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Characterization of material constants based on synthetic biaxial data

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ABSTRACT: When applying finite element analysis (FEA) to the designing of rubber products, the material constants are required as input data. To obtain sufficiently accurate material constants, combined tests and biaxial tests are recommended. However, these tests are time-consuming and sometimes require special equipment. We propose a fast and easy method to characterize the material constants. In this method, only the tensile test is required. The tensile data are used to generate biaxial data based on the constant true Young modulus with varying Poisson's ratio approach. The synthetic biaxial data are then converted into material constants by multiple regression. Compared with the material constants obtained from conventional method, those obtained from the proposed method give a better prediction of rubber behaviour under tension and simple shear modes. Even though they give a poorer prediction under compression mode, the difference between the FEA and experimental results is relatively low (∼11–12%). Thus, it could be concluded that the material constants obtained from this method could give good prediction of rubber behaviour under various types of deformation.

KEYWORDS: finite element analysis, rubber, biaxial test

INTRODUCTION

Finite element analysis (FEA), an effective engineering tool for product design, has been extensively applied to assist in the design of rubber products such as laminated rubber bearings[1–3], automotive exhaust hangers[4], and fluid seals[5]. However, the application of FEA to rubber encounters many difficulties due to the geometrical and material nonlinear behaviour of rubber-like materials. In FEA, there are many factors governing the precision of the computational result. The selection of an appropriate strain energy function (W) and the determination of material constants are the most important factors affecting the modelling success. Many theoretical models have been developed to characterize the mechanical behaviour of rubber[6–11]. One of the most important is the Mooney-Rivlin model,

$$W = C_{10}(I_1 - 3) + C_{01}(I_2 - 3), \qquad (1)$$

where C_{10} and C_{01} are material constants and I_1 and I_2 are strain invariants. This model is extensively used for stress analysis of rubber components and is incorporated in most commercial FEA programs, and is therefore the focus of the present study.

To predict the rubber behaviour based on the Mooney-Rivlin model, the values of C_{01} and C_{10} must be determined. FEA programs can be used to approximate these constants from experimental data. However, it has been reported that the material constants derived from uniaxial tensile data alone are insufficient to describe the behaviour of rubber subjected to multiaxial deformations[12]. Combined tests or biaxial tests are always recommended. Unfortunately, these tests are time-consuming and sometimes require specially designed equipment. Due to these hurdles, we propose an alternative method to characterize and to validate material constants derived from synthetic biaxial data. In this method, the uniaxial tensile data are used to generate the biaxial data based on the constant true Young modulus with varying Poisson's ratio approach[13]. The synthetic biaxial data are subsequently employed to characterize the material constants.

To test our proposed method, the material constants were derived from both combined test data (conventional method) and synthetic biaxial data (proposed method). The material constants obtained from the two methods were then fed into the FEA program to predict rubber behaviour. The prediction accuracy was then compared.

CONSTANT TRUE YOUNG MODULUS WITH VARYING POISSON'S RATIO APPROACH

In uniaxial tension, the true elastic stress varies linearly with strain over the strain range encountered in most engineering applications (up to 100% strain). Assuming incompressibility, the relationship between true stress (t_i) and engineering stress (σ_i) is defined as

$$t_i = \frac{\sigma_i}{\lambda_j \lambda_k} = \sigma_i \lambda_i \quad (2)$$

where λ is the extension ratio and the subscripts refer to the directions of the principal stresses. Based on the standard equations of elasticity at low strain where the Young modulus (E) is constant, the nonlinear multi-axial behaviour of rubber can be accommodated by considering Poisson's ratio to be a function of the principal extension ratios. Thus, the stress-strain relations are given by

$$\varepsilon_1 = \lambda_1 - 1 = (1/E)[t_1 - \nu(t_2 + t_3)] \quad (3)$$
$$\varepsilon_2 = \lambda_2 - 1 = (1/E)[t_2 - \nu(t_3 + t_1)] \quad (4)$$
$$\varepsilon_3 = \lambda_3 - 1 = (1/E)[t_3 - \nu(t_1 + t_2)] \quad (5)$$

where ν is Poisson's ratio and ε_i is the strain. For plane stress, when $t_3 = 0$, eliminating t_2/E or t_1/E from (3) and (4) give, respectively,

$$t_1 = \frac{E[(\lambda_1 - 1) + \nu(\lambda_2 - 1)]}{1 - \nu^2}, \quad (6)$$
$$t_2 = \frac{E[(\lambda_2 - 1) + \nu(\lambda_1 - 1)]}{1 - \nu^2}. \quad (7)$$

Since incompressibility is assumed, $\lambda_3 = (\lambda_1 \lambda_2)^{-1}$, and (5)–(7) then yield

$$\nu = \frac{\lambda_1 \lambda_2 - 1}{\lambda_1 \lambda_2 (\lambda_1 + \lambda_2 - 1) - 1}. \quad (8)$$

Based on this approach, only a single material property (a true Young modulus obtained from tensile test) is required to describe the rubber elastic behaviour at low strains. Unfortunately, most FEA programs dealing with nonlinear hyperelastic analysis only accept material data inputted in terms of the material constants. Consequently, conversion of the true Young modulus into the material constants is necessary.

CONVERSION OF TRUE YOUNG MODULUS INTO MATERIAL CONSTANTS

Conversion of true Young modulus into material constants was done by using a specially written program called elastic.exe. With a given specified maximum strain, the program generates a series of the two extension ratios λ_1 and λ_2. Initially, six equally spaced values of λ_1 (excluding 0% strain, $\lambda_1 = 1$) up to a value corresponding to the maximum strain are selected. For the kth value of λ_1 (arranged lowest first), $4+k$ equally spaced values of λ_2 are selected ($k = 1, \ldots, 6$). The program then employs the true Young modulus to calculate the principal true stresses, based on (6)–(8), for each of the 45 combinations of the extension ratios.

The stress-strain relation of rubber subjected to biaxial deformation is given by

$$t_i - t_j = 2(\lambda_i^2 - \lambda_j^2) \left[\frac{\partial W}{\partial I_1} + \lambda_k^2 \frac{\partial W}{\partial I_2} \right]. \quad (9)$$

Differentiating (1) with respect to I_1 and I_2 it can be seen that $\partial W / \partial I_1 = C_{10}$ and $\partial W / \partial I_2 = C_{01}$. Using these results in (9) gives

$$t_i - t_j = 2(\lambda_i^2 - \lambda_j^2) \left[C_{10} + \lambda_k^2 C_{01} \right]. \quad (10)$$

Hence if the relationship between true stress and extension ratio is known, the material constants can be obtained by a multiple regression technique.

EXPERIMENTAL TESTS

Materials

The rubber compound used in this study was supplied by NCR rubber industry (Thailand) Co., Ltd., and consisted of a carbon black-filled natural rubber compound typically employed in the production of bridge bearing.

Determination of material constants from the combined test data

To derive the material constants from the combined test data, the tension and simple shear deformation modes were studied. The tests were carried out using a universal testing machine (Instron 4301).

For the tension test, dumbbell specimens (Die type 1 according to ISO 37) were prepared. As FEA can only simulate the elastic response, the viscous response must be eliminated. To achieve this, the specimens were initially extended using a crosshead speed of 50 mm/min to approximately 80% strain and then allowed to return to their original state with the same crosshead speed. This testing procedure was

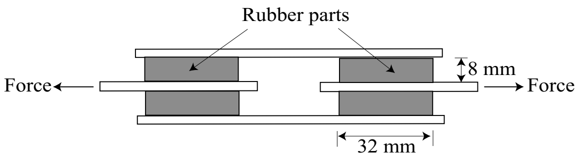

Fig. 1 Quadruple specimen for simple shear test.

Table 1 Equilibrium stress-strain data obtained from tensile and simple shear tests.

Strain (%)	Stress (MPa)	
	Tension	Shear
10	0.361	0.053
20	0.686	0.163
30	0.975	0.270
40	1.251	0.365
50	1.529	0.453
60	1.828	0.536
70	2.155	0.617
80	2.528	0.699

repeated until the equilibrium stress-strain curve was obtained.

For the simple shear test, the quadruple specimens (Fig. 1) were prepared and tested with a crosshead speed of 50 mm/min. As with the tension test, the specimens were deformed several times prior to recording the equilibrium stress-strain relation. Table 1 shows the equilibrium stress-strain relation obtained from tensile and simple shear tests.

After inputting the data from the tests into the FEA program, MSC.MARC, a curve fitting process was carried out with the constraint that only non-negative coefficients are allowed. This yielded the values $C_{10} = 0.43058$ MPa and $C_{01} = 0$. Since the value of C_{01} is zero, the material model then becomes the widely known neo-Hookean model.

Determination of material constants from synthetic biaxial data

Based on the equilibrium tensile stress-strain data, the true stress at any given strain was calculated using (2) and the plot of true stress against strain was made (Fig. 2). At low strain (up to 50% strain), a straight line was obtained. Its slope is equal to the true Young modulus (3.15 MPa). Using this value in elastic.exe gives $C_{10} = 0.40444$ MPa and $C_{01} = 0.10607$ MPa.

FINITE ELEMENT ANALYSIS

To validate the obtained material constants, finite element analysis was performed to predict the rubber behaviour subjected to various deformation modes (tension, simple shear, and compression). The computed results were then compared with the experimental results. In the present study, all FEA analysis was performed using the MSC.MARC 2005 program which uses the large strain-updated Lagrangian formulation to solve such nonlinear static mode problems.

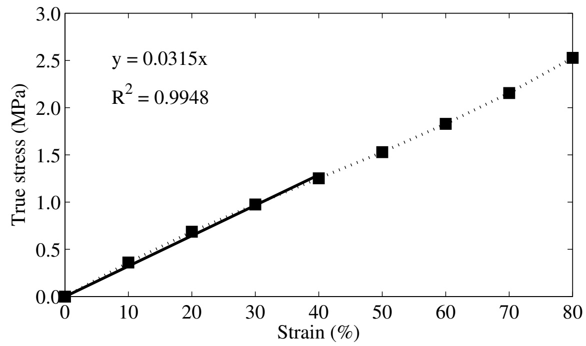

Fig. 2 Plot of true stress against strain

Tension

By taking advantage of symmetry, only a quarter of domain was modelled (Fig. 3). This model consisted of 150 elements and 182 nodes. We chose to analyse plane stress to reduce computational time. The boundary conditions were applied as follows. The left and bottom edges of the model were fixed. A variable tension force was then applied at the right edge and the corresponding displacement was calculated.

Simple shear

Only one forth of the quadruple specimen (one cylinder) was modelled (Fig. 4). This model consisted of 3000 elements and 3312 nodes. A contact boundary

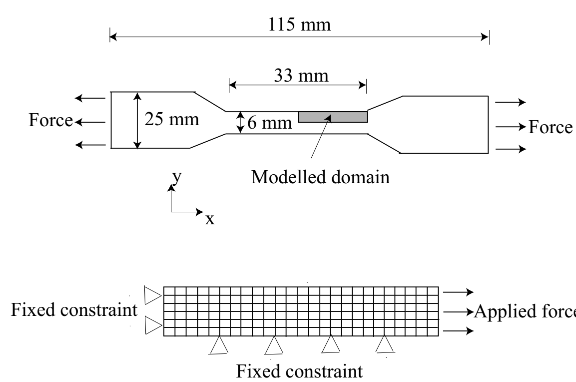

Fig. 3 Selected domain and finite element model of the specimen for tensile test.

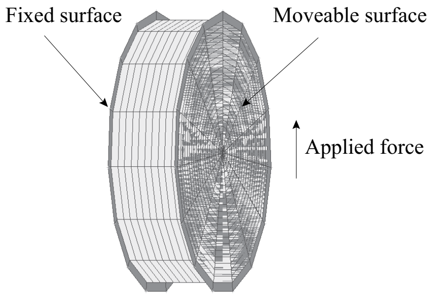

Fig. 4 Finite element model of the specimen for simple shear test.

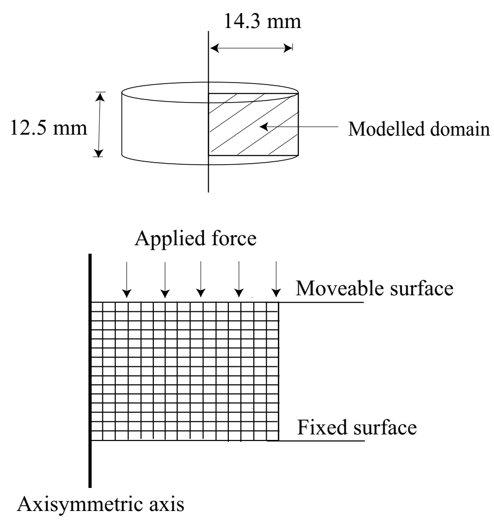

Fig. 5 Selected domain and finite element model of the specimen for compression test.

condition was used to represent the bonded surfaces between rubber and steel parts. The variable shear force was then applied to one of the contact surfaces while the other contact surface was fixed. The corresponding displacement was then computed.

Compression

Since the specimen is symmetric along the centre axis, only half of the cross-section was modelled (Fig. 5). The model consisted of 225 axisymmetric elements and 257 nodes. A contact boundary condition was used to represent the bonded surfaces. The variable compressive force was then applied to one of the contact surfaces while the other was fixed. Finally, FEA was performed to calculate the corresponding displacement.

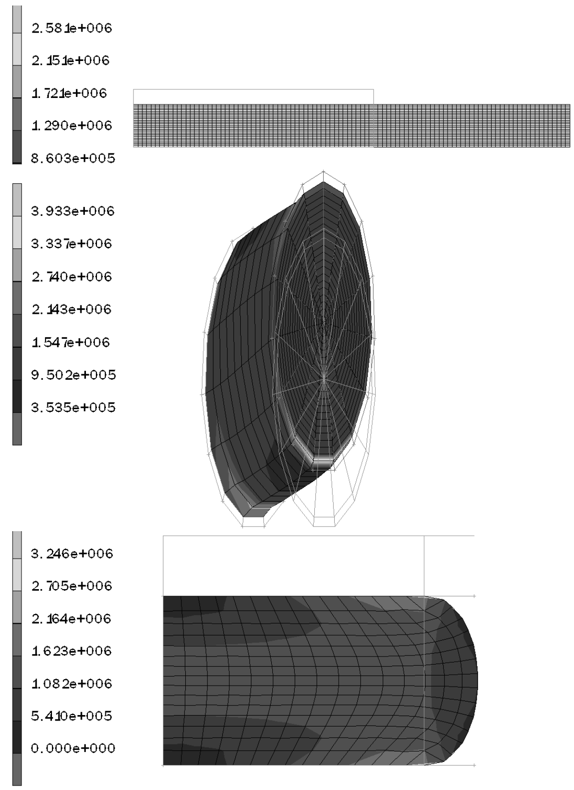

Fig. 6 Examples of deformed shapes and stress distribution of the specimens subjected to tension, simple shear, and compression.

RESULTS AND DISCUSSION

Examples of the deformed shape of the specimens subjected to various deformation modes and the resulting distribution of stress are shown in Fig. 6. To check the accuracy of FEA results, we compare the predicted force-displacement data with the experimental data (Fig. 7). For tensile and simple shear tests, the material constants derived from the synthetic biaxial data give significantly better prediction than those derived from the combined test data. It can also be observed that the material constants derived from the synthetic biaxial data could be used to describe the rubber behaviour under tension and shear almost throughout the tested strain range, except at very low strains where the predicted and experimental results are noticeably different. However, for the compression test, the material constants derived from the combined test data give better prediction than those derived from the synthetic biaxial data. Again, the predicted results are significantly different from the experimental results at very low strains.

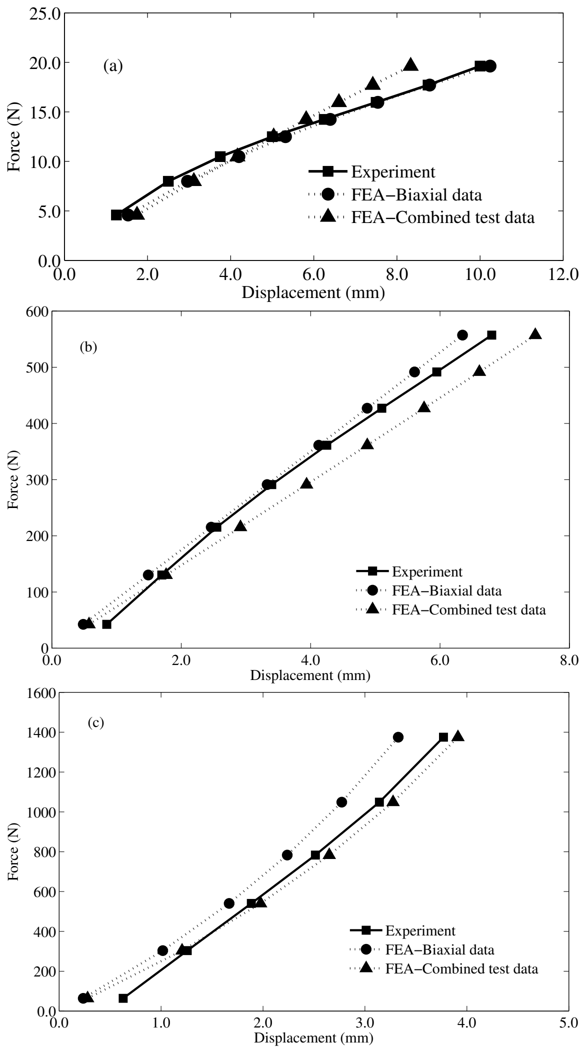

Fig. 7 Force-displacement relationships obtained from FEA and experiment: (a) tension (b) simple shear (c) compression.

CONCLUSIONS

The results reveal that the material constants derived from either the combined test data or the synthetic biaxial data could be used to describe the rubber elastic behaviour under various deformation modes. Compared to the material constants derived from the combined test data, the material constants derived from the synthetic biaxial data provide better prediction of rubber behaviour in both tension and simple shear modes. Even though the material constants derived from the synthetic biaxial data give poorer prediction under the compression mode, the difference between the predicted and experimental results, except at very low strain, is still relatively low (∼11–12%) and is considered to be within the acceptable range. As a consequence, it could be said that the proposed method could be used as an alternative to characterize the material constants.

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