Calculating the Process Capability Ratio for Weibull Data

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Abstract

The process capability ratio is one of many statistical process control widely used in manufacturing and service engineering based on normality assumption. If the data do not correspond with the assumption, they can lead to erroneous conclusions. So the wrong conclusions would cost manufacturing and service organization big financial losses and lost customers to competitors. One approach to dealing with this situation is to transform the data so that in the new, the transformed data have a normal distribution appearance. Many authors investigated the standard transformation such as square root, logarithmic, and so on including the well known Box-Cox transformation to transform data are not normally distributed to normality. However, transformations may behave sufficiently normal for statistical process control but they do not yield accurate process performance index. In this paper, the use of Manly transformation, Yeo-Johnson transformation, and Nelson transformation to transform Weibull data are investigated in sense of calculating the process capability ratio and the coefficient of variation. It is found that all of three transformations can be used for transforming Weibull data to data that are normally distributed and the average of the process capability ratio of transformed data via all of them is not different at significant level 0.05 although the average of coefficient of variation of data transformed by Nelson transformation is the lowest. However, Nelson transformation is easy to work because it does not need the transformation parameter.

Keywords: Process capability ratio, Manly transformation, Yeo-Johnson transformation, Nelson transformation

Introduction

In statistical quality control, the basic assumptions are that the quality characteristic of product has a normal distribution and the process mean is centered between the lower and upper specification limits. In many industrial situations, the process outputs are not normally distributed and heavy tail such as chemical processes parameters, cutting tool wear processes and some concrete production processes. In these cases, they can lead to erroneous conclusions. Control charts and process capability ratio are widely used in quality control. The process capability ratio (PCR) is a measure of the ability of the process to manufacture product that meets the specifications. It is a quantitative method to express process capability used extensively in industry (Montgomery, 2001). An important assumption of PCR is based on a normal distribution of process output. If the distribution is non-normal, then the expected process fallout attributed to a particular value of PCR may be in error. When the assumption cannot be warranted, either capability index should be computed based on the other distributions than normal distribution, or the data should be transformed so that it conforms better to the normal distribution (Farnum as cited in Aichouni, Al-Ghonamy, & Bachioua, 2014). There are various graphical and analytical approaches to select a transformation such as reciprocal, logarithm, square root and so on. The usability of four types of transformations (Box-Cox, exponential, power and logarithmic) for transformation of data sets with four non-normal distributions (logarithmic-normal, exponential, gamma and Weibull) toward to normally distributed data was investigated. It is also possible to meet with data sets, which are not normally distributed, and which cannot be transformed to normally distributed data (Mach, Thuring, & Samal, 2006). The possibility to use exponential, Weibull and Lognormal distribution for transforming non-normal data by using the Box-Cox and Johnson transformation would help the quality professional to perform correct process analysis in control charts for computing the process capability ratio to meet customer specifications (Sherrill & Johnson as cited in Aichouni et al., 2014). However, the Box-Cox transformation should be used with caution in some cases such as failure time and survival data (Doksum & Wong, 1983). Moreover, the Box-Cox transformation was not satisfactory even when the best value of transformation parameter had been chosen (John & Draper, 1980). Furthermore, transformations may behave sufficiently normal for statistical process control but they do not yield accurate process performance index, for example, Johnson transformation of a gamma distribution yields a beautiful normal probability plot, but the estimated nonconforming fraction is four times too large (Levinson, 2010).

The objective of this article is to investigate the use of Manly transformation, Yeo-Johnson transformation and Nelson transformation to transform Weibull data for computing the PCR.

Methods and Materials

To investigate the use of transformations to transform Weibull data for computing the PCR. Some transformations, estimation of transformation parameter, Weibull distribution, and the process capability ratio were reviewed. The methodology is next.

1. Some Transformations

Let X is a random variable distributed as non-normal, Y the transformed variable of X, x the value of X, and λ a transformation parameter.

1.1 Box-Cox Transformation

Box-Cox Transformation is a power transformation that it has a simple modified form to avoid discontinuity at $\lambda = 0$. It can be defined by equation

$$Y = \begin{cases} \dfrac{X^{\lambda} - 1}{\lambda}, & \lambda \neq 0 \\ \ln X, & \lambda = 0 \end{cases} \quad \text{for } x > 0 \qquad (1)$$

(Box & Cox, 1964). This transformation is limited for positive data only so it is fitted for right skew data.

1.2 Manly Transformation

A family of exponential transformations in this form

$$Y = \begin{cases} \dfrac{\exp(\lambda X) - 1}{\lambda}, & \lambda \neq 0 \\ X, & \lambda = 0. \end{cases} \qquad (2)$$

This is a useful alternative to Box-Cox transformation because negative x values are also allowed. It has been found in particular that this transformation is quite effective at turning skew unimodal distributions into nearly symmetric normal distributions (Manly, 1976).

1.3 Yeo-Johnson Transformation

A new family of distributions that can be used without restrictions on that have many of the good properties of the Box-Cox power family. These transformations are defined by:

$$Y = \begin{cases} \dfrac{[X+1]^{\lambda}-1}{\lambda} & ,x \geq 0, \lambda \neq 0 \\ \ln[X+1] & ,x \geq 0, \lambda = 0 \\ -\left[(-X+1)^{2-\lambda}-1\right] \Big/ 2-\lambda & ,x < 0, \lambda \neq 2 \\ -\ln(-X+1) & ,x < 0, \lambda = 2 \end{cases} \tag{3}$$

It is modified from the Box-Cox transformation (Yeo & Johnson, 2000). Although interpretation of the Yeo-Johnson transformation parameter is difficult, this family can be useful in procedures for selecting a transformation for linearity or normality (Weisberg, 2001).

1.4 Nelson Transformation

The transformation is an alternative method to solve the problem that the exponential distribution (it is a special case of Weibull distribution when the shape parameter is zero) is highly skewed. For transforming the exponential random variable to normal distribution, the appropriate transformation is

$$Y = X^{1/3.6}$$

(Nelson as cite in Montgomery, 2001). It is easy to work because it does not need the transformation parameter.

2. Estimation of Transformation Parameter

For quality control, it is supposed that m samples are available, each containing n observations on the quality characteristic. So several groups of data, the value of λ in (2) and (3) need to be found so that the transformed variables will be independently normal distributions. The probability density function of each Y_{ij} is in the form

$$f(y_{ij} \mid \mu, \sigma^2) = \frac{1}{(2\pi\sigma^2)^{1/2}} \exp\left\{-\frac{1}{2\sigma^2}(y_{ij}-\mu)^2\right\} \text{ for } i=1,\ldots,m \text{ and } j=1,\ldots,n, \tag{4}$$

where μ is the mean of the transformed population data, σ^2 the pooled variance of all transformed population data and y_{ij} the observed value of Y_{ij}. For (2), the likelihood function in relation to the observations x_{ij} is given by

$$L(\mu, \sigma^2, \lambda \mid x_{ij}) = \frac{1}{(2\pi\sigma^2)^{nm/2}} \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\exp(\lambda x_{ij})-1}{\lambda}-\mu\right]^2\right\}.J(y;x)$$

(5)

where $J(y;x)=\prod_{i=1}^{m}\prod_{j=1}^{n}\left|\frac{\partial y_{ij}}{\partial x_{ij}}\right|$. For a fixed λ, the MLE's for μ and σ^2 are $\hat{\mu}=\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\exp(\lambda x_{ij})-1}{\lambda}\right]$ and

$$\hat{\sigma}^2=\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left\{\frac{\exp(\lambda x_{ij})-1}{\lambda}-\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\frac{\exp(\lambda x_{ij})-1}{\lambda}\right)\right\}^2$$

Substitute $\hat{\mu}$ and $\hat{\sigma}^2$ into the likelihood equation (5). Thus for fixed λ, the maximized log likelihood is

$$\ln L(\lambda|x_{ij})=$$

$$-\frac{mn}{2}\ln 2\pi-\frac{mn}{2}\ln\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left\{\frac{\exp(\lambda x_{ij})-1}{\lambda}-\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\frac{\exp(\lambda x_{ij})-1}{\lambda}\right)\right\}^2-\frac{mn}{2}+\lambda\sum_{i=1}^{m}\sum_{j=1}^{n}x_{ij}, \quad (6)$$

except for a constant, the maximum likelihood estimate of λ is obtained by solving the likelihood equation

$$\frac{d}{d\lambda}\ln L(\lambda)=$$

$$\frac{-mn\left[\sum_{i=1}^{m}\sum_{j=1}^{n}e^{2\lambda x_{ij}}x_{ij}-\frac{1}{mn}\left(\sum_{i=1}^{m}\sum_{j=1}^{n}e^{\lambda x_{ij}}\right)\left(\sum_{i=1}^{m}\sum_{j=1}^{n}e^{\lambda x_{ij}}x_{ij}\right)\right]}{\sum_{i=1}^{m}\sum_{j=1}^{n}e^{2\lambda x_{ij}}-\frac{1}{mn}\left(\sum_{i=1}^{m}\sum_{j=1}^{n}e^{\lambda x_{ij}}\right)^2}+\frac{mn}{\lambda}+\sum_{i=1}^{m}\sum_{j=1}^{n}x_{ij}=0. \quad (7)$$

Similar procedures yield the same results for (3), when $x_{ij}\geq 0$ and $\lambda\neq 0$, the maximum likelihood estimate of λ is obtained by solving the likelihood equation

$$\frac{d}{d\lambda}\ln L(\lambda)=\frac{-mn\left[\sum_{i=1}^{m}\sum_{j=1}^{n}\left(x_{ij}+1\right)^{2\lambda}\ln\left(x_{ij}+1\right)-\frac{1}{mn}\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\left(x_{ij}+1\right)^{\lambda}\right)\sum_{i=1}^{m}\sum_{j=1}^{n}\left(x_{ij}+1\right)^{\lambda}\ln\left(x_{ij}+1\right)\right]}{\sum_{i=1}^{m}\sum_{j=1}^{n}\left(x_{ij}+1\right)^{2\lambda}-\frac{1}{mn}\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\left(x_{ij}+1\right)^{\lambda}\right)^2} \quad (8)$$

$$+\frac{mn}{\lambda}+\sum_{i=1}^{m}\sum_{j=1}^{n}\ln\left(x_{ij}+1\right)=0.$$

Since λ appears on the exponent of the observations, it is considered to be too complicated for solving it. The maximized log likelihood function is a unimodal function so the value of the transformation parameter is obtained when the slope of the curvature of the maximized log likelihood function is nearly zero. Hence we can also use the numerical method such as bisection for finding the suitable value of λ.

3. Weibull Distribution

The Weibull distribution is often used to model the time until failure of many different physical systems. The probability density function of a two parameter Weibull random variable *X* is

$$f(x)=\begin{cases}\dfrac{\alpha}{\beta}\left(\dfrac{x}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^{\alpha}} & ,x\geq 0;\alpha,\beta>0\\ 0 & ,x<0\end{cases}$$

where α is the shape parameter and β is the scale parameter. The mean and variance are $E(X)=\beta\cdot\Gamma\left(\frac{1}{\alpha}+1\right)$ and $V(X)=\beta^{2}\left[\Gamma\left(\frac{2}{\alpha}+1\right)-\left\{\Gamma\left(\frac{1}{\alpha}+1\right)\right\}^{2}\right]$, respectively (Montgomery, Runger, & Hubele, 2004).

4. Process Capability Ratio

The process capability ratio (C_p) is calculated from

$$C_p=\frac{USL-LSL}{6\sigma}$$

where USL and LSL are the upper and lower specification limits, respectively. In a practical application, the standard deviation σ is almost always unknown and must be replaced by estimating σ that we use either the sample standard deviation $\hat{\sigma}=S$ or $\hat{\sigma}=\frac{\bar{S}}{c_4}=\frac{\bar{R}}{d_2}$.

5. Methodology

The steps to investigate this work are summarized as follows:

5.1 Determine the number of sample (m) = 20, 25 and the sample size (n) =4, 5, 6.

5.2 Generate Weibull populations of size $N_i=1{,}000$ where i=1,…,m with the different values of scale parameter β_i and shape parameter α_i defined as Table 1.

Table 1 Shape parameter, scale parameter, USL, and LSL of life years for some items

Item	shape parameter	scale parameter	USL	LSL
Pump	1.2	4	15	2
Clutch	1.4	11	15	2
Gear	2	9	15	2

Source: http://reliabilityanalyticstoolkit.appspot.com/mechanical_reliability_data (Retrieved on 2 February, 2016)

5.3 From each generated Weibull population, 1,000 random samples, each of size n that is equal are drawn.

5.4 Estimate the transformation parameter of Manly transformation and Yeo-Johnson transformation by maximum likelihood method. The value of transformation parameter is obtained by bisection method.

5.5 Each set of the sample data was transformed to normality by Manly transformation, Yeo-Johnson transformation, and Nelson transformation and computed the process capability ratio and coefficient of variation.

5.6 Find the average of process capability ratio and the average of coefficient of variation for each situation.

5.7 Compare the average of process capability ratio and the average of coefficient of variation of three. Transformations at significant level 0.05.

The best transformation is that the average of coefficient of variation should be the lowest.

Results

The results of this paper are considered from an example and a numerical study as follows.

1. An Example

The data shown here are donated by Prof. I-Cheng Yeh, Department of Information Management, Chung-Hua University, Taiwan (UCI Machine Learning Repository, 2007).

One hundred observations of the concrete compressive strengths (in MPa, megapascals) are drawn from 1,030 observations and they are arranged as twenty five samples of four observations in the following Table 2.

These data are not normally distributed with mean= 44.61 and standard deviation = 18.82 by Kolmogorov-Smirnov test and test statistic = 0.142 at significant level 0.05 as Figure1.

```
> mean(Original)
[1] 44.6105
> sd(Original)
[1] 18.81718
> lillie.test(Original)

        Lilliefors (Kolmogorov-Smirnov) normality test

data:  Original
D = 0.14244, p-value = 3.348e-05
```

Figure 1 Result of Kolmogorov-Smirnov test for checking normal distributions

Table 2 Original data of the concrete compressive strengths (in MPa)

Sample Number	X_1	X_2	X_3	X_4
1	45.85	28.02	80.32	37.43
2	52.91	42.13	39	33.02
3	71.99	28.8	28.1	46.8
4	61.09	56.4	71.3	72.3
5	69.3	77.3	71.7	57.6
6	65.2	68.1	79.3	76.8
7	79.3	73.3	29.45	37.34
8	39.61	30.39	25.69	24.92
9	26.31	47.74	17.22	27.77
10	35.57	35.23	33.36	82.6
11	60.32	38.77	51.33	36.99
12	33.7	63.14	15.34	51.02
13	39.38	44.33	41.37	14.94
14	15.82	76.24	33.01	24.28
15	51.72	39.64	44.28	53.39
16	55.45	62.05	23.25	41.68

Table 2 (Cont.)

Sample Number	X_1	X_2	X_3	X_4
17	22.49	27.04	27.63	32.92
18	17.34	75.5	16.11	43.38
19	81.75	39.7	32.1	39.66
20	37.91	70.7	42.13	61.92
21	79.99	33.8	33.42	40.87
22	52.42	38.46	37.26	31.42
23	19.01	29.72	79.3	25.1
24	29.07	33.8	40.93	25.56
25	37.43	29.87	43.58	33.76

They have Weibull distributions with scale parameter 48 and shape parameter 2.5 by Kolmogorov-Smirnov statistic = 0.099 as Figure 2.

```
> ks.test(Original,"pweibull",2.5,48)

        One-sample Kolmogorov-Smirnov test

data:  Original
D = 0.099058, p-value = 0.2803
alternative hypothesis: two-sided
```

Figure 2 Result of Kolmogorov-Smirnov test for checking Weibull distributions

To illustrate the calculation of PCR, we suppose that the specification on the concrete compressive strengths are the upper specification limit (USL) = 80 MPa and the lower specification limit (LSL) = 10 MPa (WSDOT FOP for AASHTO T 22, 2013). From Figure 2, we estimate $\hat{\sigma} = s = 18.82$. Thus, our estimate of the PCR is $\text{PCR} = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \frac{80 - 10}{6(18.82)} = 0.62$. However, the distribution has right skewness. Thus, this estimate of capability is unlikely to be correct. To compare the efficiency of Manly transformation, Yeo-Johnson Transformation, and Nelson transformation, the data are transformed by all of them. By Bisection Method, the values of Manly transformation parameter and Yeo-Johnson transformation are - 0.016163 and 0.187, respectively. For Nelson transformation, the value of transformation parameter does not need.

So Manly transformation is in the form $y = \frac{\exp(-0.016163X) - 1}{(-0.016163)}$ and Yeo-Johnson transformation is in the form $y = \frac{[X+1]^{0.187} - 1}{0.187}$.

Weibull data in Table 2 are transformed by Manly transformation and the results are in Table 3.

Table 3 Transformed data by Manly transformation

Sample Number	X_1	X_2	X_3	X_4
1	32.38	22.53	44.98	28.08
2	35.56	30.56	28.93	25.59
3	42.54	23.03	22.58	32.83
4	38.82	37.01	42.33	42.64
5	41.68	44.13	42.45	37.48
6	40.30	41.29	44.70	43.99
7	44.70	42.95	23.43	28.03
8	29.25	24.01	21.02	20.51
9	21.43	33.27	15.03	22.37
10	27.05	26.86	25.79	45.59
11	38.53	28.81	34.88	27.84
12	25.98	39.57	13.59	34.75
13	29.13	31.65	30.17	13.27
14	13.96	43.83	25.58	20.08
15	35.05	29.27	31.62	35.77
16	36.62	39.18	19.38	30.33
17	18.86	21.91	22.28	25.53
18	15.12	43.61	14.18	31.18
19	45.36	29.30	25.04	29.28
20	28.34	42.14	30.56	39.13
21	44.89	26.04	25.82	29.91
22	35.35	28.64	27.99	24.64
23	16.37	23.60	44.70	20.63
24	23.20	26.04	29.94	20.94
25	28.08	23.69	31.28	26.02

These transformed data are checked the normality assumption by using Kolmogorov-Smirnov statistic = 0.086 as Figure 3.

```
> mean(Manly)
[1] 30.4613
> sd(Manly)
[1] 8.782152
> lillie.test(Manly)

        Lilliefors (Kolmogorov-Smirnov) normality test

data:  Manly
D = 0.085517, p-value = 0.06846
```

Figure 3 Result of Kolmogorov-Smirnov test for checking normality assumption of transformed data by Manly transformation

From Figure3, we estimate $\hat{\sigma} = S = 8.78$. Thus, our estimate of the PCR is $PCR_M = \frac{USL^* - LSL^*}{6\hat{\sigma}} = \frac{44.89 - 9.23}{6(8.78)} = 0.68$.

Note that the USL* and LSL* are the USL and LSL transformed by Manly Transformation (Montgomery, 2001). This estimate of process performance is clearly much more realistic than the one resulting from transformation.

Furthermore, Weibull data in Table 2 are transformed by Yeo-Johnson transformation and the results are in Table 4.

Table 4 Transformed data by Yeo-Johnson transformation

Sample Number	X_1	X_2	X_3	X_4
1	5.63	4.69	6.82	5.23
2	5.92	5.46	5.31	4.99
3	6.58	4.74	4.70	5.67
4	6.23	6.06	6.56	6.59
5	6.50	6.74	6.57	6.10
6	6.37	6.46	6.80	6.72
7	6.80	6.62	4.78	5.23
8	5.34	4.84	4.54	4.48
9	4.58	5.71	3.85	4.68
10	5.13	5.12	5.01	6.89
11	6.20	5.30	5.86	5.21
12	5.03	6.30	3.67	5.85
13	5.33	5.56	5.43	3.63
14	3.72	6.71	4.99	4.44
15	5.88	5.34	5.56	5.94
16	6.02	6.26	4.36	5.44
17	4.30	4.63	4.67	4.99
18	3.87	6.69	3.75	5.52
19	6.86	5.35	4.94	5.34
20	5.26	6.54	5.46	6.25
21	6.82	5.04	5.02	5.40
22	5.90	5.29	5.22	4.90
23	4.02	4.80	6.80	4.49
24	4.76	5.04	5.41	4.53
25	5.23	4.81	5.53	5.04

These transformed data are checked the normality assumption by Kolmogorov-Smirnov statistic = 0.078 as Figure 4.

```
> mean(YJ)
[1] 5.4354
> sd(YJ)
[1] 0.8513322
> lillie.test(YJ)

        Lilliefors (Kolmogorov-Smirnov) normality test

data:  YJ
D = 0.078474, p-value = 0.1358
```

Figure 4 Result of Kolmogorov-Smirnov test for checking normality assumption of transformed data by Yeo-Johnson transformation

From Figure 4, we estimate $\hat{\sigma} = s = 0.85$. Thus, our estimate of the PCR is

$$PCR_{YJ} = \frac{USL^{**} - LSL^{**}}{6\hat{\sigma}} = \frac{6.82 - 3.03}{6(0.85)} = 0.74.$$

Note that the USL^{**} and LSL^{**} are the USL and LSL transformed by Yeo-Johnson Transformation (Montgomery, 2001). We see that the PCR calculated form data transformed by Yeo-Johnson transformation is higher than the PCR calculated form data transformed by Manly transformation.

Last, Weibull data in Table 2 are transformed by Nelson transformation and the results are in Table 5.

These transformed data are checked the normality assumption by Kolmogorov-Smirnov statistic = 0.078 as Figure 5.

```
> mean(Nelson)
[1] 2.8198
> sd(Nelson)
[1] 0.3385232
> lillie.test(Nelson)

        Lilliefors (Kolmogorov-Smirnov) normality test

data:  Nelson
D = 0.077981, p-value = 0.1415
```

Figure 5 Result of Kolmogorov-Smirnov test for checking normality assumption of transformed data by Nelson transformation

Table 5 Transformed data by Nelson transformation

Sample Number	X_1	X_2	X_3	X_4
1	2.89	3.01	3.28	3.13
2	3.24	3.19	3.37	2.78
3	2.48	2.70	3.12	2.66
4	2.77	2.15	2.99	3.05
5	2.37	2.21	3.40	2.74
6	3.38	3.00	2.27	2.55
7	2.73	2.52	2.83	2.54
8	3.06	3.34	3.23	3.30

Table 5 (Cont.)

Sample Number	X_1	X_2	X_3	X_4
9	2.58	2.93	2.69	2.76
10	3.16	2.87	3.33	2.78
11	3.15	2.50	3.32	2.78
12	3.26	2.66	2.76	2.56
13	2.66	2.57	3.38	2.77
14	2.53	3.27	3.28	3.37
15	2.56	2.46	2.20	2.65
16	2.99	2.13	2.81	2.64
17	2.87	2.40	2.51	2.16
18	2.62	2.83	2.65	2.73
19	3.37	2.80	2.85	2.73
20	2.64	2.91	3.28	3.08
21	3.34	2.73	2.44	2.52
22	3.41	2.73	2.98	2.12
23	2.42	3.02	2.82	2.64
24	2.85	2.78	3.14	2.80
25	2.60	2.45	2.46	2.66

From Figure 5, we estimate $\hat{\sigma} = s = 0.34$. Thus, our estimate of the PCR is $PCR_N = \frac{USL^{***} - LSL^{***}}{6\hat{\sigma}} = \frac{3.38 - 1.90}{6(0.34)} = 0.73$. Note that the USL^{***} and LSL^{***} are the USL and LSL transformed by Nelson Transformation (Montgomery, 2001). We see that the PCR calculated form data transformed by Nelson transformation is higher than the PCR calculated form data transformed by Manly transformation and it is nearly the PCR calculated form data transformed by Yeo-Johnson transformation.

2. A Numerical Study

From the steps of methodology, for each item in Table 1, results of the average process capability ratios and the average coefficient of variation with 1,000 replicated samples of each situation are shown in Table 6-8.

Table 6 The average of coefficient of variation and the average of process capability ratios of Pump

number of sample	sample size	The average of coefficient of variation (%)			The average of process capability ratios		
		Manly transformation	Yeo-Johnson transformation	Nelson transformation	Manly transformation	Yeo-Johnson transformation	Nelson transformation
20	4	56.1186	54.3242	25.3058	0.4867	0.5606	0.4399
	5	48.8507	49.5812	22.6809	0.4512	0.5550	0.4885
	6	54.2055	54.1663	23.9307	0.4715	0.6142	0.4807
25	4	56.7901	55.9227	26.5075	0.4089	0.4925	0.4422
	5	52.5374	51.9347	23.4885	0.4843	0.5663	0.4649
	6	52.6663	53.6741	24.5671	0.4446	0.5397	0.4631

Table 7 The average of coefficient of variation and the average of process capability ratios of Clutch

number of sample	sample size	The average of coefficient of variation (%)			The average of process capability ratios		
		Manly transformation	Yeo-Johnson transformation	Nelson transformation	Manly transformation	Yeo-Johnson transformation	Nelson transformation
20	4	46.8597	42.6949	21.3015	0.4268	0.4077	0.4050
	5	52.8858	47.2532	22.5115	0.3492	0.3435	0.3633
	6	55.2542	40.9508	23.4955	0.3560	0.3612	0.3607
25	4	49.9826	42.1897	20.8472	0.4028	0.4066	0.4078
	5	48.0286	40.3907	21.3568	0.4003	0.3865	0.3945
	6	49.5435	41.4978	21.8658	0.3937	0.3803	0.3871

Table 8 The average of coefficient of variation and the average of process capability ratios of Gear

number of sample	sample size	The average of coefficient of variation (%)			The average of process capability ratios		
		Manly transformation	Yeo-Johnson transformation	Nelson transformation	Manly transformation	Yeo-Johnson transformation	Nelson transformation
20	4	45.5666	42.7075	18.0881	0.4905	0.4883	0.4847
	5	42.3013	36.4616	16.0328	0.5317	0.5363	0.5452
	6	38.8246	33.2505	15.4931	0.5606	0.5669	0.5726
25	4	41.2695	34.8153	16.3418	0.5375	0.5408	0.5433
	5	39.6444	35.4077	15.6198	0.5385	0.5404	0.5542
	6	43.5490	39.9437	16.6372	0.5229	0.5230	0.5256

Discussion

From the results of a numerical method, we see that the average of coefficient of variation of the data transformed by Nelson transformation in every case is the lowest and it differs from the others at significant level 0.05. Hence, Nelson transformation is the most *efficient* in sense of dispersion measure. The average of coefficient of variation of the data transformed by Manly transformation and Yeo-Johnson transformation is not different at significant level 0.05. For the average of process capability ratios computed from the data transformed by all of them is not different at significant level 0.05. Furthermore, the difference of upper specification limit (USL) and lower specification limit (LSL) has affected the PCR computed from the transformed data including USL and LSL that they are transformed too.

Conclusion and Suggestion

All of three transformations can be used for transforming the Weibull data to data that are normally distributed. When the transformed data are applied to calculate the PCR, the results of them are not different at significant level 0.05 although the average of coefficient of variation of data transformed by Nelson transformation is the lowest. However, Nelson transformation should be used to transform Weibull data to normality because it does not need the value of transformation parameter.

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