PROBLEM SOLVING: A BASIS TO REFORM MATHEMATICS INSTRUCTION
การแก้ปัญหา: พื้นฐานสำหรับการปฏิรูปการสอนคณิตศาสตร์

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ABSTRACT

This paper is a documentary research. It aims to provide a perspective to reform mathematics instruction in Thailand by dealing particularly with theme of problem solving. The specific goals of this research are as follows: To analyze the problem solving appearing in mathematics textbooks in order to distinguish between problems and routine exercises; to analyze and provide the evidence of the relation between the structure and assumption underlying such routine-exercise problems, and mathematics instruction in the classroom; to provide a perspective on genuine problem solving; to provide a perspective on problem solving with comments on research in mathematics education in Thailand; and to provide an alternative view of classroom instruction that fosters problem solving. The methodology used in this study was an investigation and examination of related documents. It was found that Thai mathematics textbooks from first grade to ninth grade contain mostly routine

exercises aiming at drilling computation skills or revising some rules or principles which have been learnt in the class. There is a general structure and a basic pedagogical and epistemological assumption underlying Thai mathematics textbooks. For the structure, it contains examples that are used to introduce techniques, rules or principles including a conclusion about those techniques, rules or principles. There are a lot of exercises provided for students to practice those techniques, rules or principles. The pedagogical assumption is that students are expected to master the techniques, rules or principles as required in the curriculum. In this sense, the epistemological assumption (i.e. how students come to know mathematics) is that the students have mastered the set of such techniques, rules or principles which comprise their mathematical knowledge and understanding.

Polya[20], Lester[8] and Kroll & Miller[7] suggested that genuine problem solving is a process consisting of four phases: understanding the problem, devising a plan, carrying out the plan and looking back. Moreover, the feature of the problem should not be solved by simply using computation expertise. To provide a perspective in problem solving with comments on research in mathematics education in Thailand the research revealed that problem solving is a very complex phenomenon and there have been many research approaches and methodologies. However, it is found that research approaches in Thailand have still studied the relationship between some factors and the student's mathematics achievement and the research methodologies were dominantly the experimental, survey or correlational method.

Lastly, this paper proposed two research projects in order to provide an alternative view of classroom instruction that fosters problem solving. The first project conducted in Japan suggests using open-ended problems to evaluate higher-order thinking, an important aspect of problem solving. The second is Lester's project[9] which provided a perspective on using metacognitive instruction to improve the student's problem solving behavior. The aims of this project were to 1) assess 7th

graders' metacognitive beliefs and processes and investigate how they affect problem solving behaviors; and 2) explore the extent to which these students can be taught to be more strategic and aware of their own problem-solving behaviors. Research results from these two projects suggest three considerable issues which can be used to improve mathematics teaching in Thai classrooms such as selecting mathematical problems, classroom organization and teachers' roles.

The research results can be concluded as an integrative perspective that problem solving can be used as a basis to reform mathematics instruction and can be contributed to an attempt for educational reform as a bottom-up process in the sense that "bottom" means classroom and "up" means society.

บทคัดย่อ

บทความนี้เป็นการวิจัยเอกสารเพื่อเสนอทัศนะเกี่ยวกับการแก้ปัญหา (problem solving) สำหรับการปฏิรูปการสอนคณิตศาสตร์ในโรงเรียนในประเทศไทย โดยมีวัตถุประสงค์เพื่อวิเคราะห์การแก้ปัญหาที่ปรากฏอยู่ในหนังสือเรียนวิชาคณิตศาสตร์ เพื่อแยกความแตกต่างระหว่างปัญหา (problem) กับแบบฝึกหัด (routine exercise) ที่ใช้ฝึกทักษะ เพื่อวิเคราะห์โครงสร้างและสมมติฐานเกี่ยวกับการสอนและการรู้ที่มีอยู่ในหนังสือเรียนวิชาคณิตศาสตร์ของไทย และเสนอให้เห็นความสัมพันธ์ของโครงสร้างและสมมติฐานดังกล่าวกับการสอนคณิตศาสตร์ในชั้นเรียน เพื่อเสนอทัศนะเกี่ยวกับการแก้ปัญหาที่แท้จริง (genuine problem solving) เพื่อเสนอมุมมองเกี่ยวกับการแก้ปัญหา รวมทั้งข้อสังเกตเกี่ยวกับการวิจัยด้านการศึกษาคณิตศาสตร์ของไทย และเพื่อเสนอทางเลือกสำหรับการสอนคณิตศาสตร์ในชั้นเรียนที่สนับสนุนการแก้ปัญหา ผลการวิจัยสรุปได้ดังนี้

จากการวิเคราะห์หนังสือเรียนวิชาคณิตศาสตร์ของไทยตั้งแต่ชั้นประถมศึกษาปีที่ 1 ถึงชั้นมัธยมศึกษาปีที่ 3 พบว่า ส่วนใหญ่ประกอบด้วยแบบฝึกหัดที่ใช้ฝึกทักษะโดยเฉพาะทักษะการคิดคำนวณ และแบบฝึกหัดเพื่อใช้ทบทวนกฎหรือหลักการที่ได้เรียนไปแล้ว โดยส่วนของโครงสร้างประกอบด้วย ตัวอย่างที่ใช้สำหรับแนะนำเทคนิค รวมทั้งกฎและหลักการที่หลักสูตรกำหนด บทสรุปเกี่ยวกับเทคนิค กฎ หรือหลักการดังกล่าว มีแบบฝึกหัดสำหรับฝึกทักษะเกี่ยวกับเทคนิค

กฎ หรือหลักการที่ได้เรียนไปแล้ว ทั้งนี้โดยมีสมมติฐานเกี่ยวกับการสอนคือ นักเรียนได้รับการคาดหวังว่าจะต้องรอบรู้เทคนิค กฎ หรือหลักการต่าง ๆ ที่หลักสูตรกำหนดไว้ว่าจะต้องเรียนในแต่ละชั้นเรียน กล่าวคือนักเรียนจะต้องรอบรู้ชุดของเทคนิค กฎหรือหลักการต่างๆ ที่กล่าวมาแล้ว

จากแนวคิดของโพลยา (Polya)[20] เลสเตอร์ (Lester)[8] และโครลกับมิลเลอร์ (Kroll & Miller)[7] ความหมายของการแก้ปัญหาที่แท้จริงคือ กระบวนการที่ประกอบด้วยวิธีการแก้ปัญหา 4 ระยะได้แก่ ทำความเข้าใจปัญหา วางแผนการแก้ปัญหา ลงมือแก้ปัญหาตามแผนที่วางไว้ ทบทวนไตร่ตรองวิธีการแก้ปัญหาในระยะต่าง ๆ ที่ผ่านมา นอกจากนี้ ลักษณะของปัญหาไม่ควรจะเป็นแบบที่นักเรียนสามารถแก้ปัญหาได้โดยใช้เพียงแค่ความสามารถในการคำนวณเท่านั้น สำหรับมุมมองในการแก้ปัญหา พบว่าการแก้ปัญหาเป็นปรากฏการณ์ที่สลับซับซ้อน และแนวทางการเข้าสู่ปัญหาวิจัย รวมทั้งวิธีการในการวิจัยหลากหลาย อย่างไรก็ตาม บทความนี้ได้ชี้ให้เห็นว่าการเข้าสู่ปัญหาวิจัยของการวิจัยในเมืองไทยส่วนใหญ่ยังเน้นการศึกษาความสัมพันธ์ระหว่างตัวแปรบางประการกับผลสัมฤทธิ์ทางการเรียนวิชาคณิตศาสตร์ของนักเรียน ซึ่งวิธีการวิจัยที่ใช้ส่วนใหญ่ได้แก่การวิจัยเชิงทดลอง การสำรวจ และการหาความสัมพันธ์ นอกจากนี้ บทความนี้ยังได้เสนอทัศนะของโครงการวิจัยสองโครงการ เพื่อเป็นทางเลือกสำหรับการสอนวิชาคณิตศาตร์ในชั้นเรียนที่จะสนับสนุนการแก้ปัญหา โดยโครงการวิจัยโครงการแรกทำในประเทศญี่ปุ่น มีเป้าหมายเพื่อใช้ปัญหาปลายเปิด (open–ended problems) ในการประเมินการคิดในระดับสูง (higher–order thinking) ของนักเรียน ซึ่งถือเป็นส่วนหนึ่งของการแก้ปัญหา ส่วนอีกโครงการหนึ่งทำในประเทศสหรัฐอเมริกาเป็นงานวิจัยของเลสเตอร์และคณะ[9] ซึ่งศึกษาใน 2 ประเด็น คือ 1) ประเมินความเชื่อและกระบวนการเชิงเมตะค็อกนิชัน (Metacognitive beliefs and processes) ของนักเรียนชั้นเกรด 7 รวมทั้งศึกษาอิทธิพลของความเชื่อและกระบวนการดังกล่าวที่มีต่อพฤติกรรมการแก้ปัญหา 2) สำรวจขอบเขตของนักเรียนกลุ่มที่ศึกษา เพื่อสอนให้มีกลยุทธ์ในการแก้ปัญหามากขึ้นกว่าเดิม รวมทั้งให้ตระหนักถึงพฤติกรรมการแก้ปัญหาของตนเอง ซึ่งจากการศึกษางานวิจัยทั้งสองโครงการพบประเด็นที่น่าจะนำมาพิจารณาในการปรับปรุงการสอนวิชาคณิตศาสตร์ในโรงเรียน 3 ประเด็นคือ การเลือกปัญหาคณิตศาสตร์ การจัดชั้นเรียน และบทบาทของครูในชั้นเรียน

ผลจากการวิจัยสามารถสรุปเป็นภาพรวมได้ว่า การแก้ปัญหาเป็นพื้นฐานในการปฏิรูปการสอนวิชาคณิตศาสตร์ได้ โดยมุมมองนี้มีส่วนสนับสนุนความพยายามในการปฏิรูปการศึกษาในฐานะที่เป็นกระบวนการจากล่างขึ้นบน (bottom–up process) ซึ่ง “ล่าง” ในที่นี่หมายถึงชั้นเรียน และ “บน” หมายถึง สังคมโดยรวม

INTRODUCTION

From the report for national assessment of educational progress of sixth grade students in Thailand in 1988, it was found that the average percentage of mathematics achievement ranked lowest at 44.80% when compared with other subjects[12]. To be more specific, when using tenth-scale criteria mathematical performance ranked highest at 5.70 and problem-solving skills ranked lowest at 3.50. Worse still, the report across all Educational Regions showed that except for two regions (i.e. Educational Region II and III), the rates of progress were lower than those of 1987. In particular, when mathematical skills were compared, it was found that except for one province, problem-solving skill was the most crucial one. How can we call this as assessment of educational progress? Subsequently, there was also such a report for ninth and tenth grade students of secondary schools in Educational Region X[28]. The results are similar. For ninth grade students, the achievement of the two mathematics courses (M311-312 and M321-322) ranked lowest at 43.06% and 41.18% respectively. In detail, performance in the probabilistic topic in M311-312 was the worst (at 29.50%), and performance in applying addition, subtraction, multiplication and division of fraction in M321-322 was also the worst (at 31.93%). For tenth grade students, the achievement for M021-022 ranked highest among other subjects, however, it was not more than 50% (49.38%). For M011-012, it was the second rank from the lowest at 36.15%.

Overall, the quality of education in Thailand is clearly stated in the Eighth National Education Development Plan (1997-2001)[5]. It states that:

"The quality of education is considered to be a rather critical problem in Thai education system as evidenced by unsatisfactory achievement levels both in analytical thinking, analyzing and synthesizing processes, creativity, initiative-taking, problem solving, and in students' academic knowledge in science, mathematics, and the Thai language."

It also suggests the major reasons for such low quality as follows:

"The major reasons for low quality were the inefficient processes of teaching-learning, the inappropriateness of the curriculum, the lack of teachers in some areas and the lack of qualified teachers as the consequences of teacher production and the development process, and the teachers' self-development opportunities."[5]

Needless to say, Thai education is at risk. In particular, Thai school mathematics is in a crucial state. With such undesirable educational outcomes, it is hopeless to expect that the students' mathematical knowledge can promote the educational goals such as to intelligently solve problems; to be creative and possess an inquiring mind and to keep up with technological progress, etc., as stated in the National Scheme of Education 1992. Undoubtedly, redefinition of Thai school mathematics is urgently needed, particularly in the perspective of redefining classroom instruction. This should also be included in the current educational reform movement. The author wishes to reform mathematics instruction in Thailand by focusing on problem solving. The specific goals of this research are as follows: 1) To analyze the problem solving appearing in mathematics textbooks in order to distinguish between problems and routine exercises, 2) To analyze and provide the evidence of the relation between the structure and assumptions underlying such routine exercises, problems, and mathematics instruction in the classroom, 3) To provide a perspective on genuine problem solving, 4) To provide a perspective in problem solving with comments on research in mathematics education in Thailand, and 5) To provide an alternative view of classroom instruction that fosters problem solving.

Problem solving as appearing in mathematics textbooks

For most students and teachers in Thailand, problem solving might mean to solve some mathematical tasks in the classroom or do some exercises as assigned homework. For some students who enjoy mathematics contests, it might mean to solve some non-routine mathematical tasks, which are mostly difficult and challenging. For entrance examinees it might mean to solve as many mathematical tasks as they can in a fixed time. So what does problem solving mean?

According to Webster's Dictionary[24], the definitions of a problem are:

Definition 1: 'In mathematics, anything required to be done, or requiring the doing of something.'

Definition 2: 'A question ... that is perplexing or difficult.'

For Schoenfeld[24], Webster's first definition captures the sense of the term *problem* as it has traditionally been used in mathematics instruction. That is, for nearly as long as we have written records of mathematics, sets of mathematics tasks have been with us–as vehicles of instruction, as a means of practice, and as yardsticks for the acquisition of mathematical skills.

However, it often appears that collections of tasks are anything but problems in the sense of the second definition. That is, they are rather routine exercises organized to provide practice on a particular mathematical technique that, typically, has just been demonstrated to the student.

To illustrate what a problem in the second sense means, the following tasks are examples Schoenfeld excerpted from a late 19th century text, W.J. Milne's A Mental Arithmetic[13].

52. How much will it cost to plow 32 acres of land at $3.75 per acre?
    Solution: $3.75 is 3/8 of $10. At $10 per acre the plowing would cost $320, but since $3.75 is 3/8 of $10, it will cost 3/8 of 320, which is $120. Therefore, etc.
53. How much will 72 sheep cost at $6.25 per head?
54. A baker bought 88 barrels of flour at $3.75 per barrel. How much did it all cost?
55. How much will 18 cords of wood cost at $6.66 per cord?
    [These exercises continue down the page and beyond][13, 14]

According to Schoenfeld[24] the particular technique students are intended to learn from this body of text is illustrated in the solution of problem 52. In all of the exercises, the student is asked to find the product $(A \times B)$, where A is given as a two-digit decimal that corresponds to a price in dollars and cents. The decimal values have been chosen so that a simple ratio is implicit in the decimal form of A. That is, $A = r \times C$, where r is a simple fraction and C is a power of 10. Hence, $(A \times B)$ can be computed as $r \times (C \times B)$. Thus, working from the template provided in the solution to solve problem 52, the student is expected to solve problem 53 as follows:

$$(6.25 \times 72) = ([5/8 \times 10] \times 72) = (5/8 \times [10 \times 72])$$
$$= (5/8 \times 720) = 5 \times 90 = 450.$$

The student can obtain the solutions to all the problems in this section of the text by applying this algorithm. When the conditions of the problem are changed ever so slightly (in problems 52 to 60 the number C is 10, but in problem 61 it changes from 10 to 100), students are given a suggestion to help extend the procedure they have learned:

61. The porter on a sleeping car paid $37.50 per month for 16 months. How much did he earn?

    SUGGESTION: $37.50 is 3/8 of $100.

Using an analysis of Milne's textbook as a criterion, the author explored Thai mathematics textbooks from first grade to ninth grade. The results are as follows:

Characteristics of textbooks for elementary school

1. The first part of each unit contains routine exercises for drilling computational skills.
2. Word problems appeared at the end of each unit, and these word problems are routine exercises like that of Milne's textbook.
3. Most of the word problems in the first and second grades require the students to write symbolic sentence before solving the problem.

4. Almost all of routine exercises and word problems have one and only one correct answer, and are well formulated.

The following are some examples of such of exercises and word problems. These examples are excerpted from Unit 11 of the textbook for second grade.[15]

Find the answer and check it (p. 161)

Example 1: 48 ÷ 8 = ☐

Ans. 6

Check 6 × 8 = 48

Example 2: 66 ÷ 9 = ☐

Ans. 7, remainder 3

Check (7 × 9) + 3 = 66

1. 21 ÷ 3 = ☐
2. 36 ÷ 4 = ☐
3. 35 ÷ 7 = ☐
4. 69 ÷ 8 = ☐

Multiplicative word problems

Example: How much do we have to pay for two pairs of shoes at 38 bath per pair?

Symbolic sentence: 2 × 38 = ☐

Method:	One pair of shoes costs	38	baht
	Total bought	2	pairs
	Total paid	76	baht

Ans. 76 baht

Write symbolic sentences and show the method

1. How much do we have to pay for 2 handbags at 49 baht each?
2. How much do we have to pay for 3 blouses at 35 baht each?
3. How much do we have to pay for 4 pants at 55 baht each?
4. How much money will we get from selling 5 pairs of socks at 14 baht each pair?
5. How much money will we get from selling 8 water bottles at 20 baht each?[15]

From these examples, it is seen that the routine exercises appearing in the first part of this unit are designed to drill computational skills. It is expected that students will apply these skills to solve word problems at the end of the unit. In fact, as we clearly see, all the word problems mentioned above are merely routine exercises that require only computational skills. That is, even though students may not understand the meaning of these word problems, they can still solve the problems by just putting the two numerals from the problem into the symbolic sentence and computing the answer. This is what the author means by saying that the problems are well formulated.

Furthermore, the author also explored the characteristics of mathematics textbooks as described in the *Handbook for Mathematics Teachers.* It does not describe directly the characteristics of problems in textbooks. However, regarding the teaching of problem solving, it suggests that the problem-solving process is composed of the following four phases: 1) make profound sense of the problem, 2) devise a plan for solving the problem, 3) carry out the plan, and if it fails, try another plan, and 4) check the solution. Unfortunately, when looking into suggested activity, most of it consists of the teacher simply asking a series of questions. Thus, it is not very likely that this teaching/learning activity will enhance problem-solving behavior as desired.

Similarly, an analysis of the mathematics textbooks for the lower secondary grades revealed the following results.

Characteristics of textbooks for lower secondary grades

1. Most of the classroom exercises are designed for leading to conclusion of some rules or principles.
2. The exercises at the end of each section are designed to drill some rules or principles which have been learnt.

3. Word problems at the end of each section are designed for students to apply some rules or principles.
4. Tests at the end of each unit are designed to revise the rules or principles.

The Following are some examples of exercises and word problems as mentioned above. These examples are excerpted from Unit 1 of the textbook for seventh grade.[15]

Classroom exercises (p. 5)

Answer the following questions

1. How do you write $2^5 \times 2^3$ in terms of the multiplication of 2?
2. How many 2's are there in such a multiplication?
3. How do you write 2^8 in terms of the multiplication of 2?
4. Is it $2^5 \times 2^3 = 2^8$? Why?
5. Is it $5^4 \times 5^2 = 5^6$? Why?
6. Let a be any number. Is it $a^4 \times a^2 = a^6$? Why?
7. Let m and n be positive integers, and a be any number. Is it $a^m \times a^n = a^{m+n}$? Why?

The above series of questions is designed to lead to the conclusion of the rule that $a^m \times a^n = a^{m+n}$, when a is any number and m and n are positive integers.

Exercises at the end of the section

Example 1: Find the multiplication of 2.5×10^5 and 69,000 and write the outcome in the form of $A \times 10^n$ when $1 \leq A < 10$, and n is an integer.

Method: $69{,}000 = 6.9 \times 10^4$

$$(2.5 \times 10^5) \times (6.9 \times 10^4) = 2.5 \times 6.9 \times 10^9$$
$$= 17.25 \times 10^9$$
$$= 1.725 \times 10^{10}$$

Ans. 1.725×10^{10}

Example 2: The Earth weighs 5×10^{24} kg and the Sun weighs 4×10^{5} times the Earth; find the weight of the Sun.

Method: The Earth weighs 5×10^{24} kg

The Sun weighs 4×10^{5} times the Earth

Thus, the Sun weighs about $(4 \times 10^{5}) \times (5 \times 10^{24})$ kg

$= 20 \times 10^{29}$ kg

$= 2 \times 10^{30}$ kg

Ans. The Sun weighs about 2×10^{30} kg

Exercise 1.2

1. Find the following multiplications and write them in the form of $A \times 10^{n}$ when $1 \leq A < 10$, and n is an integer.

   (1) $(2 \times 10^{16}) \times (5 \times 10^{30})$

   (2) $(7.2 \times 10^{30}) \times (3.09 \times 10^{18})$

   (3) $(4.1004 \times 10^{102}) \times (6.05 \times 10^{99})$

2. In 1986, the population of Asia (excluding Russia) was 2.876×10^{9}.

   (1) The population of Europe (excluding Russia) was 0.171 times that of Asia. Find approximately the number of population of Europe.

   (2) The population of North America was 0.093 times that of Asia. Find approximately the population of North America.

   (3) The number of population of Oceania (Australia, New Zealand, and Islands in the South Pacific Ocean) was 0.009 times that of Asia. Find approximately the population of Oceania.

3. Write the answers of (1) - (3) in the form of $A \times 10^{n}$ when $1 \leq A < 10$, and n is an integer.

   [These exercises continue until item 8][15]

This exercise is designed to drill the rule that we can write any number in terms of $A \times 10^n$ when $1 \leq A < 10$, and n is an integer. Even though word problems as in item 2 are designed for students to apply some rules or principles, in effect, the students can directly use the rules as in item 1 without understanding the meaning in the word problems. Again, tests at the end of each unit drill students with some rules or principles they have been taught in class.

The characteristics of mathematics textbooks for the lower secondary grades as mentioned concur with those described in the *Handbook for Mathematics Teachers*. It classifies the category of problems in mathematics textbooks as follows: 1) classroom exercises designed for guiding students to find some rules or principles by themselves with the teacher's suggestion, 2) exercises at the end of each section designed for classroom assignment or home assignment with the aim of drilling accurately and fast use of skills, 3) mixed exercises designed for revision or drilling extra skills.

From the above-mentioned exploration, it is seen that most of the exercises and word problems in our mathematics textbooks are merely routine exercises similar to those appearing in Milne's textbook. Thus, the author will elaborate on the relationship between classroom instruction and structure and some assumptions underlying those exercises in textbooks.

Relationship between problem structure and classroom instruction

According to Schoenfeld[24], there is a general structure of mathematical tasks mentioned above and some basic pedagogical and epistemological assumptions underlying the design of such routine exercises.

STRUCTURE:

1. A task is used to introduce a technique (including rules and principles).
2. The technique is illustrated.
3. More tasks are provided so that the student may practice the illustrated skills.

BASIC ASSUMPTION:

Having completed this cluster of exercises, the students will have a new technique in their mathematical tool-kit. Presumably, the sum total of such techniques (the curriculum) reflects the corpus of mathematics the student are expected to master; the set of techniques the students have mastered comprises the students' mathematical knowledge and understanding.

In Thailand, since textbooks are the most frequently used instructional materials in classrooms[22], it can be inferred that our classroom instruction is consistent with this analysis. The results of Ryan's work prove the evidence. Ryan *et al.*[22] describe what the emphasis and intended objective of the lesson are as follows. They said that in Thailand the vast majority of the lesson emphasized the teaching of new content (both introduction and expansion), and that a somewhat greater percentage was spent on comprehending concepts and the use of rules but not on problem solving. Furthermore, they describe classroom activities in schools in Thailand as follows. "The most frequently occurring format in the communication category was either lecture or discourse. In essence, both of these formats are teacher-directed communication. In discourse, there is somewhat more opportunity for students to talk, although most of the student talk is of short duration and much is in response to a directive or question from the teacher. Furthermore, in mathematics class written seatwork predominated among activities."[22]

Repeatedly, recent research results from IEA's Third International Mathematics and Science Study (TIMSS)[1] confirmed the above-mentioned issues. Here are some of the research results. In Japan, Korea, the Netherlands, Sweden, and Thailand, in order to organize their mathematics lesson, teachers used textbooks more than any other sources of information. Internationally, the textbook appears to play a role in mathematics classrooms in many countries. For nearly all students in all countries, teachers reported using a textbook in their mathematics classes. Particularly, in Thailand, there were up to 95–99% of students whose teachers reported that they use a textbook in teaching their mathematics class (p. 157). Unfortunately, as illustrated above, Thai textbooks consist mainly of routine exercises. Moreover, the research results also revealed that there were 87% of students whose teachers reported that they ask students to practice computational skills in most or every lesson (p. 159). This is not an unexpected result because to do the exercises as appearing in the textbook requires such skills. In contrast, there were only 17% of students whom the teachers ask to do reasoning tasks in every lesson while there were 49% of students whom the teachers ask to do reasoning tasks in some lessons (p. 160). This was also an anticipated result since teachers mainly use the exercises in textbooks in their mathematics class, and there is thus no room for having students to do such reasoning tasks. From this view, it is seen that how the exercises in mathematics textbooks influence mathematics instruction in the actual classroom. Coupled with the above-mentioned issue, out-of-classroom instruction seems to contribute to classroom instruction. The case of cramschools is a good example. As it can be seen, all cramschools widely spread across the country prepare their students with shortcut techniques so that they can pass the entrance examination. Therefore, if someone is a teacher of mathematics, being asked what technique should be used with this and that problem is very common. A number of mathematics teachers have complained that now students do not want to learn how to think or to deal with mathematical problems by themselves. They just want to know a list of techniques for each mathematical topic. Thus, classroom instruction, which does not meet their demand, is not sufficient for them. This also contributes to many other problems our schools are facing at present.

From this perspective, it is seen that Thai mathematics textbooks and the situations of classroom instruction are somewhat outdated and stagnant. Consequently, this indicates a gap between the social and human needs of Thai students, as stated in the educational goals, and what the students actually do in classrooms. That is, while stated in "Vision of Education Development and Desirable Thai Society" that education will be based on a *learner-centered system*[3], the present classroom instruction, at least in the mathematics classroom, does the opposite.

How can we bridge this gap? The author argues that Thai school mathematics has to emphasize problem solving both in the curriculum and in classroom instruction. Nevertheless, the problem solving that the author is talking about does not mean to solve a problem in the sense mentioned earlier. The author will address the meaning of problem solving that should be used to reform Thai classroom instruction.

What is genuine problem solving?

The mathematician best known for his conceptualization of mathematics as problem solving and for his work in making problem solving the focus of mathematics instruction is Polya. Therefore, we will begin with his comment on distinguishing between routine exercises and problem solving. Polya[20] wrote that:

"A teacher of mathematics has a great opportunity. If he fills his allotted time with drilling his students in routine operations he kills their interest, hampers their intellectual development, and misuses his opportunity. But if he challenges the curiosity of his students by setting them problems proportionate to their knowledge, and helps them to solve their problems with stimulating questions, he may give them a taste for, and some means of, independent thinking" (p.v.).

Polya[20] also argues that to solve a problem is to find a way where no way is known off-hand, to find a way out of a difficulty, to find a way around an obstacle, to attain a desired end that is not immediately attainable by appropriate means. Here, it is worth noting, there is no mention of a particular technique at all.

In addition, Polya[20] suggests that trying to find the solution, we may repeatedly change our point of view, our way of looking at the problem. We have to shift our position again and again. Our conception of the problem is likely to be rather incomplete when we start the work; our outlook is different when we have made some progress, and it is again different when we have almost obtained the solution.

Thus, to solve a problem in Polya's sense is seen as a process, and this well-known problem-solving process consists of four phases: understanding the problem, devising a plan, carrying out the plan, and looking back. It is seen that to do problem solving and to do some exercises as mentioned earlier is quite different.

To illustrate the genuine problem solving, below are some examples. Lester[8] used the following task with third and fifth grade students in his research on mathematical problem solving, and he also called this task a process problem.

"Tom and Sue visited a farm and noticed there were chickens and pigs. Tom said,
'There are 18 animals.'
Sue said, 'Yes, and they have 52 legs in all.'
How many of each kind of animal were there?"

He found that most third-graders "solved" this problem by adding 18 and 52, and most fifth graders attempted to solve it by dividing 52 by 18. These results have shown that the students in his research did not grasp the task feature as a process in Polya's sense.

Inprasitha[5] also used this problem in his study with the third grade students in an elementary school in Japan. This problem was posed to three individual students outside the classroom. Each student solved the problem by way of thinking aloud. Even though the study aimed to investigate the origin of mathematics anxiety generated during mathematical problem solving, it was found that one of the three children generated a creative method by using a figure to proceed until he solved the problem by himself. Inprasitha[6] then adapted the above problem and used it with 15 students in an elementary school in Song Phi Nong District, Suphan Buri Province. The problem was posed to the students in pairs and to individuals outside the classroom. They were allowed to solve the problem with no time limit by way of thinking aloud. After they said that they had worked out an answer or had given up, they were then interviewed about what they had just done. The aim of this study was to investigate the students' metacognitive experiences occurring during the mathematical problem solving. During the interview session, it was found that with appropriate intervention by the teacher, most students had engaged in the problem solving process.

Similarly, Kroll and Miller[7] gave an example of a genuine problem for most middle grades students who have not yet studied algebra:

The Boys' Club held its annual carnival last weekend. Admission to the carnival was $3 for adults and $2 for children under 12. Total attendance was 100 people and $232 was collected. How many adults and how many children attended the carnival?

They describe why this is called a genuine problem as follows: This problem requires no computational expertise beyond addition and multiplication of whole numbers. On the other hand, merely performing some operations on numbers plucked from the problem cannot solve it. In this respect, it is different from many textbook story problems.

According to Kroll & Miller[7], for a student who can write and solve systems of equations, a solution to the carnival problem may be routine. However, without algebra a more innovative approach is required. For example, a fifth grader might guess numbers of adults and children that sum to 100, check to see how much money would result, and systematically adjust successive guesses according to whether more or less money is required. Alternatively, students might make an organized list showing dollars collected from a child and 99 adults, 2 children and 98 adults, and so on. After completing a small number of entries, they might observe a pattern and use that observation to extrapolate the required result.

The author argues that it is genuine problems like the one above that should receive increased emphasis in today's school mathematics, both in the curriculum and in classroom instruction. In the next section the author will describe the situations around the world, and why problem solving should be emphasized in school mathematics.

Research in problem solving

The promotion of thinking has for a long time been one of the goals of the teaching undertaken in schools. Recently, it has been regarded that the teaching of problem solving has also generally been accepted as a method of developing thinking skills[19]. A movement to emphasize problem solving started in the United States when NCTM[16] heralded a popular theme for the 1980s that problem solving must be the focus of school mathematics, and it is mainly emphasized as a method to improve the level of mathematics teaching. Since then, similar movements have been occurring worldwide. For example, in Australia, Stacey[27] asserts that the most important goal of problem solving is to encourage a deeper understanding of mathematical ideas, to foster attitudes which empower students to use mathematics and to build knowledge, habits and strategies which help students employ their mathematical thinking wherever it may be useful.

Apart from this, Becker and Shimada[2] claimed that research in problem solving in school mathematics was being addressed in Japan starting in the early 1970s. One example is the developmental research concerned with methods of evaluating higher-order thinking in mathematics education using open-ended problems as a theme. Nohda[17], one of the project team, has extended the concept of open-ended approach to the open-approach method. The method of using the open-approach depends on the problem, which consists of problem situations, process problems and open-ended problems, and procedures of these problems including classroom conditions and teaching objectives. In 1995, he suggested that the students' deeper understanding of mathematical ideas and thinking will be evolved through problem solving, and a teaching approach using an "open-ended problem" is recommended as an effective method to encourage the students to recognize the problem-solving process.

In the research community, problem solving has come to be regarded as a fundamentally important aspect of mathematics education[10, 19]. According to the proceedings of recent meetings of the International Groups for the Psychology of Mathematics Education, groups of researchers in Brazil, Japan, Italy, Portugal, Sweden, the United Kingdom, and the United States have been very active studying mathematical problem solving in a systematic manner.

To date, there have been many research approaches in both emphases and methodologies that deal with problem solving. Lester[10] provides such an overview of problem-solving research as in Table 1.

Table 1. An overview of problem-solving research emphases and methodologies: 1970-1994

Dates[a]	Problem-solving research emphases	Research methodologies used
1970-1982	Isolation of key determinants of problem difficulty; identification of characteristics of successful problem solvers; heuristics training	Statistical regression analysis; early "teaching experiments"
1978-1985	Comparison of successful and unsuccessful problem solvers (experts vs. novices); strategy training	Case studies; "think aloud" protocol analysis
1982-1990	Metacognition; relation of affects/beliefs to problem solving; metacognition training	Case studies; "think aloud" protocol analysis
1990-1994	Social influence; problem solving in context (situated problem solving)	Ethnographic methods

[a]Of course, the dates shown are only approximate. However, the chronology is reasonably accurate.

(Excerpted from Lester)[10]

In contrast to the research trend on problem solving mentioned above, most of the researches in mathematics education in Thailand have been conducted in the form of theses. In addition, so far the dominantly used methodologies are quantitative methods, particularly correlation and statistical regression analysis which were used during 1970–1982 as illustrated in Table 1. Sananwai[23] analyzed 167 master's theses in mathematics education conducted during 1975–1983. The research results revealed that the characteristics of most master's theses were as follows: 61% of the researches were experimental design, 86% used tests or questionnaires for collecting data, and 78% used statistics for testing the difference of means between two groups.

Similarly, Somboon[26] analyzed 402 master's theses conducted during 1975–1986. The results showed that most of the researches (about 96%) used experimental, survey, or correlational methods. From these results, it is seen that most of the researchers in mathematics education in our country did not deal directly with the problem-solving issue. The author argues that so far we know much about the macroperspective of the influence of some factors on mathematics achievement. What we need to know from now on is the microperspective of such influence, that is, its mechanism or processes. As Mandler[11] noted, the macroanalytic approach typically generates global measures of both individual variation and task performance. The central interest is in the individual's performance on a task or test as a whole, which is often expressed in a single score on the task. There is little room in such an approach for a distinctly different goal, that is, how to design tasks for people so that performance can be optimized. In contrast, microanalytic approach deals with what happens specifically in the interaction between the individual and the task.

Classroom instruction that fosters problem solving

From the above perspective, it is the author's contention here to offer some concrete proposals on how Thai traditional classroom instruction can be changed. He will propose two perspectives from Japan and the United States on how to use problem solving to organize classroom instruction.

In Japan, one approach to use problem solving in classroom instruction is the open-ended approach. The author will illustrate how this approach was used to reform the traditional classroom in Japan. According to Becker and Shimada[2], traditional problems used in mathematics teaching in both elementary and secondary school classrooms have a common feature: that one and only one correct answer is predetermined. The problems are so well formulated that answers are either correct or incorrect (including incomplete ones) and the correct one is unique. They call these problems "complete" or "closed" problems.

In contrast, they call problems that are formulated to have multiple correct answers "incomplete" or "open-ended" problems. In the teaching method that they call an "open-ended approach," an "incomplete" problem is presented first. The lesson then proceeds by using many correct answers to the given problem to provide experience in finding something new in the process. This can be done through combining the students' own knowledge, skills, or ways of thinking that have previously been learned.

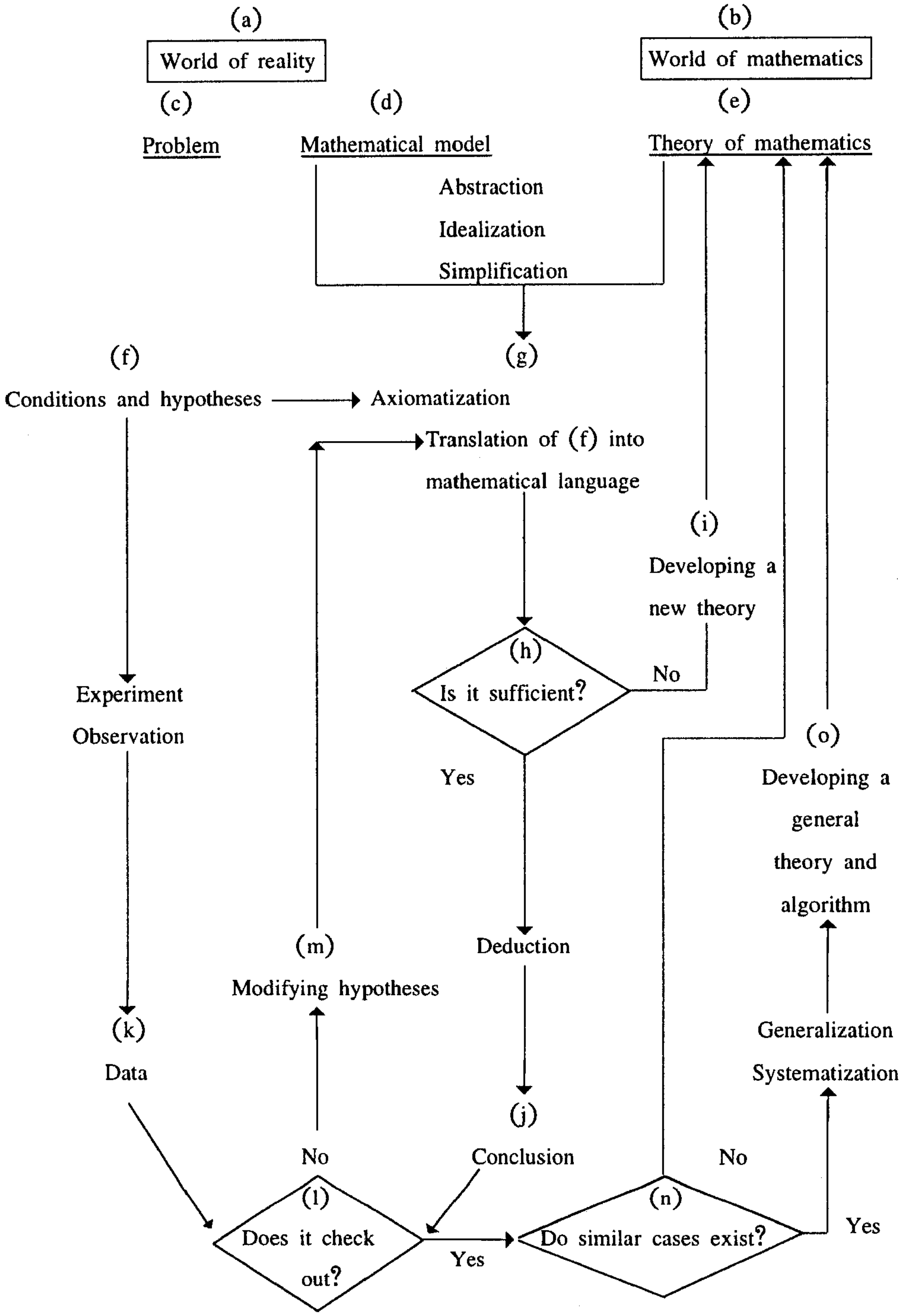

Fig. 1 A model of mathematical activities

(Excepted from Becker and Shimada)[2]

In Japan the open-ended approach is used in order to evaluate the students' achievement of the objectives of higher-order thinking. It is assumed that student activity is stimulated by this approach in various stages of mathematical activity. The model of student activities is shown in Figure 1.

The underlying basic concept in this model is that many areas of thinking are closely related to mathematics, such as understanding an existing theory of mathematics, solving a mathematical problem, constructing a new theory, or solving a problem in a non-mathematical field by applying mathematics. These areas of thinking are grouped together and called "mathematical activities." This model can be seen to reflect a historical process through which humans have developed today's mathematics as well as a developmental process of students' learning mathematics in the sense that ontogeny recapitulates phylogeny.

According to Becker and Shimada[2], the traditional classroom and the classroom using an open-ended approach can be described according to this model as follows.

In the traditional classroom in Japan, when a teacher introduces a new concept to a class, it is common to begin with introductory problems to help the students see a need for the new concept. It is also common to construct the new concept on the students' previous learning and to lead them toward a new theory. If the introductory problems are real-world problems, and if teaching starts with translating conditions or hypotheses into mathematical language, the lesson proceeds along the course from (f) to (g) and so on. However, it usually begins from stage (g), in which the translation has been done by someone else (i.e., the teacher or the author of a textbook). Afterward, it proceeds along the course from (g) to (i) to (j) and then to (n) and omits (l).

This means that the model used in (g) is actually a quasi–mathematical model.* Next, the usual teaching may proceed to (o), and a set of formal exercises is assigned for assimilation. The activities in the exercises are usually a kind of symbol game.

Furthermore, for assimilation of the new topics, the so–called application or verbal problems are assigned. Although these problems are described in terms of the world of reality, they are usually so well–structured that they have one definite answer.

A common feature through these stages is that the conclusion in each stage is predetermined, with one logical exception of the generalization from (n) to (o). Even in this exceptional case, however, many teachers use a teaching plan that predetermines the general theory. Other generalizations suggested by students are likely put aside or ignored by the teacher.

In contrast, the processes of abstraction, idealization, or simplification from (f) to (g) and of generalization in its essential sense from (n) to (o) are open–ended in such a broader sense that the result is not predetermined. The open–ended problems that are proposed here involve the processes from (f) to (g) or from (n) to (o) in a reduced form that permits their use as part of classroom teaching. However, the problems are not of the form that occurs in a real situation, while having a characteristic aspect peculiar to the processes from (f) to (g) or from (n) to (o).

In sum, Becker and Shimada[2] suggest that we may reasonably assert that activities corresponding to the processes from (f) to (g) to (h) to (j) to (l) to (m) or from (n) to (o) should be included in an entire program of mathematics education for most students.

The following excerpt is an example of lesson development explaining how the open–ended approach to teaching can be carried out in the classroom. Hashimoto[1] developed the following example to be used in the research project using the open–ended approach. An example is the water–flask problem.

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* A quasi–mathematical model is a model in which the meaning in an abstract theory is associated with words that describe the world of reality. A "fair die" used in textbook exercises in probability is simply an expression of an event with the probability of 1/6 in axiomatic probability theory.

THE WATER–FLASK PROBLEM

A transparent flask in the shape of a right rectangular prism is partially filled with water. When the flask is placed on a table and tilted, with one edge of its base being fixed, several geometric shapes of various sizes are formed by the cuboid's faces and the surface of the water. The shapes and sizes may vary according to the degree of tilt or inclination. Try to discover as many invariant relations (rules) concerning these shapes and sizes as possible. Write down all your findings.

The following examples are just a few of the possible answers to the problem:

1. Let a and b represent the lengths of the perpendicular edges from the base to the surface of the water, as in Figure 1. Then $a + b$ is constant.
2. The midpoint M of segment AC in Figure 2 is a fixed point, and the segment joining M and the midpoint N of the side opposite AC is a fixed segment.
3. The total area of the sides of the flask under the water surface is invariant.
4. When the flask is tilted as in Figure 3, $b \times c$ is a constant.
5. The shape of the water surface is a rectangle.

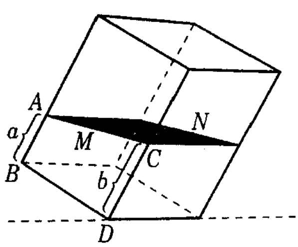

Fig. 2

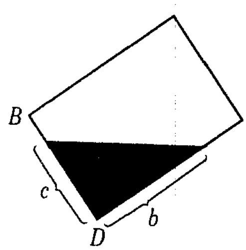

Fig. 3

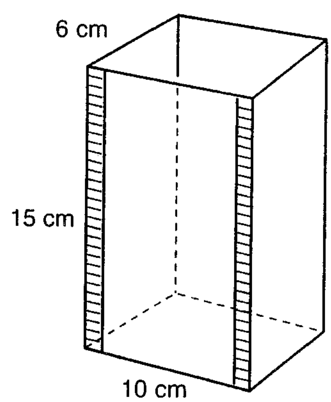

Fig. 4

This problem is an open–ended problem as defined earlier; that is, the problem is formulated to yield multiple correct answers. In order to carry out this problem in a classroom in an elementary school, here is an example of the development of a lesson on the water–flask problem in a Japanese school.

Purpose of the task and preparation

1. The task: water-flask problem
2. Subjects: Thirty-nine sixth grade students in Fukuzawa Elementary School in Setagaya Ward, Tokyo
3. Purpose: To have students discover various relations implicit in the problem situation where a flask containing water is tilted; furthermore, if possible, to have students formulate the relations they discover into mathematical expressions and explain logically the formulated relations
4. Instruments and materials:
    a) Ten water flasks (as in Figure 4)
    b) Ten beakers for pouring water
    c) A blank sheet of paper for each student
    d) Ten blank transparency sheets for presentations by groups on the overhead projector (Note: A science laboratory was used because it was more convenient than an ordinary classroom for the use of beakers, flasks, water, and discussion).

Sequence of presentation and allocation of time

The teacher used two forty-minute periods for the lesson. Two periods were used because other study-group members had found from their experience that more than one period was necessary. In most instances, one period allows insufficient time for discussion and a summary of all the students' findings, many of which are very interesting. Two periods, however, provided enough time for students to explore the problem situation and discuss most, if not all, of their findings.

In general, when teaching with the open-ended approach, the teacher must carefully allocate and manage time because students are likely to generate many responses, both expected and unexpected, and all should be discussed and summarized. The following lesson plan is designed for two periods:

The first period: individual work follows the presentation of the problem to the whole class. Each student is given a blank worksheet on which to write his or her ideas. The worksheets are collected afterward by the teacher for use in preparing a summary of the individual student's responses. In the group work that follows, each group of four students uses one set of instruments and a representative of each group writes down the results of the group's discussion and also, if possible, the process by which the group arrived at the result.

The second period: The results of each group's work are presented and discussed. Then the lesson is summarized.

The lesson plan[4]

The first period

Teacher's presentation and directions	Students' activities	Remarks	Cumulative time in minutes
1. When we tilt the flask with water in it while fixing one edge of the base on the table, we see that the shape and size of various parts are changing. Find out as many relations among the parts as possible, and write them down.	1. Understanding the question	1. Explain the problem by using a real flask with water. 2. Use Figure 5 as a poster to make sure the students understand the problem.	5

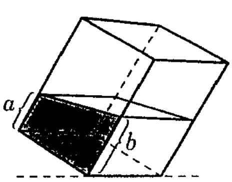

Fig. 5

2. Write down what you have noticed on the blank worksheet.	2. Trying to find various rules (individual work)	3. Distribute sheets to each student. 4. Collect sheets on which students have written their findings.	25
3. Within each group, discuss what you have found. The leader of each group should record the group's observations.	3. Discussing within groups, and discovering various rules	5. Distribute new sheets to each group.	40
The second period			
1. Please present the results of your group discussion.	1. Groups take turns presenting their results.	1. List every response from the groups on a poster.	20
2. Let's group together similar findings.	2. Rules are grouped from various viewpoints.	2. Have students group findings carefully so as not to duplicate or omit any.	30
3. We know the rule that $a + b$ is constant, where a and b are the lengths of the sides shown in Figure 5. Can we explain the figure rule?	3. Students methodically consider why the property of the sum's being constant is true.	3. Assign a and b to the bases of the shaded trapezoid as in Figure 5.	40

4. The teacher gives the reason, if necessary.	4. Students listen to the explanation.
5. Can we put in order other rules from different points of view?	5. Students summarize their findings.

(Excerpted from Hashimoto)[4]

Another perspective is from the United States. From the author's experience, most of the teachers of mathematics in Thailand rarely use the problem like the carnival or process problem mentioned earlier with elementary school students because they might believe that the students cannot solve such a problem unless their students have learnt algebra. This also reflects a consistent belief about pedagogy in that a teacher has to teach something first, then he or she can pose some mathematical tasks in which students know an underlying technique or a way to find a desired end. This belief in turn is reflected upon his or her classroom instruction. To illustrate how to organize classroom instruction in order for students to do the above-mentioned activities, the author proposes a model of teaching action that can promote problem solving. This model was developed by Lester *et al.*[9] and was used in his research on the Role of Metacognition[*] in Mathematical Problem Solving: "A Study of Two Grade Seven Classes." This study was designed to: 1) assess 7th graders' metacognitive beliefs and processes and investigate how they affect problem-solving behaviors; and 2) explore the extent to which these students can be taught to be more strategic and aware of their own problem-solving behaviors.

[*] Metacognition refers to the knowledge and control one has of one's cognitive functioning, that is, what one knows about one's cognitive performance and how one regulates one's cognitive actions during the performance of some task.

Teaching actions for problem solving[9]

Teaching action	Purpose
BEFORE	
1. Read the problem to the class or have a student read the problem. Discuss words or phrases which students may not understand.	Illustrate the importance of reading problems carefully and focus on words that have special interpretations in mathematics.
2. Use whole-class discussion about understanding the problem. Use problem-specific comments and/or the Problem-Solving Guide.	Focus attention on important data in the problem and clarify parts of the problem.
3. (Optional) Use whole-class discussion about possible solution strategies. Use the Problem-Solving Guide.	Elicit ideas for possible ways to solve the problem.
DURING	
4. Observe and question students to determine where they are in the problem-solving process.	Diagnose students' strengths and weaknesses related to problem solving.
5. Provide hints as needed.	Help students past blockages in solving a problem.
6. Provide problem extensions as needed.	Challenge the early finishers to generalize their solution strategy to a similar problem.
7. Require students who obtain a solution to "answer the question."	Require students to look over their work and make sure it makes sense.

AFTER

8. Show and discuss solutions using the Problem-Solving Guide as a basis for discussion.	Show and name different strategies used successfully to find a solution.
9. Relate the problem to previously solved problems and discuss or have students solve extensions of the problem.	Demonstrate that problem-solving strategies are not problem-specific and that they help students recognize different kinds of situations in which particular strategies may be useful.
10. Discuss special features of the problem, such as a picture accompanying the problem statement.	Show how the special features of a problem may influence the way that one thinks about a problem.

(Excerpted from Lester *et al.*)[9]

Instead of confirming that this is a complete model, the research results revealed that the relationship between problem solving and the students' mathematical activities is much more complex than it was thought. Here are some assumptions suggested by Lester *et al.*[9]

1. There is a dynamic interaction between mathematical concepts and the processes (including metacognitive ones) used to solve problems involving those concepts. That is, control processes and awareness of cognitive processes develop concurrently with the development of an understanding of mathematical concepts.

2. In order for the students' problem-solving performance to improve, they must attempt to solve a variety of types of problems on a regular basis and over a prolonged period of time.

3. Metacognition instruction is most effective when it takes place in a domain-specific context.

4. Problem-solving instruction, metacognition instruction in particular, is likely to be most effective when it is provided in a systematically organized manner under the direction of the teacher.

5. Problem-solving instruction that emphasizes the development of metacognitive skills should involve the teacher in three different, but related, roles: (a) as an external monitor, (b) as a facilitator of the students' metacognitive awareness, and (c) as a model of a metacognitively-adept problem solver.

6. The standard arrangement for classroom problem-solving activities is for students to work in small groups (usually groups of four). Small group work is especially appropriate for activities involving new content (e.g. new mathematics topics, new problem-solving strategies) or when the focus of the activity is on the process of solving problems (e.g. planning, decision making and assessing progress).

7. The teacher's instructional plan should include attention to how the students' performance is to be evaluated. It was assumed that in order for students to become convinced of the importance of monitoring their actions and being aware of their thinking, it would be necessary to use evaluation techniques that rewarded such behaviors.

From the above perspective, the implementation for improving mathematics instruction in Thailand is considered. The substantial issues among two perspectives are as follows: 1) mathematical problems, 2) classroom organization, and 3) teacher's roles.

Mathematical problems

It is seen that an open-ended problem used in the research project in Japan is quite different from the exercises commonly occurring in our textbooks, which teachers often use in their mathematics classroom. In contrast to exercises where the students' responses should be one and only one answer (either right or wrong), for open-ended problems, the research results revealed that students were able to find a variety of ideas in accordance with their abilities by determining their own viewpoints for the situations. They expressed unexpected ideas, where the teacher discovered new aspects of the students' ways of thinking that had previously not been apparent. Regarding this, we can acknowledge that open-ended problems can be uses for an evaluation of higher-order thinking as mentioned earlier. This is also an important aspect of problem solving, and in turn this approach also has implications for improving assessment in problem solving.

Similarly, using non-routine problems like the carnival problem with elementary school children as in Lester's project seems to be hard to accept by mathematics teachers in Thailand. Therefore, distinguishing between genuine problems and mere exercises needs to be looked at more closely.

Classroom organization

As illustrated in the lesson plan in the two projects, classroom organization is also an important issue in teaching problem solving. In the traditional classroom, teachers are considered the most powerful decision-makers in almost all issues. For instance, Sekiguchi[25] notices a well-known interaction pattern (called IRF) in many classrooms in Japanese schools: An initiation (often a question) by a teacher, followed by response from a student, then followed by feedback from the teacher. There are several assumptions underlying this pattern. First, it is the teacher who is supposed to initiate a question. Secondly, the teacher is presumed to know the right answer to the question beforehand, despite the fact that it is quite unusual in general society that the person asking a question already knows the answer. In Thai schools, individually motivated questions and the teacher's lecture to the whole class are very common. However, from the projects it can be seen that small group working is also appropriate for problem-solving activities. However, to deal with small working groups is not an easy task because it is also related to other issues such as time allocation, organizing group presentation, and bringing the lesson to a conclusion at the end of each period. These group projects demand a great deal of time from the teachers in the preparation of their lesson.

Teacher's roles

The teacher's roles in the two projects are the most difficult issue when teaching through problem solving. It is not such a complicated task for teachers in the traditional classroom who give talks or explain things to the whole class. In contrast, in Lester's project, the teacher's role (a) as an external monitor, (b) as a facilitator, and (c) as a model problem solver, are hard to deal with. For instance, Lester *et al.*[9] realized that when students were weak in basic skills, the teacher spent much of the "external monitoring" time explaining how to do calculations or how to reason logically. One observation on this point is that teachers should expect to provide instruction in basic skills simultaneously with instruction in control strategies and heuristics. Similarly, in the open-ended project, the teacher's role as moderator, raising questions, providing hints, and initiating discussion may be difficult to deal with. In the round-table discussion to review the open-ended project, the question, "Should the teacher have some particular teaching style?" was raised. In the open-ended style of teaching, asking the students only patterned questions such as "Are there any other opinion?" is not sufficient. It is important for the teacher to ask specific questions in order to help students find some general rules. We should study what mental attitudes the teacher should have. However, if we really want to change our classroom instruction, dealing with such group projects is inevitable.

CONCLUDING REMARKS

This paper is an attempt to provide a perspective on how to reform mathematics instruction in Thailand. In particular, problem solving has been proposed as a theme. Problem solving is a widely used term in the field of education. We often hear most of educationists say that in general our educational aim is to help students "to think, act, and solve problems." Unfortunately, it seems to be that there is little justification in what we mean by those terms. For instance, as it is seen in this paper, it is necessary to clarify first what is meant by "solve problems." The author argues that the meaning of problem solving should not be limited to only "doing routine exercises" such as those appearing in our mathematics textbooks. At least, viewing problem solving and problem-solving teaching as an open-ended approach or as metacognitive teaching provided in this paper should be an alternative for mathematics teachers to improve their teaching. For mathematics educators, viewing mathematics according to "a model of mathematics activities" should extend their perspective on teaching/learning mathematics.

From this perspective, the author argues that to change the traditional classroom instruction does not mean just simply changing methods of teaching such as from a lecture to using computer assisted instruction or other kinds of media. The important point is that we have to change our teachers' view of mathematics first. He believes that this is the only way that little by little can we change our traditional classroom instruction. Additionally, this change is an initial point for educational reform. This reform is generated by taking problem solving as a basis for classroom instruction. As stated in the Eighth National Education Development Plan (1997-2001), the major reasons for the low quality of education were the inefficient processes of teaching-learning, and the inappropriateness of the curriculum. As a response to the inefficient processes of teaching-learning, the two perspectives from Japan and the United States presented in this paper will help Thai teachers of mathematics improve the processes of teaching-learning. The author has proposed as concretely as possible how to organize classroom instruction that can foster problem solving. It is also hoped that the teachers can implement these processes immediately.

The author believes that our educational reform should start from the classroom. The teacher and students of that class have created a traditional classroom. In this sense, the classroom is a culture: a microculture. Thus, it must be the teacher and the students of that class who have the duty of changing such a traditional classroom culture. Nevertheless, such a microculture is influenced by the teacher's and students' beliefs about mathematics, mathematical values, and interests of that classroom. Thus, we should not expect to change the classroom culture by simply putting a computer or other kinds of media into the classroom. We still know little about the factors which influenced our classroom culture. Thus, changing it requires an evolutionary change in our approach to the teaching/learning process.

With the proposed perspectives, it is expected that the teacher and students in a class will interact, interpret others' or their own acts, and the rules and values they generate and maintain. This will in turn create a new classroom culture. Only in this way can we expect that our classroom instruction will be changed. In addition, this change will contribute to the actual educational reform that Thailand is urgently in need of.

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