ScienceAsia 28 (2002) : **365-376**

Use of Globally Linearizing Control with Extended Kalman Filter for pH Control of a Wastewater Treatment Process

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Received 6 Aug 2001
Accepted 29 May 2002

ABSTRACT Several chemical industrial plants such as electroplating and metal finishing plants have used strong acids and strong bases in production lines. These acids and bases are then released from the production lines to a wastewater treatment system and then treated to achieve compliance with an effluent standard. It is well known that the pH control of a wastewater treatment process is one of the most challenging control problems due to high non-linearity and time-variance of the pH value during pH titration. A conventional PID controller and an on-off controller are rarely able to handle this non-linearity resulting in poor control performances. Therefore, advanced nonlinear control techniques are needed.

This research presents simulation study of Globally Linearizing Control (GLC) together with an extended Kalman Filter to control pH of the wastewater treatment process of an electroplating plant. The GLC, one of the advanced nonlinear model-based control techniques, has been developed for both Single-Input and Single-Output (SISO) or Multi-Input and Multi-Output (MIMO) nonlinear process systems. Since the GLC is a model-based control technique, it needs measurements and values of states and parameters, which are neither all measurable nor known exactly. Therefore, the extended Kalman Filter, a state and parameter estimation technique, is applied to estimate unavailable or unknown states and parameters, and these estimates are incorporated in the control action determination of the GLC algorithm. Simulation results have shown that in a nominal case, the GLC is able to control the pH of the system to a desired set point and its control performance is equivalent to that of a PID one. In the presence of plant/model mismatch, the GLC is still able to handle this mismatch and gives good control performance whereas the PID gives poor control response; the GLC is much more robust than the PID controller.

KEYWORDS: Globally Linearizing Control (GLC), an extended Kalman Filter, Integral of Absolute Error (IAE) and plant/model mismatch.

INTRODUCTION

Electroplating plants are one of the industries that cause numerous pollution problems due to waste-water released from the metal finishing process containing heavy metals eg Nickel, Chromium -c. The heavy metals in the wastewater are a poiso in the environment. Therefore, it is necessary to ave a reliable wastewater treatment system, w ich includes a measurement monitoring device a d a controlling system. A conventional method is sed to control the pH value of the wastewate to precipitate the heavy metals by adjusting acid or ase in appropriate amounts. Either batch or contin ous processes can be used depending on the quantit of wastewater and reaction time. However, pH con of the wastewater treatment is difficult because of nonlinear behavior and sensitivity to changes in manipulated variables.[1]

Normally, several industries use linear controllers, such as PID, and on-off controllers to control the pH of wastewater to a desired set point, however the performance of such controllers are poor. Chemicals overdose can occur, therefore, the closed-loop control response oscillates and is sometimes unstable. Advanced nonlinear model-based control techniques that can handle these processes are needed.[2]

Globally Linearizing Control (GLC) is one of the advanced nonlinear model-based control techniques, that has been proposed and applied to control nonlinear system. The basic concept of GLC is to transform nonlinear process models to linear process models by input-output linearization.[3] Feedback control law[4], via external linear controller, is incorporated to reduce offset of controlled variables.[5] The GLC was applied to control batch, semi-batch and polymerization processes with great success.[6, 7]

This work presents simulation study of GLC to control pH values of wastewater from an electroplating plant. The extended Kalman Filter[8] is included to estimate unmeasured concentrations of heavy metal ions in wastewater. These estimates are incorporated into GLC control formulation to determine manipulated input in order to control the pH to a desired set point.

Process and Mathematical Modeling

A wastewater treatment process of an electroplating plant as shown in Figure 1 is studied here. Wastewater from the electroplating process consists of alkali wastewater, acid wastewater, chrome wastewater and wastewater. Each wastewater is fed into a reduction tank, then, the flow rate of hydrochloric acid is adjusted to reduce hexavalent chromium ions to be trivalent chromium ions.

The reaction in a reduction tank is as follows:

$$CrO_3 + 3HCl \rightarrow CrCl_3 + 3(H_3O)^+$$

Next, trivalent chromium ions are precipitated in the reaction tank, by adjusting pH value to over 9 with sodium hydroxide. Nickel ions are precipitated in another precipitating tank, by adjusting pH value to over 11 with sodium hydroxide. Then, the heavy metal ions-free wastewater is neutralized in a neutralizing tank.

The reaction in a reaction tank is as follows:

$$CrCl3 + 3NaOH \rightarrow 3NaCl3 + Cr(OH)_3$$

The objective of this work is to control nickel concentration and pH in the precipitating tank as well as pH in the neutralizing tank to desired set points as illustrated in Figure 2. Inlet wastewater (F_{in}) has pH value about 9 and nickel ions concentration ($C_{Ni,in}$) about 8.5179x10^{-5} mole per liter. Flow rate of sodium hydroxide is adjusted for controlling pH to 11, equivalent to hydroxide ions concentration ($C_{OH,out}$) of 10^{-3} mole per liter. With this pH, nickel ions are completely precipitated. Outlet wastewater from the precipitating tank with the pH of 11 and Nickel ions ($C_{Ni,out}$) not exceeding 1.7036x10^{-5} mole per liter is then sent to the neutralizing tank. Then, flow rate of hydrochloric acid (F_a) is used to adjust the pH to 7. The neutralized water is then released to environment.

It is assumed that both tanks are perfectly mixed and isothermal. Other assumptions made in the formulating process models include; reactions involved are irreversible reactions, all pH values are measurable, and the feed concentration is known. Under the assumptions above, material balances of the precipitating tank can be written as follows:

- Total mass balance (Density is assumed to be constant)

$$\frac{dV}{dt} = F_{in} - F_{out} + F_b \tag{1}$$

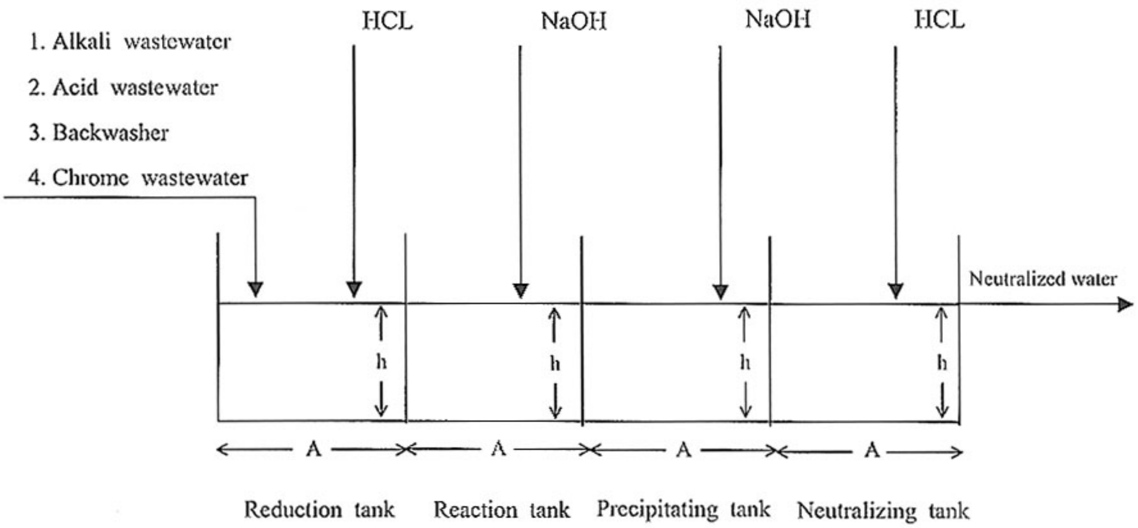

Fig 1. Wastewater treatment process of electroplating plant.

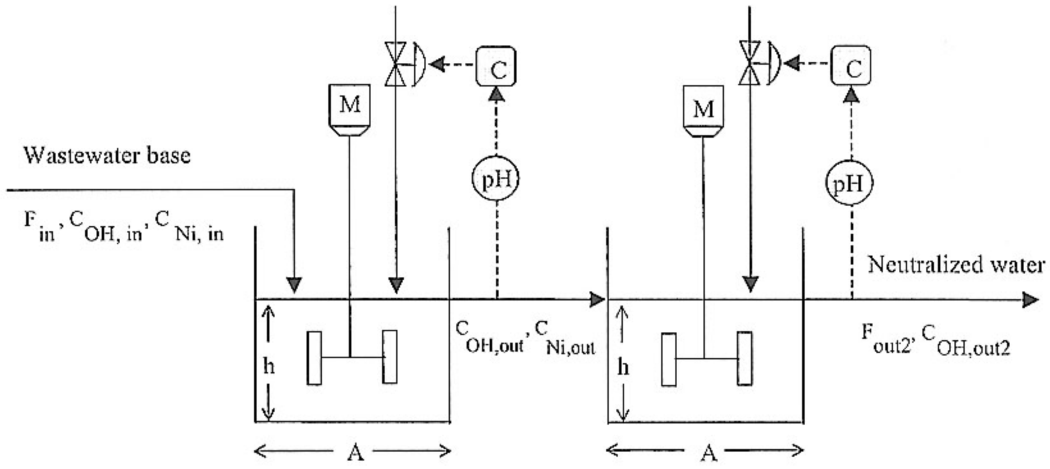

Figure 2: Precipitating tank (left) and neutralizing tank (right)

Fig 2. Precipitating tank (left) and neutralizing tank (right).

- Hydroxide ions (OH^-) balance

$$\frac{dVC_{OH,out}}{dt} = F_{in}C_{OH,in} - F_{out}C_{OH,out} + F_bC_{OH,b} - VN_2r_{OH} \tag{2}$$

- Nickel ions (Ni_2^+) balance

$$\frac{dVC_{Ni,out}}{dt} = F_{in}C_{Ni,in} - F_{out}C_{Ni,out} - VN_1r_{Ni} \tag{3}$$

The reaction rate of hydroxide ions (OH^-) per unit volume is

$$r_{OH} = k_1C_{Ni,out}C^2_{OH,out} \tag{4}$$

The reaction rate of Nickel ions ($Ni_2{}^+$) per unit volume is

$$r_{Ni} = k_1C_{Ni,out}C^2_{OH,out} \tag{5}$$

Similarly, with the assumptions above, material balances of the neutralizing tank are as follows:

- Total mass balance

$$\frac{dV}{dt} = F_{out} - F_{out2} + F_a \tag{6}$$

- Hydroxide ions (OH^-) balance

$$\frac{dVC_{OH,out2}}{dt} = F_{out}C_{OH,out} - F_{out2}C_{OH,out2} - F_aC_{H,a} \tag{7}$$

Replacing equations (1) and (4) into equation (2), equations (1) and (6) into equation (7), and equations (1) and (5) into equation (3), we obtain

$$\frac{dC_{OH,out2}}{dt} = \frac{1}{V}\Big(F_{in}(C_{OH,in} - C_{OH,out}) + F_b(C_{OH,b} - C_{OH,out}) - \cdot N_2k_1C_{Ni,out}C^2_{OH,out}\Big) \tag{8}$$

$$\frac{dC_{OH,out2}}{dt} = \frac{1}{V}\Big(F_{in}(C_{OH,out} - C_{OH,out2}) + F_b(C_{OH,out} - C_{OH,out2}) - F_a(C_{OH,out2} + C_{H,a})\Big) \tag{9}$$

$$\frac{dC_{Ni,out}}{dt} = \frac{1}{V}\Big(F_{in}(C_{Ni,in} - C_{Ni,out}) - F_bC_{Ni,out}\Big) - N_1k_1C_{Ni,out}C^2_{OH,out} \tag{10}$$

Globally Linearizing Control (GLC)

Globally Linearizing Control (GLC), one of the advanced nonlinear control techniques, uses mathematical models of a plant to determine control action. Process models used can be either linear or nonlinear. In this work, mathematical models of the system written in the ODE form are as follows:

$$\frac{dx}{dt} = f(x) + g_1(x)F_b + g_2(x)F_a \qquad (11)$$

Here, $x = [\, C_{OH,out} \quad C_{OH,out2} \quad C_{Ni,out} \,]^T$

$$f(x) = \begin{bmatrix} F_{in}(C_{OH,in} - C_{OH,out})/V - N_2 k_1 C_{Ni,out} C^2_{OH,out} \\ F_{in}(C_{OH,out} - C_{OH,out2})/V \\ F_{in}(C_{Ni,in} - C_{Ni,out})/V - N_1 k_1 C_{Ni,out} C^2_{OH,out} \end{bmatrix}$$

$$g_1(x) = \begin{bmatrix} (C_{OH,b} - C_{OH,out})/V \\ (C_{OH,out} - C_{OH,out2})/V \\ -C_{Ni,out}/V \end{bmatrix},$$

$$g_2(x) = \begin{bmatrix} 0 \\ -(C_{OH,out2} + C_{H,a})/V \\ 0 \end{bmatrix}$$

$$y_1 = C_{OH,out},\ y_2 = C_{OH,out2}\ \ h_1(x) = C_{OH,out},\ h_2(x) = C_{OH,out2}$$

The controlled variables (x) are $C_{OH,out}$, $C_{OH,out2}$, $C_{Ni,out}$, the manipulated variables (u) are F_b, F_a and the measured outputs (y) are $C_{OH,out}$, $C_{OH,out2}$,. Then, we obtain

$$F_b = \left(V/\beta_{11}(C_{OH,b} - C_{OH,out})\right) * \left(v_1 - \beta_{10}C_{OH,out} - \beta_{11}(F_{in}(C_{OH,in} - C_{OH,out})/ V - N_2 k_1 C_{Ni,out} C^2_{OH,out})\right) \qquad (12)$$

$$F_a = \left(V/\beta_{21}(C_{H,a} + C_{OH,out2})\right) * \left((\beta_{21}(C_{OH,out} - C_{OH,out2})F_b/V) - v_2 + \beta_{20}C_{OH,out2} + \beta_{21}(F_{in}(C_{OH,out} - C_{OH,out2})/V)\right) \qquad (13)$$

Where:

$$v_1 = \beta_{10}C^{sp}_{OH,out}(t) + K_{C_1}[\, C^{sp}_{OH,out}(t) - C_{OH,out}(t)\,] + \frac{K_{C_1}}{\tau_{I_1}}\int_0^t [\, C^{sp}_{OH,out}(t) - C_{OH,out}(t)\,]\,dt$$

$$v_2 = \beta_{20}C^{sp}_{OH,out2}(t) + K_{C_2}[\, C^{sp}_{OH,out2}(t) - C_{OH,out2}(t)\,] + \frac{K_{C_2}}{\tau_{I_2}}\int_0^t [\, C^{sp}_{OH,out2}(t) - C_{OH,out2}(t)\,]\,dt$$

Globally Linearizing Control with State and Parameter Estimator

The GLC technique uses process models of a system to determine control action, however, in reality neither process variables nor parameters are all measurable or known exactly. In this situation, state and parameter estimator is incorporated to estimate unmeasurable variables and unknown/ uncertain parameters. Here, the extended Kalman Filter is applied to produce estimates of true process values. These estimates are then incorporated into the GLC technique to control the tanks as well as cater for plant/model mismatch. Details regarding the extended Kalman Filter are given in Appendix A. Figure 3 shows the flowchart of Globally Linearizing Control with state and parameter estimator.

Here, the extended Kalman Filter is used to estimate unmeasured state variable $C_{Ni,out}$ based on information of measured, manipulated variables F_b and F_a, and measured process output variables $C_{OH,out}$ and $C_{OH,out2}$. Then GLC uses these estimates as well as all measured variables to calculate appropriate F_b and F_a in order to control variables $C_{OH,out}$ and $C_{OH,out2}$ to desired values.

Simulation Results

The GLC with the extended Kalman Filter is applied to control the pH in the precipitating tank to 11. With this pH, the nickel concentration of wastewater in the precipitating tank is below 1.7036x10-5 mole per liter. Simultaneously, another GLC with the extended Kalman Filter is used to control the pH of the heavy metal ions-free wastewater to 7. The performance of GLC with the extended Kalman Filter is then compared to those of conventional PID controller. To have a fair comparison of both the GLC and PID controllers, both controllers are turned to give about the same control responses in the nominal case; the performance indices (IAE) of both the GLC and the PID controllers are equivalent in the nominal case. In addition, the tuning parameters of the extended Kalman filter; P, Q and R, are tuned to have good estimates of $C_{Ni,out}$. The values of PID and GLC tuning parameters are given in Table 1 and 2, respectively and the extended Kalman filter tuning parameters are shown below.

The extended Kalman Filter parameters are as follows:

$$P = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.01 \end{bmatrix} \quad Q = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.01 \end{bmatrix}$$

Figures 4 - 7 show the control responses of GLC with the extended Kalman Filter and PID controller in nominal case with the disturbance of inlet wastewater flowrate of 30 % at time 0.5 minute. Both GLC with the extended Kalman Filter and PID controller can control the pH of wastewater in the precipitating tank at the set point with small overshoot (figures 4 and 5, respetively). With this pH, nickel ions are precipitated; what remains in the wastewater is compliance to the effluent wastewater standard. However, the performance of both controllers in the control of pH of wastewater in the neutralizing tank are different as can be seen from figures 6 and 7. The GLC with the extended Kalman Filter gives a good control response while the PID controller provides a poor control response; the PID controller gives oscillatory control response. These results show that the GLC with the extended Kalman Filter can cope with high non-linearity of the change in pH of the system because the GLC is a nonlinear controller with external linear control.

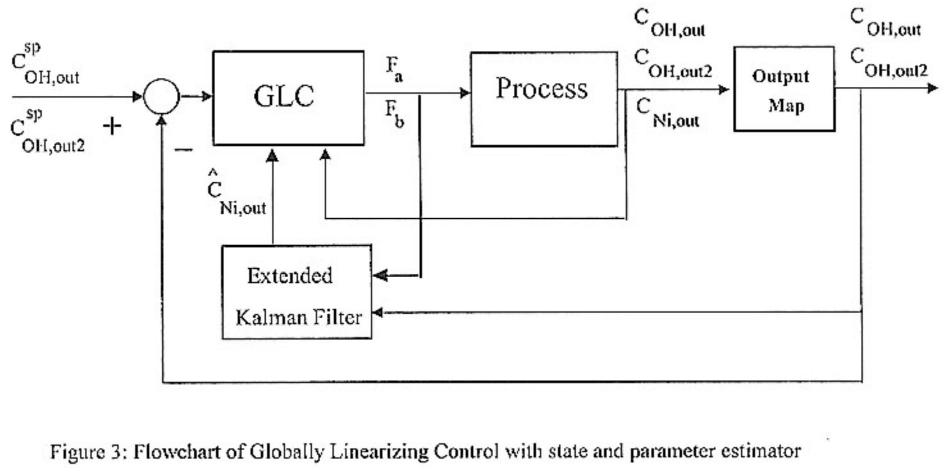

Fig 3. Flowchart of Globally Linearizing Control with state and parameter estimator.

Table 1. Tuning parameter of PID for pH and nickel concentration control of wastewater in precipitating tank and pH control of wastewater in neutralizing tank.

PID Tuning Parameters								
K_{C1}	=	45000	τ_{I1}	=	0.4728	τ_D	=	0.001
K_{C2}	=	5.0900	τ_{I2}	=	0.31	τ_D	=	0.1

Table 2. Tuning parameter of GLC for pH and nickel concentration control of wastewater in precipitating tank and pH control of wastewater in neutralizing tank.

Controller Parameters					
K_{C1}	=	0.479	K_{C2}	=	-2
τ_{I1}	=	0.505	τ_{I2}	=	0.65
β_{10}	=	1	β_{11}	=	1
β_{20}	=	1	β_{21}	=	1

The GLC uses process models of the plant to determine control action, so the nonlinear behavior of the plant is included in the GLC formulation. On the other hand, PID controller is a linear feedback control and the control performance is tuned at an operating condition, therefore, the PID controller cannot handle the nonlinear system.

Robustness Test

It is well known that not all process parameters are known exactly. Robustness tests are needed to evaluate the performance of the GLC with the extended Kalman Filter in the presence of plant/model mismatch. Here, the plant/model mismatch in the reaction rate constant and the tank volume are considered as follows:

a. The reaction rate constant (k_1) decreases 30%,
b. The tank volume (V) decreases 30%

Figures 8 - 11 show the control responses of GLC with the extended Kalman Filter and PID controller in the presence of plane/model mismatch in the reaction rate constant (k_1) (decrease 30%). In this case, both GLC with the extended Kalman Filter and PID controller can control the pH in the precipitating tank at the set point with small overshoot. They also give desired control responses for the neutralizing tank. This means meant that both controllers can handle with the plant/model mismatch in the reaction rate constant.

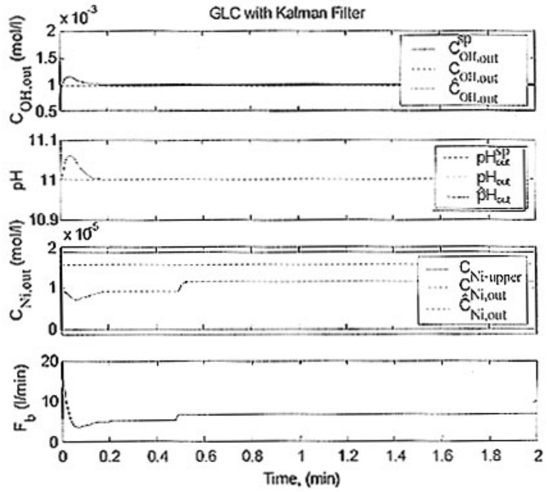

Fig 4. The pH and nickel concentration control in precipitating tank using the GLC with the extended Kalman Filter, when F_{in} is increased by 30%.

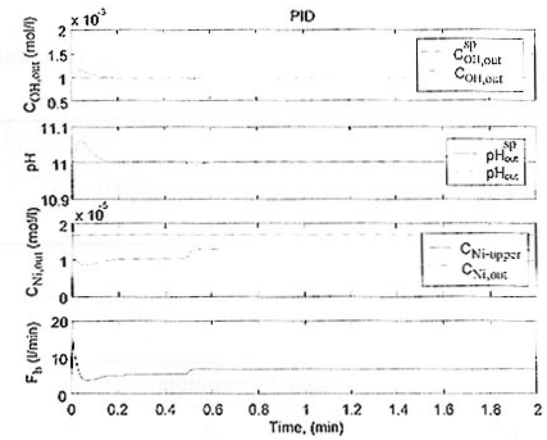

Fig 5. The pH and nickel concentration control in precipitating tank using the PID controller, when F_{in} is increased by 30%.

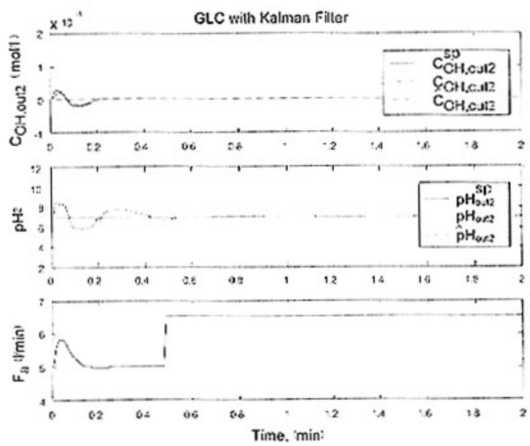

Fig 6. The pH control of wastewater in neutralizing tank using the GLC with the extended Kalman Filter, when F_{in} is increased by 30%.

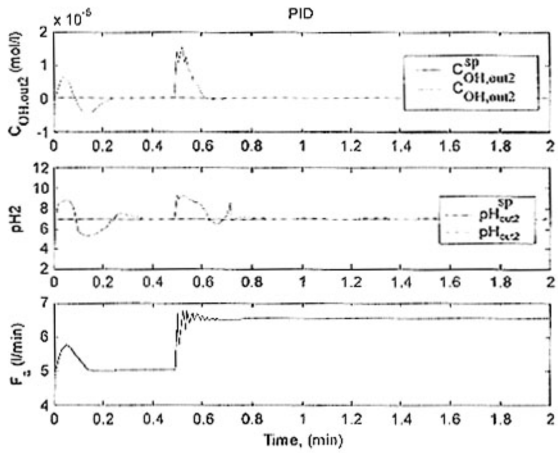

Fig 7. The pH control of wastewater in neutralizing tank using the PID controller, when F_{in} is increased by 30%.

Figures 12 - 15 show the control responses of GLC with the extended Kalman Filter and PID controller in the presence of plant/model mismatch in the volume of the tank (V) (decrease 30%). It can be seen from figures 12 and 14 that the GLC with the extended Kalman Filter is still able to give a good control response with some overshoot. The pH in the precipitating tank is controlled at 11 and the nickel ions are compliance to the wastewater effluent standard. The pH in the neutralizing tank is regulated at 7. The PID controller, on the other hand, cannot provide good control responses. Obviously, in the neutralizing tank, the PID controller cannot control the pH at the desired set point; unstable response occurs. This means that the GLC with the extended Kalman Filter can handle with the plant/model mismatch in the volume of the tank whereas the PID cannot handle this plant/model mismatch.

As mentioned above, as the GLC is a model-based controller, the performance of the GLC is poor in the presence of plant/model mismatch. However, the inclusion of the extended Kalman Filter can cater for the plant/model mismatch as seen in this simulation result.

The performances of both GLC with the extended Kalman Filter and PID controller are compared based on a performance index: Integral of Absolute Error

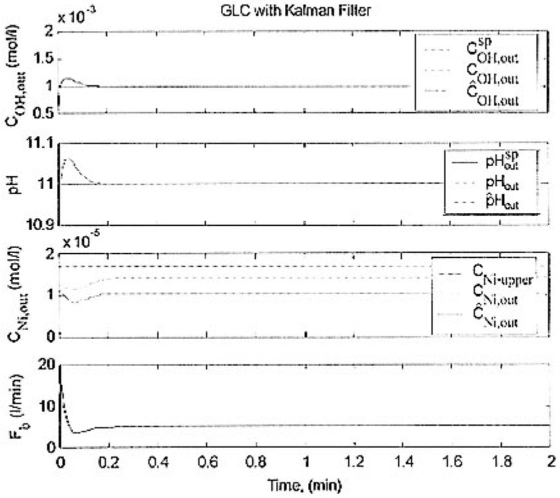

Fig 8. The pH and nickel concentration control in precipitating tank under plant-mismatch, in which k_1 decreases 30%, using the GLC with the extended Kalman Filter.

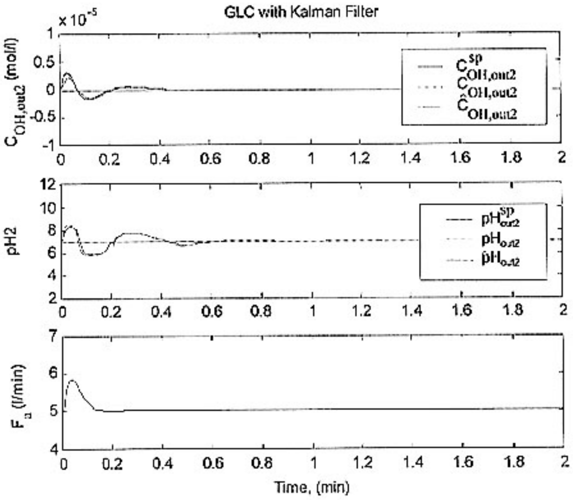

Fig 10. The pH control in neutralizing tank under plant-mismatch, in which k_1 decreases 30%, using the GLC with the extended Kalman Filter.

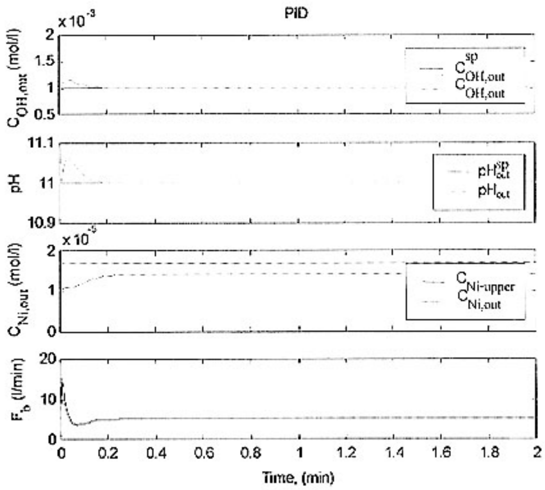

Fig 9. The pH and nickel concentration control in precipitating tank under plant-mismatch, in which k_1 decreases 30%, using the PID controller.

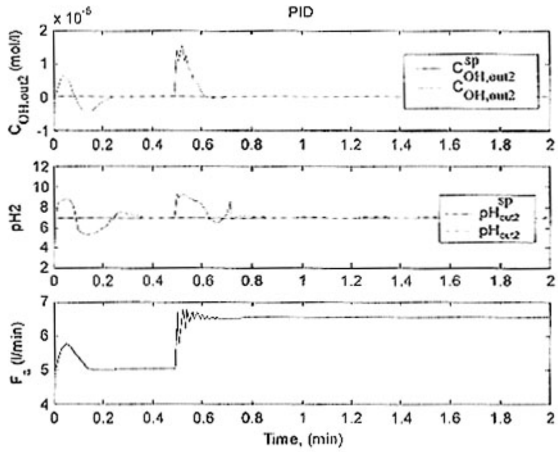

Fig 11. The pH control in neutralizing tank under plant-mismatch, in which k_1 decreases 30%, using the PID controller.

(IAE) as shown in Table 3 and Table 4. Obviously, the performances of the GLC with the extended Kalman Filter are better than those of the PID controller in the cases of feed flow rate disturbance and the plant/model mismatch in the volume of the neutralizing tank. For other cases, both controllers

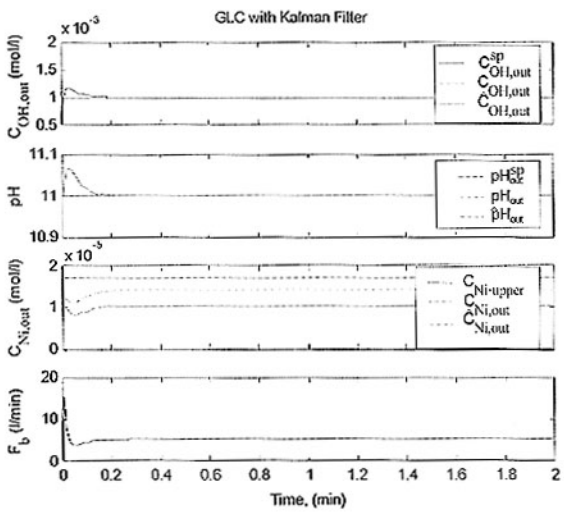

Fig 12. The pH and nickel concentration control in precipitating tank under plant-mismatch, in which V decreases 30%, using the GLC with the extended Kalman Filter.

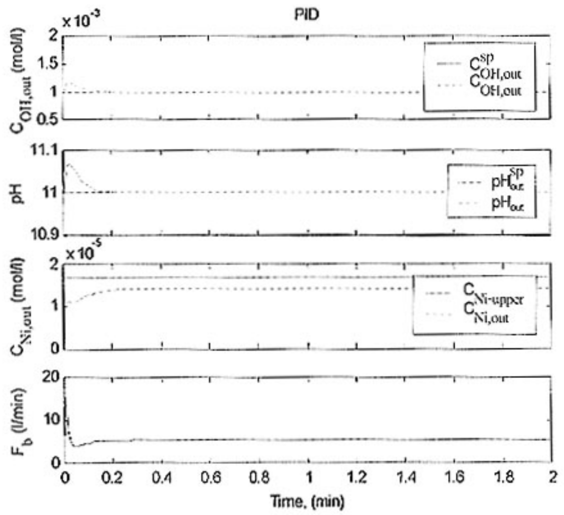

Fig 13. The pH and nickel concentration control in precipitating tank under plant-mismatch, in which V decreases 30%, using the PID controller.

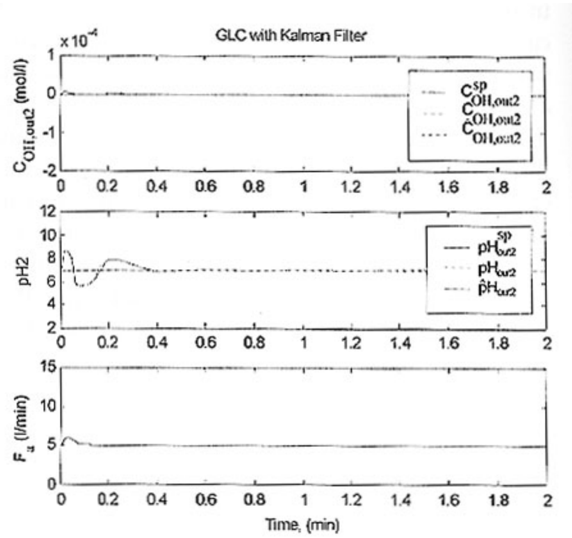

Fig 14. The pH control in neutralizing tank under plant-mismatch, in which V decreases 30%, using the GLC with the extended Kolman Filter.

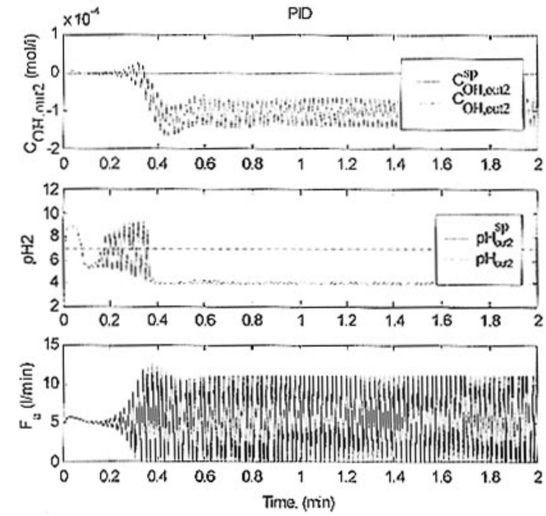

Fig 15. The pH control in neutralizing tank under plant mismatch, in which V decreases 30%, using the PID controller.

Table 3. Integral of Absolute Error (IAE) of the pH control in the precipitating tank.

Case study	GLC with Kalman	PID
1. Disturbance change		
a. F_{in} (30%increase)	4.467e-2	5.158e-2
2. Robustness test		
Plant/model mismatch case		
a. k_1 (30%decrease)	4.450e-2	4.454e-2
b. V (30%decrease)	4.427e-2	4.444e-2

Table 4. Integral of Absolute Error (IAE) of the pH control in the neutralizing tank.

Case study	GLC with Kalman	PID
1. Disturbance change		
a. F_{in} (30%increase)	3.367	6.066
2. Robustness test		
Plant/model mismatch case		
a. k_1 (30%decrease)	3.355	4.078
b. V (30%decrease)	3.657	55.075

give almost the same control performances as indicated by the values of IAE.

Conclusion

The control of a wastewater treatment process of an electroplating plant is studied in this work. This process consists of the precipitating and neutralizing tanks. The Globally Linearizing Control (GLC) with the extended Kalman Filter is applied to control the pH of the precipitating and neutralizing tanks. The performance of GLC is compared to that of the PID controller. Both controllers are tested in the presence of plant/model mismatches in the reaction rate constant and the volume of the tanks. Simulation results have shown that in a nominal case, GLC is able to control the pH of the system to a desired set point and its control performance is equivalent to that of the PID. Both controllers can also handle the plant/model mismatch in the reaction rate constant. In the presence of plant/model mismatch in the tank volume, GLC with the extended Kalman Filter is still able to handle this mismatch and gives a good control performance whereas PID gives a poor control response. By this is meant that GLC with the extended Kalman Filter can cater the plant/model mismatch and is much more robust than PID.

Nomenclature

A_k,B_k,C_k	'locally' linearized and depend upon the current estimate of x
C	concentration, mole/liter
F	volumetric rate, liter/min
k_1	reaction rate constant, $(min)^{-1}.(mole/liter)^{-2}$
K_c	gain of PI and PID controller
K	Kalman gain
N_1, N_2	stoichiometric constant, mole fraction
pH	pH value
P	the covariance of estimated errors
Q	the covariance of process noise
r	rate of reaction based on unit volume
R	the covariance of measurement noise
t	time
u	manipulated variables of process
v	manipulated variables from external linear controller
V	volume, liter
x	state variables
y	output variable

Greek Symbols

τ_D	derivative time constant of PID controller
τ_I	integral time constant of PI and PID controller
β	tuning parameter of input-output linearization
ε	a non-zero mean Gaussian process noise
η	a non-zero mean Gaussian measurement noise

Symbols

$\wedge$	estimated value

Subscripts

a	acid
b	base
H	hydrogen ions
in	inlet wastewater from the precipitating tank
Ni	nickel ions
OH	hydroxide ions
out	outlet wastewater from the precipitating tank
out2	outlet wastewater from the neutralizing tank
sp	set point
upper	upper bound

Abbreviation

IAE	integral absolute error
ODE	ordinary differential equations
PID	?????

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Appendix A

The Extended Kalman Filter

The Kalman Filter is a recursive algorithm which is easily to be implemented on a digital computer. This method determines the estimate of states which minimizes the variance of an estimation error. There are many versions of the Kalman Filter technique. Here, the extended Kalman Filter version is demonstrated.

The extended Kalman Filter is a popular technique for treating nonlinearities in the design of minimum variance estimators. Other methods of the same origin (Taylor series expansion) are iterative extended Kalman Filter, Guassian Second-Order Filter, and Linearized Kalman Filter.

By the extended Kalman Filter, the states of a system can be estimated with two steps. Firstly, using a Taylor's series expansion of the dynamic and measurement nonlinearities, neglecting second or higher-order terms, the nonlinear functions are linearized around a current point so that the nonlinear estimation problem is reduced to a linear one. Secondly, using a known solution to this linear estimation problem, the states are estimated.

The basic algorithm of the extended Kalman Filter can be summarized as follows:

Model Structure

Nonlinear models are denoted by the nonlinear differential equations:

$$\frac{dx}{dt} = f(x,u) + \varepsilon_k \tag{A1}$$

where f is a vector function of the state x and the controls u and measurements are expressed by the relation:

$$y_k = h(x) + \eta_k \tag{A2}$$

where h is a matrix function of the state variables.

ε_k is a non-zero mean Gaussian process noise representing not only actual input disturbances but also the uncertainty of process models and is a non-zero mean Gaussian measurement noise.

The covariance of ε_k and η_k are Q and R respectively. $X_{j/i}$ denotes the estimate of state x at time j given measurements at time i Similarly, $P_{j/i}$ denotes the covariance of estimated errors at time j given measurements at time i.

The linearization of these two equations (A1, A2) is produced, respectively, as follows:

$$\frac{dx_k}{dt} = A_k x_k + B_k u_k + \varepsilon_k \tag{A3}$$

$$y_k = C_k x_k + \eta_k \tag{A4}$$

where A_k, B_k, C_k are 'locally' linearized and depend upon the current estimate of x.

Given, $\hat{x}(0/0)$, $P(0/0)$, Q and R the estimation is obtained by a set of prediction and correction equations.

Prediction

Integrate the nonlinear state and covariance equations from time k to k+1 in order to acquire estimate $\hat{x}_{k+1/k}$ and $P_{k+1/k}$, we have

$$\hat{x}_{k+1/k} = f\left(\hat{x}_{k/k}, u_k\right) \tag{A5}$$

$$P_{k+1/k} = A_k P_{k/k} A_k^T + Q \tag{A6}$$

Correction

Calculate the Kalman gain matrix at time k+1

$$K_{k+1} = P_{k+1/k} C_{k+1}^T \left(C_{k+1} P_{k+1/k} C_{k+1}^T + R\right)^{-1} \tag{A7}$$

Compute the new estimate

$$\hat{x}_{k+1/k+1} = \hat{x}_{k+1/k} + K_{k+1}\left(y_{k+1} - h\left(\hat{x}_{k+1/k}\right)\right) \tag{A8}$$

Determine the new weighting matrix

$$P_{k+1/k+1} = \left(I - K_{k+1} C_{k+1}\right) P_{k+1/k} \left(I + K_{k+1} C_{k+1}\right)^T + K_{k+1} R K_{k+1}^T \tag{A9}$$

The performance of the extended Kalman Filter is relied on the choice of, $\hat{x}(0/0)$, $P(0/0)$, Q and R. $\hat{x}(0/0)$ can be determined from the initial condition, physical properties and process data of the system. $P(0/0)$ can be estimated from the uncertainty of the initial state estimate. Q and R can be chosen based on the error of process models and measurements respectively.

The extended Kalman Filter algorithm