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Full Paper

A study on Lucas difference sequence spaces $\ell_p\left(\hat{E}(r,s)\right)$ and $\ell_\infty\left(\hat{E}(r,s)\right)$

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Abstract: A new Banach sequence space established by using Lucas sequences and non-zero real numbers r and s is presented. At the same time, some inclusion relations are given and also some geometrical properties such as Banach-Saks type p, weak fixed-point property and the modulus of convexity for this space are analysed.

Keywords: difference sequence spaces, Lucas numbers, weak fixed-point property, Banach-Saks property, modulus of convexity

INTRODUCTION

Let w be the space of all real and complex valued sequences. A linear subspace of w is called a sequence space. If X is a complete linear metric space, then a K-space X is called an FK-space. An FK-space whose topology is normable is called BK-space. The symbols ℓ_∞, c, c_0 and ℓ_p for $1 \le p < \infty$ represent the sequence spaces of all bounded, convergent, null sequences and p-absolutely convergent series respectively, normed by $\|x\|_\infty = sup_k|x_k|$ and $\|x\|_p = (\sum_k|x_k|^p)^{1/p}$.

Difference sequence spaces were introduced for the first time by Kizmaz [1] in the form of $X(\Delta) = \{x \in w: x_k - x_{k+1} \in X\}$ for $X = \ell_\infty, c, c_0$. Later, Et and Colak [2] generalised these spaces such as $X(\Delta^r) = \{x \in w: \Delta^r x \in X\}, X = \ell_\infty, c, c_0$.

An infinite matrix is a double sequence $A = (a_{nk})$ of real or complex numbers defined by a function A from the set $\mathbb{N} \times \mathbb{N}$ into the complex field $\mathbb{C}(or\ \mathbb{R})$ where $\mathbb{N} = \{0,1,2,\dots\}$. The treatment of infinite matrices is absolutely different from that of finite matrices. There are various reasons for this. In some instances the most general linear operator among two sequence spaces is presented by an infinite matrix. Let X and Y be any two sequence spaces. A defines a matrix mapping from X to Y if $Ax = \{(Ax)_n\} \in Y$ for every $x = (x_k) \in X$ where

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$$(Ax)_n = \sum_k a_{nk} x_k. \tag{1}$$

The class of all matrices A such that $A: X \to Y$ is symbolised by $(X: Y)$. In this way, $A \in (X: Y)$ if and only if the series on the right hand side of (1) converges for each $n \in \mathbb{N}$ and every $x \in X$, and we get $Ax = \{(Ax)_n\} \in Y$ for all $x \in X$. The notion of matrix domain for an infinite matrix A in the sequence space μ is given by

$$\mu_A = \{x \in w: Ax \in \mu\}, \tag{2}$$

which is a sequence space [3].

During the recent years, some sequence spaces by means of the matrix domain for a triangle infinite matrix have been introduced by many mathematicians, e.g. Mursaleen and Noman [4,5], Karakas [6], Candan and Kara [7], Kara [8], Mursaleen et al. [9], Basar and Altay [10], Savas et al. [11] and Kara and Basarır [12].

In the literature, if λ is a normed or paranormed sequence space, then the matrix domain λ_Δ is said to be a difference sequence space. For $\lambda = \ell_p$, this space is called the space of sequences of p-bounded variation, i.e. bv_p. Also, it is clear that $bv_p = (\ell_p)_\Delta$.

Now let us give the following two triangle summability matrices, named as backward difference matrix and forward difference matrix for all $n, k \in \mathbb{N}$:

$$\Delta_{nk} = \begin{cases} (-1)^{n-k}, & n-1 \le k \le n \\ 0, & k > n \text{ or } 0 \le k < n-1 \end{cases} \text{ and } \Delta'_{nk} = \begin{cases} (-1)^{n-k}, & n \le k \le n+1 \\ 0, & k > n+1 \text{ or } 0 \le k < n \end{cases}$$

respectively. Kirisci and Basar [13] have lately defined and examined the difference sequence spaces $\hat{X} = \{x \in w: B(r,s)x \in X\}, 1 \le p < \infty,\ X = \ell_\infty, \ell_p, c, c_0$ where $B(r,s)x = (sx_{k-1} + rx_k); r, s \neq 0$. At the same time, difference sequence spaces have been analysed in some studies by Et [14,15], Mursaleen [16], Colak and Et [17], Et and Basarır [18], Bektas et al. [19], Et and Esi [20], Gaur and Mursaleen [21], Altın [22] and Polat et al. [23].

Now according to the well-known concept of Schauder basis, a sequence (a_n) is said to be Schauder basis for a normed sequence space X if X involves a sequence (a_n) such that there is a unique sequence of scalars (β_n) for every $x \in X$ with

$$\lim_{n\to\infty} \| x - (\beta_0 a_0 + \beta_1 a_1 + \cdots + \beta_n a_n) \| = 0.$$

The aim of this note is to introduce and investigate the new sequence spaces $\ell_p\left(\hat{E}(r,s)\right)$ and $\ell_\infty\left(\hat{E}(r,s)\right)$ by constructing the generalised Lucas difference matrix $\hat{E}(r,s)$ by the help of Lucas sequence $\{L_n\}$ and $r, s \in \mathbb{R} - \{0\}$. Besides, we show that these spaces are *BK*-spaces and linearly isomorphic to space ℓ_p for $1 \le p \le \infty$. Additionally, we study a number of inclusion relations and give the basis for the space $\ell_p\left(\hat{E}(r,s)\right), 1 \le p < \infty$ and also examine several geometric properties of $\ell_p\left(\hat{E}(r,s)\right), 1 < p < \infty$.

RESULTS AND DISCUSSION

The sequence $\{L_n\}_{n=0}^{\infty}$ of Lucas numbers given by the Fibonacci recurrence relation and different initial conditions is defined as

$$L_0 = 2, L_1 = 1 \text{ and } L_n = L_{n-1} + L_{n-2}, n \ge 2.$$

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Lucas numbers have several interesting properties and applications [24, 25]. Some of them are as follows:

$$\sum_{k=1}^{n} L_k = L_{n+2} - 3 \ ; \sum_{k=1}^{n} L_{2k-1} = L_{2n-2} \ \ n \geq 1$$

$$\sum_{k=1}^{n} L_k^2 = L_n L_{n+1} - 2 \ ; \sum_{k=1}^{2n} L_k L_{k-1} = L_{2n}^2 - 4$$

$$L_{n+1}L_{n-1} - L_n^2 = (-5)(-1)^n.$$

If the term $L_n L_{n-1}$ is deducted from both sides of the last equality, the following formula is obtained:

$$L_{n-1}^2 + L_n L_{n-1} - L_n^2 = (-5)(-1)^n.$$

In the view of the above information, we establish the generalised Lucas band matrix $\hat{E}(r,s) = \left(\hat{L}_{nk}(r,s)\right)$ by

$$\hat{E}(r,s) = \left(\hat{L}_{nk}(r,s)\right) = \begin{cases} s\frac{L_n}{L_{n-1}} & (k = n-1) \\ r\frac{L_{n-1}}{L_n} & (k = n) \\ 0 \ (k > n \ or \ 0 \leq k < n-1) \end{cases} . \quad (3)$$

Also, we define $\hat{E}$ −transform of a sequence $x = (x_n)$:

$$y_n = \hat{E}_n(r,s)(x) = r\frac{L_{n-1}}{L_n}x_n + s\frac{L_n}{L_{n-1}}x_{n-1}, n \geq 1. \quad (4)$$

Now let us introduce the following Lucas difference sequence spaces $\ell_p\left(\hat{E}(r,s)\right)$ and $\ell_\infty\left(\hat{E}(r,s)\right)$ by using (3) and (4) in the forms of

$$\ell_p\left(\hat{E}(r,s)\right) = \left\{x \in w: \sum_n \left|r\frac{L_{n-1}}{L_n}x_n + s\frac{L_n}{L_{n-1}}x_{n-1}\right|^p < \infty\right\}, 1 \leq p < \infty$$

and

$$\ell_\infty\left(\hat{E}(r,s)\right) = \left\{x \in w: sup_n \left|r\frac{L_{n-1}}{L_n}x_n + s\frac{L_n}{L_{n-1}}x_{n-1}\right| < \infty\right\}.$$

These spaces may be redefined by the help of (2) as

$$\ell_p\left(\hat{E}(r,s)\right) = (\ell_p)_{\hat{E}(r,s)} \quad \text{and} \quad \ell_\infty\left(\hat{E}(r,s)\right) = (\ell_\infty)_{\hat{E}(r,s)}. \quad (5)$$

Theorem 1. The sequence space $\ell_p\left(\hat{E}(r,s)\right)$ is a *BK*-space for $1 \leq p \leq \infty$, normed by

$$\|x\|_{\ell_p(\hat{E}(r,s))} = \left(\sum_n \left|\hat{E}_n(r,s)(x)\right|^p\right)^{1/p} \text{and } \|x\|_{\ell_\infty(\hat{E}(r,s))} = sup_n\left|\hat{E}_n(r,s)(x)\right|.$$

Proof. Since the matrix $\hat{E}(r,s)$ is a triangle, (5) holds and the spaces ℓ_p and ℓ_∞ are *BK*-spaces according to their usual norms. By the theorem 4.3.12 of Wilansky [26], the proof of theorem can be straight-forwardly obtained. So our spaces are *BK*-spaces with the above norms. This completes the proof.

Remark 1. $\ell_p\left(\hat{E}(r,s)\right)$ and $\ell_\infty\left(\hat{E}(r,s)\right)$ are the sequence spaces of non-absolute type. Indeed $\|x\|_{\ell_p(\hat{E}(r,s))} \neq \||x|\|_{\ell_p(\hat{E}(r,s))}$ and $\|x\|_{\ell_\infty(\hat{E}(r,s))} \neq \||x|\|_{\ell_\infty(\hat{E}(r,s))}$. This leads to the fact that the absolute property does not hold for the spaces $\ell_p\left(\hat{E}(r,s)\right)$ and $\ell_\infty\left(\hat{E}(r,s)\right)$ for at least one sequence where $|x| = (|x_k|)$ and $1 \le p < \infty$.

Theorem 2. The generalised Lucas difference sequence space $\ell_p\left(\hat{E}(r,s)\right)$ of non-absolute type is linearly isomorphic to the space ℓ_p in the case $1 \le p \le \infty$.

Proof. Let us consider the transformation $M: \ell_p\left(\hat{E}(r,s)\right) \to \ell_p$ defined by $x \to y = Mx$ with (4). Hence for $x \in \ell_p\left(\hat{E}(r,s)\right)$, we have $Mx = y = \hat{E}(r,s)x \in \ell_p$. So it is easy to see that M is linear and we omit it. Additionally, we can simply show that $x = 0$ whenever $Mx = 0$ and thus M is injective. Additionally, let us take $y = (y_k) \in \ell_p$ and define the sequence $x = (x_k)$ in the form of

$$x_k = \frac{1}{r}\sum_{j=1}^{k}\left(-\frac{s}{r}\right)^{k-j}\frac{L_k^2}{L_{j-1}L_j}y_j. \tag{6}$$

In that case we get

$$\|x\|_{\ell_p(\hat{E}(r,s))} = \left(\sum_k\left|r\frac{L_{k-1}}{L_k}x_k + s\frac{L_k}{L_{k-1}}x_{k-1}\right|^p\right)^{1/p}$$

$$= \left(\sum_k\left|\frac{L_{k-1}}{L_k}\sum_{j=1}^{k}\left(-\frac{s}{r}\right)^{k-j}\frac{L_k^2}{L_{j-1}L_j}y_j + \frac{s}{r}\frac{L_k}{L_{k-1}}\sum_{j=1}^{k-1}\left(-\frac{s}{r}\right)^{k-j-1}\frac{L_k^2}{L_{j-1}L_j}y_j\right|^p\right)^{1/p}$$

$$\left(\sum_k|y_k|^p\right)^{1/p} = \|y\|_{\ell_p} < \infty$$

and

$$\|x\|_{\ell_\infty(\hat{E}(r,s))} = sup_k\left|r\frac{L_{k-1}}{L_k}x_k + s\frac{L_k}{L_{k-1}}x_{k-1}\right| = sup_k|y_k| = \|y\|_{\ell_\infty} < \infty.$$

This gives us $x \in \ell_p\left(\hat{E}(r,s)\right), 1 \le p \le \infty$. Hence we see that M is surjective and norm preserving. In conclusion M is a linear bijection and so the spaces $\ell_p\left(\hat{E}(r,s)\right)$ and ℓ_p are linearly isomorphic.

Theorem 3. If $1 \le p \le \infty$, then the inclusion $\ell_p \subset \ell_p\left(\hat{E}(r,s)\right)$ strictly holds.

Proof. Let us assume that $x \in \ell_p, 1 < p \le \infty$. Due to the inequalities $\frac{L_{k-1}}{L_k} \le 2$ and $\frac{L_k}{L_{k-1}} \le 3$, we have from (4):

$$\sum_k\left|\hat{E}_k(r,s)(x)\right|^p \le \sum_k 6^{p-1}(|2rx_k|^p + |3sx_{k-1}|^p)$$

$$\le 6^{2p-1}max\{|r|,|s|\}(\textstyle\sum_k|x_k|^p + \sum_k|x_{k-1}|^p)$$

and

$$sup_k\left|\hat{E}_k(r,s)(x)\right| \le 5max\{|r|,|s|\}sup_k|x_k|.$$

From here, we obtain for $1 < p \le \infty$:

$$\|x\|_{\ell_p(\hat{E}(r,s))} \le 36max\{|r|,|s|\}\|x\|_p \quad \text{and } \|x\|_{\ell_\infty(\hat{E}(r,s))} \le 5max\{|r|,|s|\}\|x\|_\infty. \tag{7}$$

Lastly, the inclusion $\ell_p \subset \ell_p\left(\hat{E}(r,s)\right)$ is strict for $1 < p \leq \infty$ because the sequence $x = (x_k) = (1/r(-s/r)^k L_k^2)$ is in $\ell_p\left(\hat{E}(r,s)\right) / \ell_p$. It can be easily seen that (7) also holds for $p = 1$. Thus, the proof is complete.

Theorem 4. $\ell_p\left(\hat{E}(r,s)\right) \subset \ell_s\left(\hat{E}(r,s)\right)$ for $1 \leq p < s$.

Proof. Under the conditions $1 \leq p < s$ and $x \in \ell_p\left(\hat{E}(r,s)\right)$, we obtain from Theorem 1 that $y \in \ell_p$ if we consider the sequence y given by (4). In view of the fact that the inclusion $\ell_p \subset \ell_s$ holds, we have $y \in \ell_s$. This clearly implies that $x \in \ell_s\left(\hat{E}(r,s)\right)$ and so the inclusion $\ell_p\left(\hat{E}(r,s)\right) \subset \ell_s\left(\hat{E}(r,s)\right)$ is true.

Theorem 5. The sequence $\left(b^{(k)}\right)_{k=1}^{\infty}$ defined by

$$\left(b^{(k)}\right)_n = \begin{cases} \frac{1}{r}\left(-\frac{s}{r}\right)^{k-n} \frac{L_n^2}{L_{k-1}L_k}, & n \geq k \\ 0, & k > n \end{cases} \tag{8}$$

is a basis for the space $\ell_p\left(\hat{E}(r,s)\right)$ for $1 \leq p < \infty$. Also, every $x \in \ell_p\left(\hat{E}(r,s)\right)$ has a unique representation of the form

$$x = \sum_k \hat{E}_k(r,s)(x) b^{(k)}. \tag{9}$$

Proof. By use of (8), it is trivial that $\hat{E}(r,s)\left(b^{(k)}\right) = e^{(k)} \in \ell_p$ and this gives us $b^{(k)} \in \ell_p\left(\hat{E}(r,s)\right)$. Moreover, let us consider $x \in \ell_p\left(\hat{E}(r,s)\right)$ and take $x^{(m)} = \sum_{k=1}^{m} \hat{E}_k(r,s)(x) b^{(k)}$ for every non-negative integer m. Then it is obtained that

$$\hat{E}(r,s)\left(x^{(m)}\right) = \sum_{k=1}^{m} \hat{E}_k(r,s)(x)\hat{E}(r,s)\left(b^{(k)}\right) = \sum_{k=1}^{m} \hat{E}_k(r,s)(x) e^{(k)}$$

and also,

$$\hat{E}_n(r,s)\left(x - x^{(m)}\right) = \begin{cases} \hat{E}_n(r,s)(x), & n > m \\ 0, & 0 \leq n \leq m \end{cases}.$$

So there is a non-negative integer m_0 such that $\sum_{n=m_0+1}^{\infty} \left|\hat{E}_n(r,s)(x)\right|^p < \left(\frac{\varepsilon}{2}\right)^p$ for any $\varepsilon > 0$. Hence we have that

$$\left\|x - x^{(m)}\right\|_{\ell_p(\hat{E}(r,s))} = \left(\sum_{n=m+1}^{\infty} \left|\hat{E}_n(r,s)(x)\right|^p\right)^{1/p} \leq \left(\sum_{n=m_0+1}^{\infty} \left|\hat{E}_n(r,s)(x)\right|^p\right)^{\frac{1}{p}} \leq \frac{\varepsilon}{2} < \varepsilon$$

for every $m \geq m_0$, i.e.

$$\lim_{m\to\infty} \left\|x - x^{(m)}\right\|_{\ell_p(\hat{E}(r,s))} = 0.$$

Finally, to show the uniqueness of (9), let us assume that $x = \sum_k v_k(x) b^{(k)}$ for $x \in \ell_p\left(\hat{E}(r,s)\right)$. From the continuity of the linear transformation predefined in Theorem 2, we get

$$\hat{E}_n(r,s)(x) = \sum_k v_k(x) \hat{E}_n(r,s)\left(b^{(k)}\right) = \sum_k v_k(x) \delta_{nk} = v_n(x).$$

This completes the proof.

Now we examine the geometrical properties of $\ell_p\left(\hat{E}(r,s)\right)$. Let X be a normed linear space and S_X, B_X be the unit sphere and unit ball of X respectively. Clarkson [27] defined the modulus of convexity as follows:

$$\delta_X(\varepsilon) = inf\left\{1 - \frac{\|x+y\|}{2} : x, y \in S_X, \|x-y\| = \varepsilon\right\}, \varepsilon \in [0,2].$$

In the literature there are a few equivalent definitions for modulus of convexity. One of them is Gurarii's modulus of convexity, which was defined by Gurarii [28] and Sanchez and Ullan [29] in the form

$$\beta_X(\varepsilon) = inf\left\{1 - inf_{t\in[0,1]}\|tx + (1-t)y\| : , y \in S_X, \|x-y\| = 2\right\}, \varepsilon \in [0,2].$$

If $0 < \beta_X(\varepsilon) < 1$, then X is uniformly convex and if $\beta_X(\varepsilon) \leq 1$, then X is strictly convex.

Theorem 6. The Gurarii's modulus of convexity for the space $\ell_p\left(\hat{E}(r,s)\right), 1 \leq p < \infty$ is

$$\beta_{\ell_p(\hat{E}(r,s))}(\varepsilon) \leq 1 - \left(1 - \left(\frac{\varepsilon}{2}\right)^p\right)^{\frac{1}{p}}, \varepsilon \in [0,2].$$

Proof. Let $x \in \ell_p\left(\hat{E}(r,s)\right)$. Then

$$\|x\|_{\ell_p(\hat{E}(r,s))} = \left\|\hat{E}(r,s)x\right\|_{\ell_p} = \left(\sum_n \left|\hat{E}_n(r,s)(x)\right|^p\right)^{1/p}. \tag{10}$$

Let $\varepsilon \in [0,2]$ and consider the sequences below where $\hat{E}^{-1}(r,s)$ is the inverse of the matrix $\hat{E}(r,s)$:

$$u = (u_n) = \left(\hat{E}^{-1}(r,s)\left(1 - \left(\frac{\varepsilon}{2}\right)^p\right)^{\frac{1}{p}}, \hat{E}^{-1}(r,s)\left(\frac{\varepsilon}{2}\right), 0,0,\dots\right),$$

$$v = (v_n) = \left(\hat{E}^{-1}(r,s)\left(1 - \left(\frac{\varepsilon}{2}\right)^p\right)^{\frac{1}{p}}, \hat{E}^{-1}(r,s)\left(-\frac{\varepsilon}{2}\right), 0,0,\dots\right). \tag{11}$$

Therefore, it is easy to see that the $\hat{E}(r,s)$ −transforms of the sequences given by (11) are

$$\hat{E}(r,s)u = \left(\left(1 - \left(\frac{\varepsilon}{2}\right)^p\right)^{\frac{1}{p}}, \frac{\varepsilon}{2}, 0,0,\dots\right) \text{ and } \hat{E}(r,s)v = \left(\left(1 - \left(\frac{\varepsilon}{2}\right)^p\right)^{\frac{1}{p}}, -\frac{\varepsilon}{2}, 0,0,\dots\right).$$

Hence $\left\|\hat{E}(r,s)u\right\|_{\ell_p} = \|u\|_{\ell_p(\hat{E}(r,s))} = 1$ and $\left\|\hat{E}(r,s)v\right\|_{\ell_p} = \|v\|_{\ell_p(\hat{E}(r,s))} = 1$. This means that $u, v \in S\left(\ell_p\left(\hat{E}(r,s)\right)\right)$ and $\left\|\hat{E}(r,s)u - \hat{E}(r,s)v\right\|_{\ell_p} = \|u - v\|_{\ell_p(\hat{E}(r,s))} = \varepsilon$.

Now for $\lambda \in [0,1]$,

$$\begin{aligned}\|\lambda u + (1-\lambda)v\|^p_{\ell_p(\hat{E}(r,s))} &= \left\|\lambda\hat{E}(r,s)u + (1-\lambda)\hat{E}(r,s)v\right\|^p_{\ell_p} \\ &= 1 - \left(\frac{\varepsilon}{2}\right)^p + |2\lambda - 1|^p\left(\frac{\varepsilon}{2}\right)^p.\end{aligned} \tag{12}$$

From here,

$$inf_{\lambda\in[0,1]}\|\lambda u + (1-\lambda)v\|_{\ell_p(\hat{E}(r,s))} = \left(1 - \left(\frac{\varepsilon}{2}\right)^p\right)^{\frac{1}{p}}. \tag{13}$$

Therefore, for $1 \leq p < \infty$,

$$\beta_{\ell_p(\hat{E}(r,s))}(\varepsilon) \leq 1 - \left(1 - \left(\frac{\varepsilon}{2}\right)^p\right)^{\frac{1}{p}}.$$

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Corollary 1. i) If $\varepsilon = 2$, then $\beta_{\ell_p(\hat{E}(r,s))}(\varepsilon) \leq 1$ and thus $\ell_p\left(\hat{E}(r,s)\right)$ is strictly convex.

ii) If $0 < \varepsilon < 2$, then $0 < \beta_{\ell_p(\hat{E}(r,s))}(\varepsilon) < 1$ and thus $\ell_p\left(\hat{E}(r,s)\right)$ is uniformly convex.

A Banach space X is said to have Banach-Saks property if any bounded sequence in X admits a subsequence whose arithmetic mean converges in norm. Similarly, we say that a Banach space X has weak Banach-Saks property if any weakly null sequence in X admits a subsequence whose arithmetic mean strongly converges in norm.

Let X be a Banach space. Garcia-Falset [30] defined the coefficient $R(X)$ as follows:

$$R(X) = sup(\lim_{n\to\infty} inf\|x_n + x\|),$$

where the supremum is taken over all weakly null sequences (x_n) of the unit ball and all points x of the unit ball. He also proved that a Banach space X with $R(X) < 2$ has the weak fixed point property.

Some studies on geometrical properties of a sequence space have been done by Karakas et al. [31], Et et al. [32], Mursaleen et al. [33] and Karakas et al. [34].

Theorem 7. The space $\ell_p\left(\hat{E}(r,s)\right)$ has the Banach-Saks property of type p.

Proof. It can be demonstrated with a standard method.

Remark 2. $R\left(\ell_p\left(\hat{E}(r,s)\right)\right) = R(\ell_p) = 2^{1/p}$ by reason that the space $\ell_p\left(\hat{E}(r,s)\right)$ is linearly isomorphic to ℓ_p.

Now we point out the following result due to the fact that $R\left(\ell_p\left(\hat{E}(r,s)\right)\right) < 2$.

Corollary 2. The Lucas difference sequence space $\ell_p\left(\hat{E}(r,s)\right)$ has weak fixed-point property for $1 < p < \infty$.

CONCLUSIONS

An approach to constructing a new sequence space using matrix domain of a triangular infinite matrix defined by Lucas numbers has been presented.

REFERENCES

1. H. Kizmaz, "On certain sequence spaces", *Canad. Math. Bull.*, **1981**, *24*, 169-176.
2. M. Et and R. Colak, "On some generalized difference sequence spaces", *Soochow J. Math.*, **1995**, *21*, 377-386.
3. F. Başar, "Summability Theory and Its Applications", Bentham e-Books, Istanbul, **2011**.
4. M. Mursaleen and A. K. Noman, "On the spaces of $\lambda -$convergent and bounded sequences", *Thai J. Math.*, **2010**, *8*, 311-329.
5. M. Mursaleen and A. K. Noman, "On some new sequence spaces of non-absolute type related to the spaces ℓ_p and ℓ_∞ I", *Filomat*, **2011**, *25*, 33-51.
6. M. Karakas, "A new regular matrix defined by Fibonacci numbers and its applications", *BEU J. Sci.*, **2015**, *4*, 205-210.
7. M. Candan and E. E. Kara, "A study on topological and geometrical characteristics of new Banach sequence spaces", *Gulf J. Math.*, **2015**, *3*, 67-84.
8. E. E. Kara, "Some topological and geometrical properties of new Banach sequence spaces", *J. Inequal. Appl.*, **2013**, *2013*, *38*.

9. M. Mursaleen, F. Basar and B. Altay, "On the Euler sequence spaces which include the spaces ℓ_p and ℓ_∞ II", *Nonlinear Anal. Theory Meth. Appl.*, **2006**, *65*, 707-717.
10. F. Basar and B. Altay, "On the space of sequences of p −bounded variation and related matrix mappings", *Ukr. Math. J.*, **2003**, *55*, 136-147.
11. E. Savas, V. Karakaya and N. Simsek, "Some $\ell(p)$-type new sequence spaces and their geometric properties", *Abstr. Appl. Anal.*, **2009**, *2009*, Art.ID 696971.
12. E. E. Kara and M. Basarir, "An application of Fibonacci numbers into infinite Toeplitz matrices", *Caspian J. Math. Sci.*, **2012**, *1*, 43-47.
13. M. Kirisci and F. Basar, "Some new sequence spaces derived by the domain of generalized difference matrix", *Comput. Math. Appl.*, **2010**, *60*, 1299-1309.
14. M. Et, "On some difference sequence spaces", *Turk. J. Math.*, **1993**, *17*, 18-24.
15. M. Et, "Strongly almost summable difference sequences of order m defined by a modulus", *Studia Sci. Math. Hungar.*, **2003**, *40*, 463-476.
16. M. Mursaleen, "Generalized spaces of difference sequences", *J. Math. Anal. Appl.*, **1996**, *203*, 738-745.
17. R. Colak and M. Et, "On some difference sequence sets and their topological properties", *Bull. Malays. Math. Sci. Soc.*, **2005**, *28*, 125-130.
18. M. Et and M. Basarir, "On some new generalized difference sequence spaces", *Period. Math. Hungar.*, **1997**, *35*, 169-175.
19. C. A. Bektas, M. Et and R. Colak, "Generalized difference sequence spaces and their dual spaces", *J. Math. Anal. Appl.*, **2004**, *292*, 423-432.
20. M. Et and A. Esi, "On Köthe-Toeplitz duals of generalized difference sequence spaces", *Bull. Malays. Math. Sci. Soc.*, **2000**, *23*, 25-32.
21. A. K. Gaur and M. Mursaleen, "Difference sequence spaces", *Int. J. Math. Math. Sci.*, **1998**, *21*, 701-706.
22. Y. Altın, "Properties of some sets of sequences defined by a modulus function", *Acta Math. Sci.*, **2009**, 29, 427-434.
23. H. Polat, V. Karakaya and N. Simsek, "Difference sequence spaces derived by using a generalized weighted mean", *Appl. Math. Lett.*, **2011**, *24*, 608-614.
24. T. Koshy, "Fibonacci and Lucas Numbers with Applications", John Wiley and Sons, Hoboken (NJ), **2001**.
25. S. Vajda, "Fibonacci and Lucas Numbers, and the Golden Section: Theory and Applications", Dover Publications, New York, **2008**.
26. A. Wilansky, "Summability through Functional Analysis", Elseiver Science Publishers B.V., Amsterdam, **1984**.
27. J. A. Clarkson, "Uniformly convex spaces", *Trans. Amer. Math. Soc.*, **1936**, *40*, 396-414.
28. V. I. Gurarii, "On differential properties of the convexity moduli of Banach spaces", *Mat. Issled.*, **1967**, *2*, 141-148.
29. L. Sanchez and A. Ullan, "Some properties of Gurarii's modulus of convexity", *Archiv Math.*, **1998**, *71*, 399-406.
30. J. Garcia-Falset, "Stability and fixed points for nonexpansive mappings", *Houston J. Math.*, **1994**, *20*, 495-506.
31. M. Karakas, M. Et and V. Karakaya, "Some geometric properties of a new difference sequence space involving lacunary sequences", *Acta Math. Sci.*, **2013**, *33*, 1711-1720.

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32. M. Et, M. Karakas and V. Karakaya, "Some geometric properties of a new difference sequence space defined by de la Vallee-Poussin mean", *Appl. Math. Comput*., **2014**, *234*, 237-244.
33. M. Mursaleen, R. Colak and M. Et, "Some geometric inequalities in a new Banach sequence space", *J. Inequal. Appl.*, **2007**, *2007*, Art. ID 86757.
34. M. Karakas, M. Cinar and M. Et, "Some geometric properties of a new sequence space", *J. Comput. Anal. Appl.*, **2013**, *15*, 23-31.