractal objects. The paper focuses on two keys aspects: (1) scaling relations between the number of trees and some of their properties, (2) a geometrical description using fractal dimensions.

MATERIALS AND METHODS

Study area and data collection

Khao Yai National Park is located about 200 km northeast of Bangkok. The tree data we used was from a complete survey of all trees with diameter above breast height (dbh) larger than 10 cm in a 30 ha plot in the park, and was carried out by the Center for Conservation Biology, Mahidol University from April 2001 to April 2002. The survey plot (14°26′ N, 101°22′ E, elevation 730–860 m) is a large area in a primary tropical rain forest. The climate is cool and humid, with a distinct summer and receives about 2 m of rain a year mostly between April and September. The average temperature ranges from 17 °C in December and January to 28 °C in April and May.

The distribution of all the trees in the area studied is shown on Fig. 1. The plot is divided into 30 100 m × 100 m quadrats. The quadrats are labelled $Q(i,j)$ where $i = 1, \ldots, 6$ and $j = 1, \ldots, 5$. The total number of trees located in the study plot is 16 375 divided into 63 families and 182 species.

Measurements and analysis

We used MAPINFO to generate the geographic pictures needed to illustrate different aspects of our study. The maps are of the tree distribution in the entire area and the geographical distribution (in the total area of study) of the trees belonging to the six most abundant families and to the six most abundant species (Fig. 2). These maps show how the trees (of a given species) segregate and the patterns they form. The fractal dimensions of tree pattern structure were

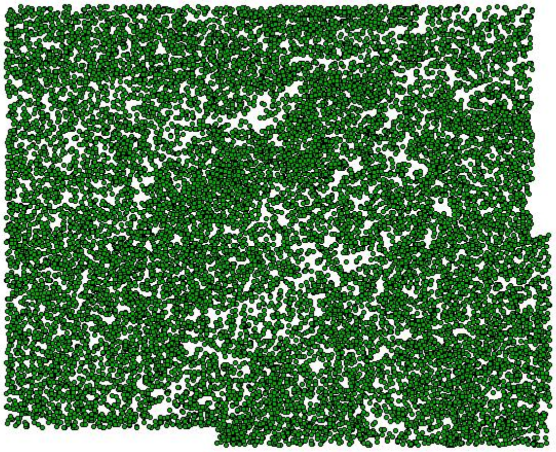

Fig. 1 Spatial distribution of all the trees in the 30 ha plot.

determined by the box-counting method (BCM) using the BENOIT program on images of maps of the points. The reliability of the method was tested with fractals (Koch snowflake and Sierpinski gasket) of known d_f. The BCM is one of the most widely used methods[25] due to the relative ease of mathematical calculations and computations involved. It consists basically of drawing successively larger boxes and counting the number of boxes that touch the objects being looked at. The slope of the log-log plot of the number of boxes versus their respective size is the fractal dimension.

RESULTS

As can be seen from Figs. 1 and 2, the distributions of the total number of trees and of the individual species are not homogeneous. While not shown, the distributions of the trees belonging to the six most abundant families are also inhomogeneous. Instead, the properties of the trees and the patterns they form exhibit great variations in the different quadrats of the plot. In other words, this particular plot of forest exhibits an enormous complexity which we hope to understand.

Spatial distribution pattern of trees

The spatial distribution of all the trees is shown in Fig. 1. To see the tree patterns at a more local level, we have divided the total area of 30 ha into 30 quadrats of one ha each and have analysed the fractal nature of the tree distribution within each quadrat. We have listed in Table 1 the number of trees (400–700), the number of species (60–90), the most abundant family and most abundant species in each of the thirty quadrats. These numbers indicate that the number of trees do not differ significantly from quadrat to quadrat. However, the spatial patterns of the individual species of trees were different in each of the quadrats. The heterogeneity in the patterns of the trees belonging to the six most abundant families in the different quadrats was also seen.

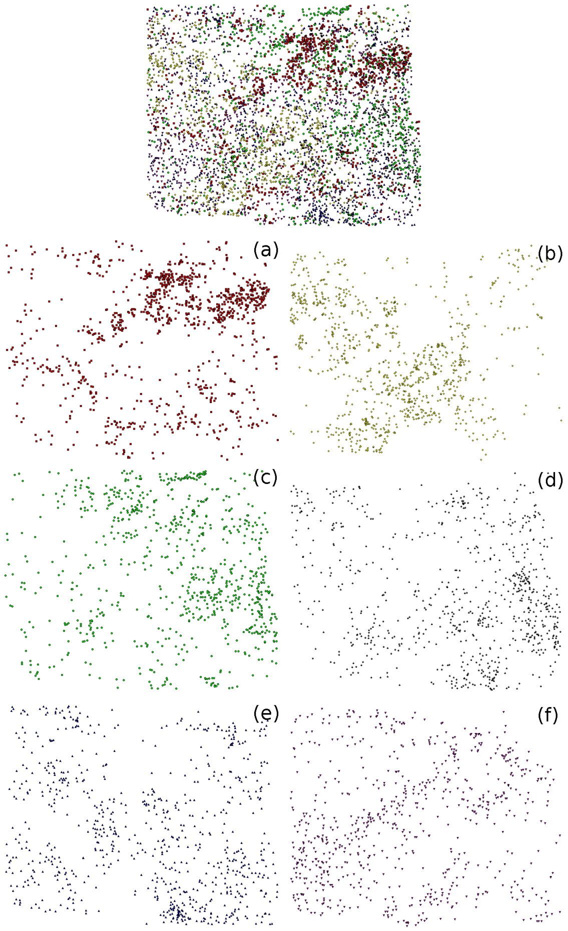

Fig. 2 Spatial patterns of trees belonging to the six most abundant species. The top frame shows the overlap of the six patterns of these species. The bottom frames are the spatial patterns: (a) *Ilex chevalieri* (b) *Sloanea sigun* (c) *Symplocos cochinchinensis* (d) *Mastixia pentandra* (e) *Dipterocarpus gracilis* (f) *Nephelium melliferum*.

Size

I. chevalieri was the most abundant species having 1021 individuals. *S. sigun*, *S. cochinchinensis*, *M. pentandra*, *D. gracilis*, and *N. melliferum* had 1007, 808, 768, 762, and 736 trees, respectively. The six most abundant families are Lauraceae, Meliaceae, Elaeocarpaceae, Aquifoliaceae, Icacinaceae, and Sapindacea. The numbers of trees belonging to

Table 1 Fractal dimension, exponent, most abundant family and species and total number of trees and species in each quadrat

Quadrat	d_f (SD)	Exponent	Family (No. of trees[a])	Species (No. of trees[b])	trees	species
1,1	1.408 (0.043)	−2.29	Lauraceae (75)	*N. melliferum* (37)	431	71
1,2	1.396 (0.035)	−1.66	Elaeocarpaceae (84)	*S. sigun* (84)	392	61
1,3	1.453 (0.015)	−1.97	Lauraceae (59)	*Ficus nervosa* (42)	536	76
1,4	1.456 (0.025)	−2.07	Dipterocarpaceae (57)	*D. gracillis* (57)	563	82
1,5	1.437 (0.023)	−1.95	Cornaceae (65)	*M. pentandra* (65)	496	72
1,6	1.478 (0.026)	−2.32	Lauraceae (120)	*Knema elegans* (56)	647	66
2,1	1.446 (0.025)	−2.21	Lauraceae (86)	*N. melliferum* (69)	542	85
2,2	1.462 (0.024)	−1.91	Lauraceae (87)	*N. melliferum* (37)	591	67
2,3	1.430 (0.023)	−2.05	Elaeocarpaceae (123)	*S. sigun* (122)	484	71
2,4	1.427 (0.023)	−1.55	Elaeocarpaceae (84)	*S. sigun* (83)	465	71
2,5	1.443 (0.026)	−1.98	Lauraceae (76)	*F. nervosa* (47)	528	68
2,6	1.464 (0.026)	−2.15	Lauraceae (101)	*M. pentandra* (72)	592	63
3,1	1.442 (0.026)	−1.49	Lauraceae (106)	*F. nervosa* (44)	521	76
3,2	1.459 (0.022)	−1.93	Lauraceae (66)	*S. sigun* (56)	572	91
3,3	1.470 (0.028)	−2.10	Lauraceae (69)	*S. sigun* (52)	613	86
3,4	1.419 (0.018)	−1.92	Elaeocarpaceae (90)	*S. sigun* (88)	434	79
3,5	1.450 (0.022)	−2.03	Symplocaceae (60)	*S. cochinchinensis* (62)	538	74
3,6	1.461 (0.024)	−2.10	Cornaceae (80)	*M. pentandra* (80)	566	74
4,1	1.412 (0.026)	−1.37	Elaeocarpaceae (95)	*S. sigun* (94)	414	73
4,2	1.425 (0.023)	−1.92	Euphorbiaceae (68)	*S. sigun* (57)	465	83
4,3	1.444 (0.018)	−2.52	Rubiaceae (45)	*I. chevalieri* (43)	520	90
4,4	1.478 (0.023)	−2.42	Aquifoliaceae (101)	*I. chevalieri* (101)	660	82
4,5	1.450 (0.023)	−2.42	Aquifoliaceae (118)	*I. chevalieri* (118)	552	74
4,6	1.466 (0.027)	−2.12	Aquifoliaceae (164)	*I. chevalieri* (163)	611	68
5,1	1.445 (0.027)	−1.54	Lauraceae (115)	*Cinnamomum subavenium* (45)	529	77
5,2	1.452 (0.026)	−1.97	Lauraceae (66)	*Gonocaryum lobbianum* (42)	561	81
5,3	1.456 (0.026)	−2.33	Lauraceae (61)	*G. lobbianum* (51)	577	80
5,4	1.477 (0.023)	−2.50	Theaceae (104)	*I. chevalieri* (93)	651	84
5,5	1.453 (0.022)	−2.39	Symplocaceae (59)	*S. cochinchinensis* (56)	577	89
5,6	1.429 (0.019)	−2.26	Myrtaceae (66)	*Cleistocalyx operculatus* (46)	485	91

[a] number of individuals in dominant family
[b] number of individuals of dominant species

these families of trees are 1873, 1083, 1082, 1024, 911, and 808 trees, respectively. The histograms of the frequency distribution of the diameters for all the trees, of the most abundant species and of the most abundant family are shown in Fig. 3. The histograms for the remaining species and families look similar. The frequency distribution of the diameters for all six species and six families are described by a nonlinear monotonically decreasing function. Among these trees, the dbh ranged from 10 cm to 70 cm.

Allometric scaling of the total number of trees as a function of trunk diameter

We have looked for an allometric scaling relationship between the numbers of trees (N) and the diameter (D), i.e., $N \propto D^x$. The power exponents were obtained from the log-log plot of the number of trees (N) as a function of the diameter (D) of the tree trunk. Fig. 4 shows the log-log plot for all the trees in the 30 ha plot. Fig. 5 shows the log-log plot for the trees found in two selected quadrats. All of the log-log plots gave an exponent value of about -2. The values of the exponents in the relations for each quadrat are listed in Table 1.

In Table 2 and Table 3, we have also listed the exponents in the allometric relation for the trees belonging to the six most abundant species and most abundant families.

Fractal analysis of the spatial patterns of the trees

We used the box-counting determination of the fractal dimensions (d_f) to quantify the pattern of the tree distribution in the area of study. Looking at the distribution of all the trees catalogued in this study, we

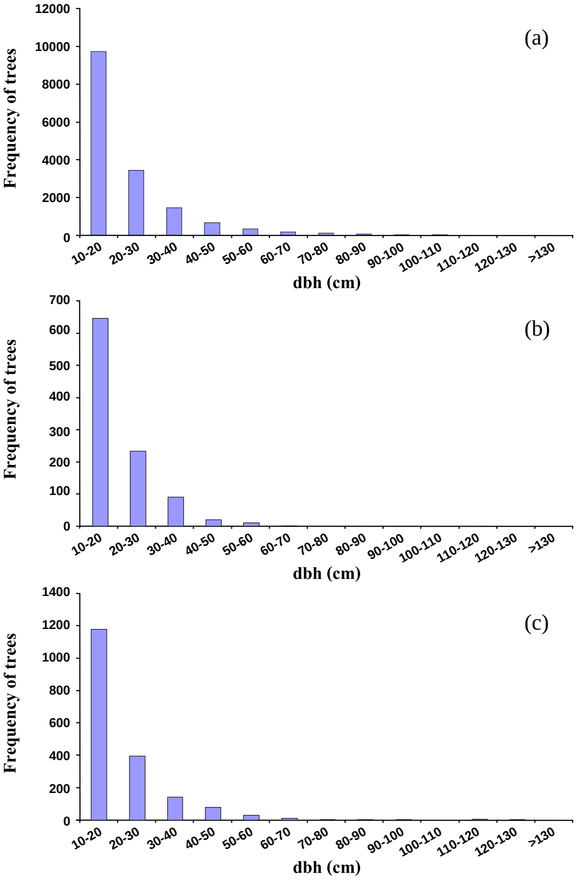

Fig. 3 Histograms of the frequency of the dbh values of (a) all trees (b) *I. chevalieri* (c) Lauraceae trees.

obtained $d_\mathrm{f} \approx 1.81 \pm 0.03$. We then determined the fractal dimensions of the six most abundant species and families in the 30 ha plot. The results are given in Table 2 and Table 3. The fractal dimensions of the

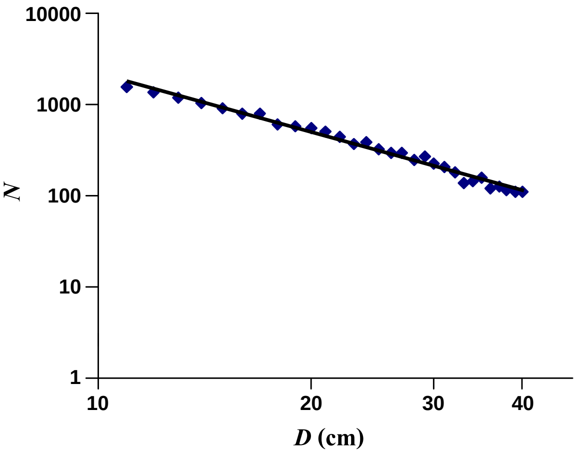

Fig. 4 Log-log plot of the number of all species of the trees versus their diameters. Straight line: $N = 3.24\times10^5 D^{-2.15}$.

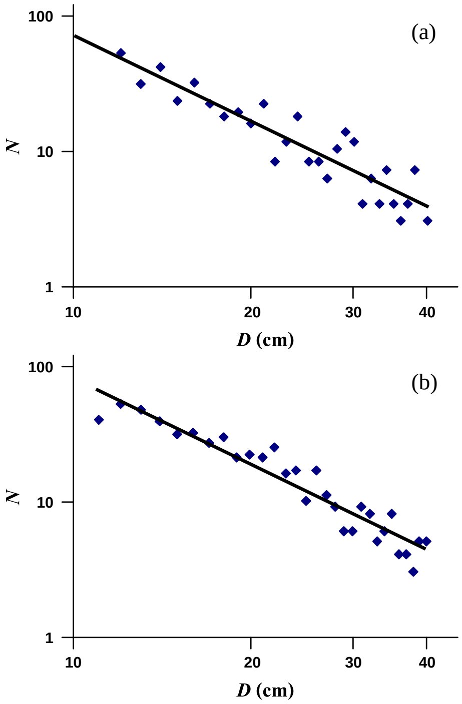

Fig. 5 Log-log plots of N versus D for two selected quadrats within the 30 ha plot: (a) $Q(2,3)$: straight line is $N = 7.26 \times 10^3 D^{-2.05}$ (b) $Q(4,6)$: straight line is $N = 1.08 \times 10^4 D^{-2.12}$.

Table 2 Fractal dimension and scaling exponent of the six most abundant species.

Species	d_f (SD)	Exponent	R^2
I. chevalieri	1.746 (0.076)	−2.337	0.918
S. sigun	1.754 (0.069)	−1.019	0.855
S. cochinchinensis	1.743 (0.080)	−1.659	0.811
M. pentandra	1.748 (0.076)	−1.826	0.894
D. gracilis	1.738 (0.084)	−2.305	0.850
N. melliferum	1.731 (0.091)	−1.993	0.903

Table 3 Fractal dimension and scaling exponent of the six most abundant families.

Family	d_f (SD)	Exponent	R^2
Lauraceae	1.742 (0.041)	−2.404	0.953
Maliaceae	1.737 (0.062)	−3.337	0.869
Elaeocarpaceae	1.751 (0.064)	−1.079	0.869
Aquifoliaceae	1.743 (0.077)	−2.340	0.918
Icacinaceae	1.735 (0.072)	−3.210	0.915
Sapondaceae	1.747 (0.072)	−2.085	0.906

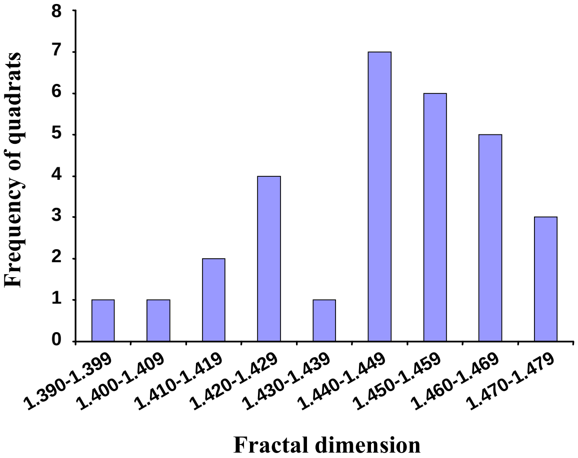

Fig. 6 Histogram of frequency distribution of fractal dimension values.

distributions of the six most abundant families and the most abundant species are more or less the same. This finding points to the development of the complexities in the patterns for the individual species or families being independent of each other.

To see the complexity, we have also determined the fractal dimensions of the trees within each of the thirty quadrats. The fractal dimensions are listed in Table 1 with the other characteristics of the tree distribution in the quadrat. The fractal dimensions range from 1.39 to 1.48. The departure from the value of d_f found for the whole area seems to indicate that tree patterns are not self-similar and tend to a homogeneous distribution ($d_\mathrm{f} = 2$) at large scales.

The quadrats $Q(1,6)$ and $Q(4,4)$ had the maximum d_f (1.478). The densities of trees in these two quadrats were 647 and 660 trees per ha. The minimum d_f (1.396) was obtained for $Q(1,2)$. The density of trees in this quadrat was 392 trees per ha.

The frequency of the values of the d_f's is shown in Fig. 6. The exponents in the allometric relations between N and D for these three quadrats were -2.32, -2.42 and -1.66, respectively. The above three examples are evidence of a possible correlation between the fractal dimensions of the tree patterns in each quadrat and the number of trees in the quadrat.

DISCUSSION

From the enormous physical and biological diversity occurring in ecosystems, patterns of ecological organization emerge. The emerging patterns represent the outcome of the fundamental law-like processes in physics, chemistry, and biology. We have focused on one class of emerging ecological phenomena: patterns that are self-similar or fractal-like over a wide range of spatial scales. Ecologists have used indices of spatial diversity to describe the richness, abundance, and evenness among species in defined geographical areas. One way to do this is to determine the fractal dimensions of the patterns formed by plant or animal species in the ecosystem. This, however, is not the only way. Parrott[5] has suggested the use of a 'Shannon entropy'[26,27]. A low Shannon entropy value indicates that the data are ordered while a high value indicates disordered data.

We still need to establish a connection between the higher fractal dimension and more complexity or between higher Shannon entropy and more disorder. One possible connection is the size. Size is probably the single most obvious feature of any organism that can profoundly affect the structure and function of the pattern. On average, larger trees have higher metabolic rates. They also have a lower population density than smaller ones. This is partly supported by our data (Figs. 4 and 5) if we take D to be a measure of the size. The reason for this has been touched on by West et al[14]. They reasoned that nature would develop a network to feed the different parts of the system in such a way that all parts receive the same nutrients. The network (vascular system) would be a self-similar one. Thus nature would develop a fourth dimension. This would then require a new geometry to describe the system, a fractal geometry[28,29].

In this paper, we have presented some examples which show the relative usefulness of scaling and fractal analysis in the characterization of spatial patterns of trees in an ecosystem. Although natural objects are never true fractals, many of them have fractal-like features. The ecological significance of scaling and fractals is that they describe very compactly the relation between the spatial variability of the patterns. A key problem is to understand how and why nature gives rise to fractal structures or power-law distributions.

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