RESEARCH ARTICLE

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Control of Spatiotemporal Disorder in an Excitable Medium

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ABSTRACT We consider partial differential equations that describe excitable media (phase variables: activator *u* and inhibitor *v*) and their disordered solutions consisting of erratically moving wave pieces. A pulse of a control parameter that displaces the *u*-nullcline parallely to the *v*-axis can split a wave into two waves moving in opposite directions. A properly applied train of such pulses completely destroys disorder. There exists a pulsing period for which the total time for disorder destruction is minimum. We discuss the application of this method to control heart fibrillation.

KEYWORDS: spatiotemporal chaos, turbulence, chaos control, excitable media

INTRODUCTION

The partial differential equations considered in this work are used to describe so-called excitable media. These media are characterized by a stable stationary state that is abandoned by a perturbation exceeding a given threshold (excitation); afterwards, the system slowly returns to the stationary state. Examples are the Belousov-Zhabotinsky (BZ) reaction, the oxidation of CO on catalytic surfaces, vesicles leading to calcium waves within living cells, the spread of epidemics, heart muscle and neural tissue. (See 9, 26 and references in 12, 13, 14) Amazingly, the general behaviour of these media can be described by differential equations that are common for the different systems (phase variables: activator *u* and inhibitor *v*). Solutions of these equations are circular, spiral and erratic waves. All these waves consist of a *u*-wavefront, which is followed by a *v*-waveback. Due to this inhibiting waveback, two colliding waves annihilate each other.

We will concentrate on solutions of the differential equations that consist of erratically moving wave pieces that result from breakup of wavefronts. We conjecture that the method of control that we will present here is generic, so as to be applicable to other excitable systems with different properties and with breakups occurring by different mechanisms. Similar erratic wavebreaks as those investigated here have been observed experimentally in the BZ reaction[14], in heart muscle[3] and on the surface of catalysts that oxidize CO[4]; mechanisms proposed for wavebreaks are: subcritical pitchfork bifurcation[8], lateral inhibition[11], discreteness of the system[10], inhomogeneities[1], or meandering of a spiral tip at such a short wavelength that the tip disrupts the wave and thus leads to new tips that erratically propagate outwards.[2, 22]

In heart muscle, spirals are associated with tachycardia (very fast heart beating) that may be followed by fibrillation (disordered mode causing erratic beating). The latter[23] is immediately fatal, accounting in the US for approximately one death out of ten.

The differential equations investigated in the present work do not describe a particular system, but are prototypical for excitable media (see eg 2, 16, 22 and references therein). We will simulate disordered modes and show how proper pulses of a control parameter can destroy these modes via wave splitting. We have shown before (experimentally and in PDE simulations of the BZ reaction[15]) that applying a control parameter pulse that displaces the *u*-nullcline parallely to the *v*-axis, can cause splitting into a forwards and a backwards moving wave. This "backfiring" is possible because the pulse causes the *v*-waveback to fail preventing backwards excitation by the *u*-wavefront. If it is a spiral wave that splits, then the spiral disappears because the two new spirals destroy each other upon collision.[15] This implies a possible way to control tachycardia, since for heart muscle, a pulse of electrical current displaces the voltage (*u*) nullcline parallely to the conductivity (*v*) axis.

The question we pose in the present work is if wave splitting can also be used to eliminate the erratically moving wave pieces that constitute a disordered mode. One can well anticipate that annihilation may not always be complete because of the disordered arrangement of the wave pieces. Thus, we shall consider sequences of pulses and ask if a finite number of pulses will cause complete annihilation. This is interesting insofar as one expects, in general, such a spatial system to be diffcult to stabilize because of its high phase space dimensionality.[22]

Methods

We use the model introduced by Bär and Eiswirth[2, 22] which is a modified version of a piecewise linearized FitzHugh-Nagumo (FHN) model[16]:

$$\frac{du}{dt} = \frac{1}{\varepsilon} u(1-u)\left[u - \frac{v+b}{a}\right] + D_u \nabla^2 u, \quad (1)$$

$$\frac{dv}{dt} = f(u) - v + D_u \nabla^2 v, \quad (2)$$

Here, $f(u) = 0$ for $u < 1/3$, $f(u) = 1\text{-}6.75\, u\,(u\text{-}1)^2$ for $1/3 \le u \le 1$ and $f(u) = 1$ for $u > 1$. We set $\varepsilon = 0.074$, $a = 0.84$, $b = 0.12$ (without pulse), $D_u = 1$ and $D_v = 0$. In Fig 1 we show the nullclines given by $\dot{u} = 0$, $\dot{v} = 0$ on the u-v-plane for the cases $b = 0.12$ (pulse off; Fig 1a) and $b = 0.02$ (pulse on; Fig 1b). In this work, all pulses have length $\tau = 2$. The pulse shifts the u - nullcline upwards. In the case of the BZ reaction[15] we could displace this nullcline by changing the light intensity. In the case of heart cell membranes (see eg 16) one could shift the u-nullcline by adding a stimulating current I_{st} to du/dt.

For numerical computations we used the explicit Euler method in a rectangular grid. Eqs (1) and (2) were integrated assuming a space step $dh = 0.5$ and a time step $dt = 0.0025$, imposing the condition that u and v are kept at the stationary state $u = v = 0$ at the boundary. This condition causes waves to vanish upon reaching the boundary; disappearance at the boundaries is observed for BZ waves as they reach glass walls, or waves in the heart as they reach the atrio-ventricular ring or the epi- or endocardial surface.[23] The medium is a square of side length $L = 70$. Note that L is well above the value $L \approx 20$, for which it has been shown[22] that disorder is only a transient.

Results

A pulse that drives the nullclines from those in Fig 1a to those in Fig 1b causes splitting due to the strongly diminished excitation gap AB and the enhanced width of the excitation front (longer CD in Fig 1b, as compared to that in Fig 1a). The v-waveback (corresponding mainly to EA) remains here almost unchanged. The enhanced u diffusing backwards along with a diminished excitation gap can thus excite the stationary state behind the wave. We illustrate the resulting splitting for one-dimensional simulations in Fig 2.

Results of simulations in two dimensions are shown in Figs 3 through 5. Fig 3a shows a snapshot of spatiotemporal disorder, which was obtained numerically as in.[2, 22] The pulse is turned on before Fig 3b, which shows splitting, analogously to the one-dimensional case (Figs 2b and 2c). Collision of waves then causes mutual annihilation (Figs 3c and 3d), which completely destroys the excited (white) wave fronts (shortly after Fig 3e). Depending on the initial disordered configuration, the scenario leading to destruction of disorder may be more

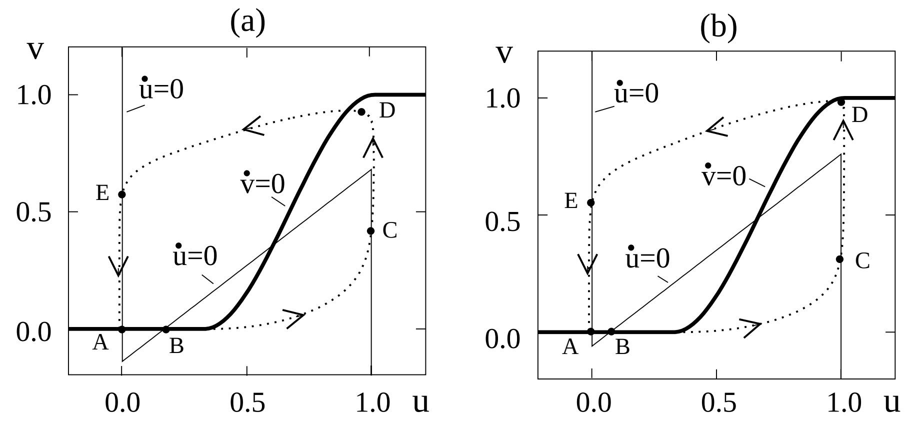

Fig 1. Nullclines corresponding to Eqs. (1) and (2). Fat curve: $\dot{v}$=0; thin straight lines: $\dot{u}$=0; dotted curve: trajectory after excitation. AB: Excitation threshold. CD: excited state. a) Pulse off; b=0.12. b) Pulse on; b=0.02.

complicated and take a longer time, as illustrated in Fig 4. Figs 4a and 4b show two different snapshots of the disordered state. Splitting after turning on the pulse is seen in Fig 4c. Waves then annihilate each other (Figs 4d through 4f) leading to a few small excited spots (Fig 4f). In contrast to the excited spots in Fig 3e, these spots are large enough to overcome the threshold AB (see Fig 1a), and thus trigger the formation of circular waves (Fig 4g), which grow and annihilate each other (Figs 4h through 4j); Fig 4j shows the situation shortly before complete annihilation. Using the same parameters as for Fig 3 and Fig 4, but turning on the pulse after a different configuration, one may not obtain complete annihilation after one pulse. This is illustrated in Fig 5, Figs 5a and 5b are snapshots of the disordered state. Splitting (Fig 5c) causes wave annihilation (Figs 5d through 5f) leaving open wave

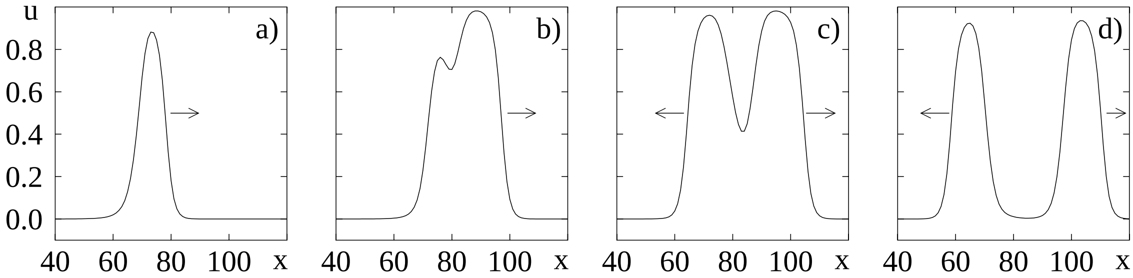

Fig 2. Splitting in 1D with a b-pulse. Ordinate scales are the same for a) - d). (a) t=0. The pulse is turned on at t=1 and off at t=3; (b) t=2.8 (c) t=3.8 (d) t=4.8.

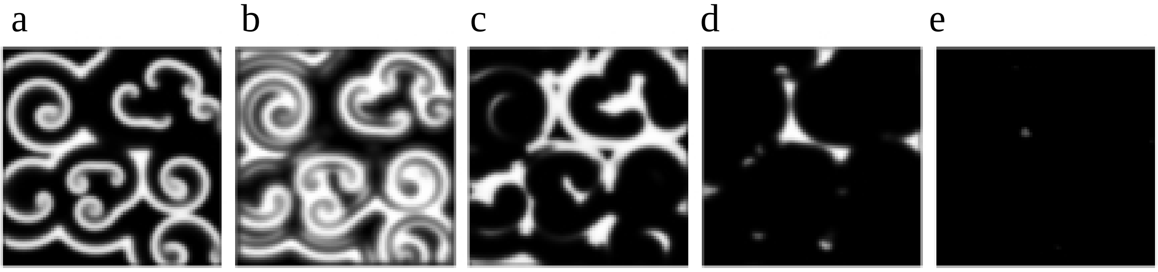

Fig 3. Splitting in 2D. Size of squares: 70 x 70. The grey levels display the variable u. White: excited state. (a) Disordered state; t=0. The pulse is turned on at t=0 and off at t=2. (b) Wavefronts split; t=1.5. (c)-(e): Wave annihilations due to collisions. (c) t=3.5 (d) t=5 (e) t=6. The remaining excited spots are too small to trigger new waves.

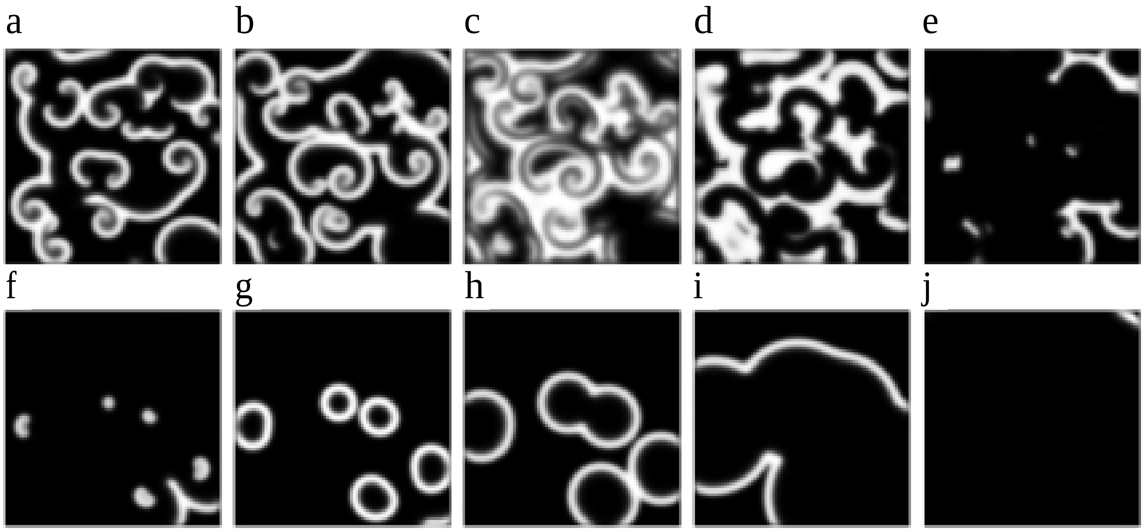

Fig 4. As Fig 3, but annihilation occurs after the formation of nearly circular waves (g) triggered by sufficiently large excited spots (f). (a) Disordered state; t=0. (b) Disordered state; t=3. The pulse is turned on at t=3 and off at t=5. Times t for (c)-(j): 4.5, 6, 7.5, 10, 13, 15.5, 22 and 33.

ends that curl into spirals (Fig 5f), so that disorder can develop anew (Figs 5g and 5h). A second pulse causes splitting again (Fig 5i), which finally causes annihilation of all waves, following a similar scenario as in Figs 4d through 4j.

In general, more than two pulses may be necessary. Each of the diagrams in Fig 6 was obtained by examining a large number of initial disordered configurations. These diagrams show what fraction P of these configurations are destroyed (ordinate) after a given number N of pulses (abscissa). In Fig 6a (time between pulses T = 30) we found a substantial number of configurations (last bar on the right of Fig 6a) leading to the periodicity illustrated in Fig 7d, so that $N \to \infty$, ie no destruction takes place. The reason is that splitting occurs before the waves have time to annihilate each other via collisions. In Fig 6b (T = 60) the mean time for wave destruction $\langle T_d \rangle$ vs. T (see Fig 8) has its minimum; only 1 or 2 pulses are needed for destruction. For T = 90 (Fig 6c) more pulses (3 or 4) may be needed for destruction. In Fig 7 we illustrate how destruction occurs after 1 pulse (Fig 7a), 2 pulses (Fig 7b) or 3 pulses (Fig 7c). We show in this figure the fraction E of excited medium (corresponding to the white regions in Figs 3, 4 and 5) as a function of time, under the influence of a periodic train of pulses (T = 50). Note that a pulse substantially decreases E, which can destroy disorder (as the first pulse in Fig 7a or the second one in Fig 7b) or does not destroy disorder (as the first pulse in Fig 7b). In some cases, a pulse destroys disorder, although destruction may as well have been attained as a result of the preceeding pulse (example: third pulse in Fig 7c).

A quantity that is interesting if one aims for rapid disorder destruction, particularly in cardiology, is the mean time $\langle T_d \rangle$ for destruction, as given by

$$\langle T_d \rangle = \sum_{N=1}^{\infty} T\, N\, P(N) \qquad (3)$$

Here, $P(N)$ is the probability that N pulses control disorder (see Fig 6). In Fig 8 we show $\langle T_d \rangle$ vs T. At the left of the dashed vertical line at $T = T_{crit}$, we found initial configurations leading to periodicity, such as that exemplified in Fig 7d (T_{crit} = 52). On

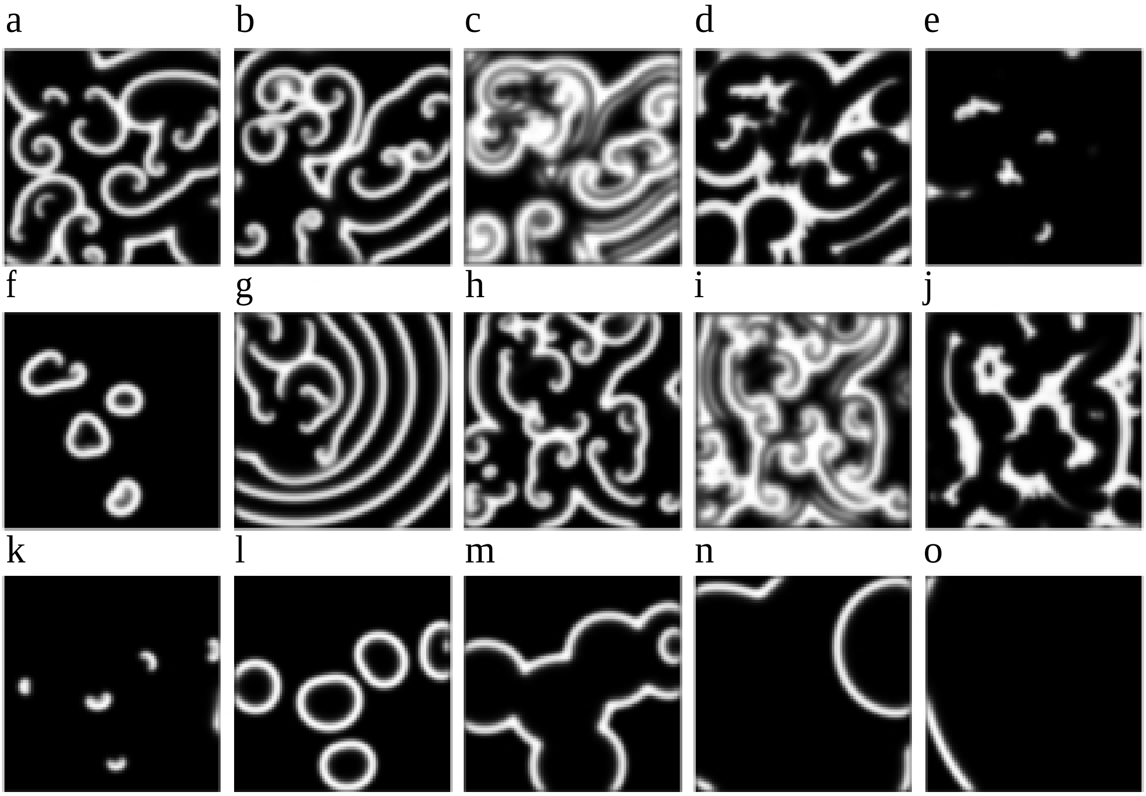

Fig 5. As Fig 4, but annihilation fails after one pulse. Two pulses are necessary to destroy disorder. (a) Disordered state; t=0. (b) Disordered state; t=9.5. The first pulse is turned on at t=10 and off at t=12. Times t for (c)-(g): 11.5, 13.5, 16, 19 and 57. (h) Newly developed disorder; t=110. The second pulse is turned on at t=110 and off at t=112. Times t for (i)-(o): 111.5, 113.5, 116.5, 121, 125, 136 and 165.

the other hand, we found no such cases at the right of the dashed line, ie destruction of disorder is always attained within a finite time for $T > T_{crit}$. The minimum seen in Fig 8 indicates the pulse period T for optimal operation if fast control is required.

DISCUSSION

We have shown that disordered modes consisting of erratically moving wavepieces can be destroyed by a periodic train of pulses. Each of these pulses must properly displace the activator nullcline parallely to the inhibitor axis. Such displacement is possible in a wide range of systems. For the BZ reaction it is accomplished using light.[15] For a highly simplified model of heart muscle membranes[16, 20] the displacement would be accomplished using an external current.

This method is much easier to implement than the control of spatiotemporal chaos using time series analysis.[21] Pulsing, as proposed here, is thus a promising alternative to complicated devices that have been proposed to defibrillate the heart[5]; such devices involve the registration of a long time series, the reconstruction of a phase space, the detection of a saddle point and its invariant manifolds in a Poincaré section, and the application of the so-called OGY-method (proposed by Ott et al[17]).

However, application to the heart muscle implies future work, as well as caution in the following aspects: i) Spiral breakup as presented here may occur in 3D, but not in 2D heart tissue[18, 23], so that

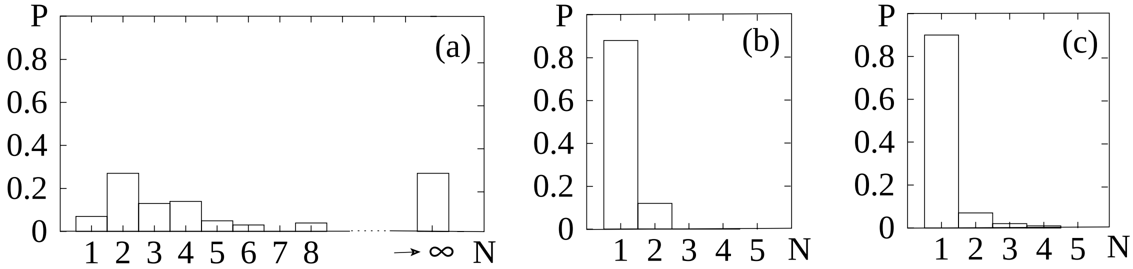

Fig 6. Probability P (ordinate) that destruction of disorder occurs after N pulses (abscissa). P was determined by considering 100 different initial disordered configurations in a square (size: 70 x 70). The diagrams differ by the pulsing period T. (a) T=30 (b) T=60 (c) T=90.

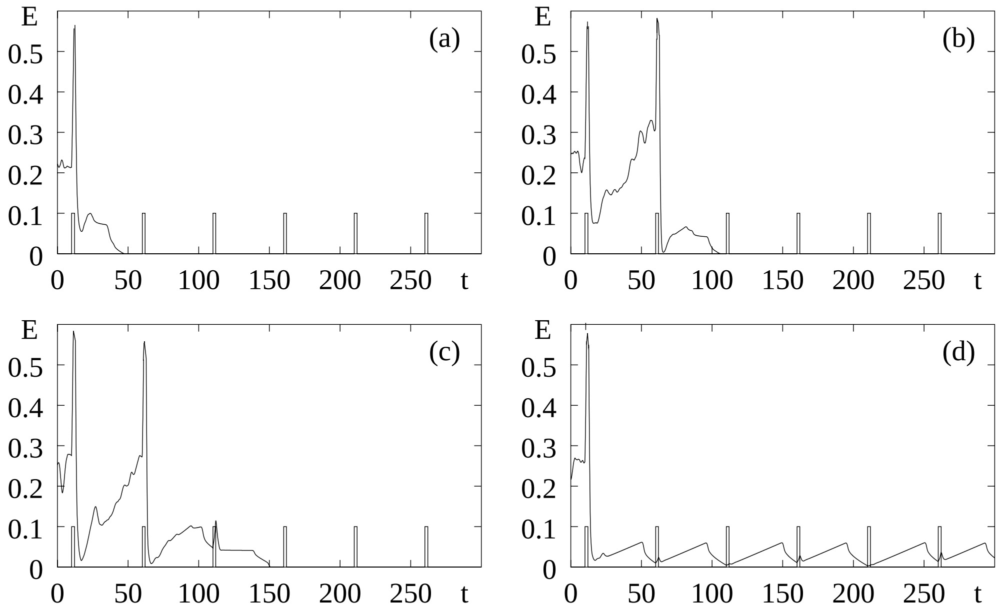

Fig 7. Time course of the effect of pulsing with period T=50. E (ordinate): fraction of excited state (white in Figs 3 through 5) in a 70 x 70 square. Note that the pulses in this figure are not quantified by the numbers in the ordinate; their height is 0.1. Pulse duration: τ=2. Disorder is destroyed after one pulse (a), two pulses (b) and three pulses (c). Disorder destruction fails in (d) since pulsing entrains the system into a limit cycle ($N \to \infty$; see Fig 6a).

our work would have to be redone in 3D; ii) the electrical current, which displaces the u-nullcline is the current through one heart cell membrane; it would have to be related to the total externally applied current (see 20); iii) inhomogeneity, anisotropy and the particular geometry of the heart should be considered; iv) a distinction should be made between atrial fibrillations (non lethal) and ventricular fibrillation (see 7); v) it must be clarified, whether fibrillation is related to spiral breakup (see 19), as assumed here, or to erratic meandering of the tip of a single spiral (see 6), or if both phenomena are possible, depending on conditions, the erratic meandering being the simplest kind of fibrillation.[24] Note that erratic meandering has recently also been found in the analogously behaving BZ reaction.[25]

We want to finish by stressing that the present work does not give specific recipes for a specific system. It merely teaches us that, in principle, spatiotemporal chaos in an excitable medium can be controlled by applying proper periodic pulses to the medium. We conjecture that this method is generic, so that it remains a technical matter to make it applicable other excitable systems with different properties and different mechanisms of wave breakdown.

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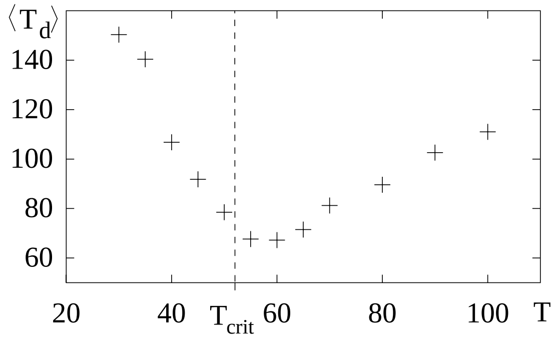

Fig 8. Mean time for disorder destruction $\langle Td \rangle$ (ordinate) vs. pulsing period T (abscissa). Depending on the initial configuration a periodic behaviour (as in Fig 7d) may be possible if $T<T_{crit}$ (left of vertical dashed line) implying failure of disorder control. The fastest destruction of disorder is given by the minimum at $T \approx 60$.

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