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Full Paper

A new approach to inverse and direct systems of soft topological spaces

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Abstract: The inverse (direct) systems in the category of soft topological spaces and soft continuous mappings between them are described. Structures of limits in this systems are defined and a series of their properties are discussed. Further, it is proven that the limit of the inverse system of soft compact spaces is soft compact.

Keywords: soft set, soft topology, inverse system, inverse limit, direct system, direct limit.

INTRODUCTION

The theory of soft set was introduced by Molodtsov in 1999 [1] as a new mathematical tool for dealing with uncertainties. After Molodtsov's work, some different applications of soft sets were studied. Maji et al. [2,3] published a detailed theoretical study on soft sets. Up to the present, the research on soft sets has been very active and many important results have been achieved in the theoretical aspect. Aktas and Cagman [4] introduced the basic properties of soft sets and compared soft sets with the related concepts of fuzzy sets and rough sets. At the same time, they gave a definition of soft groups and derived their basic properties. Sun et al. [5] defined soft modules and investigated their basic properties. Gunduz and Bayramov [6] introduced intuitionistic fuzzy soft modules and investigated some of their important properties.

The idea of soft topological spaces was first given by Shabir and Naz [7] and mappings between soft sets were described by Majumdar and Samanta [8]. Later, many researches on soft topological spaces were reported [9-12]. In these studies the concept of soft point was expressed by different approaches. In the present study we use the concept of soft point which was given earlier [13]. Soft topological spaces and soft continuous mappings form a category and this category is an extension of the category of topological spaces.

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It is well known that the inverse limit is not only an important concept in the category theory, but also plays an important role in some areas of mathematics, e.g. group theory, homology theory and algebraic topology. Up to the present, the inverse system and its limit have been defined in different categories. Furthermore, some of its properties have been investigated. Li [14-15] defined the inverse (direct) system of fuzzy topological spaces and their limits and obtained their properties for the case of category $L-Top$. Ghadiri and Davvaz [16] introduced the direct system and direct limit of H_v −modules. Gunduz and Bayramov [17] defined the inverse (direct) system in the category of fuzzy modules and their limits and obtained their properties.

In this paper after beginning with the basic concepts of a soft set, we introduce inverse and direct systems in the category of soft topological spaces and prove that their limits exist in this category. It is proven that the limit of the inverse system of soft compact spaces is soft compact. The direct limit is defined by a universal property and so it is unique. To use this method we first consider the fundamental equivalent relation and then investigate the results of the connection between fundamental soft sets, soft direct systems and direct limits.

PRELIMINARIES

In this section we recall the necessary information commonly used in the soft set theory.

Definition 1 [1]. Let X be an initial universal set and E be a set of parameters. A pair (F,E) is called a soft set over X if and only if F is a mapping from E into the set of all subsets of the set X, i.e. $F: E \to P(X)$, where $P(X)$ is the power set of X.

Definition 2 [2]. The intersection of two soft sets (F,A) and (G,B) over X is the soft set (H,C), where $C = A\cap B$, $\forall \varepsilon \in C$ and $H(\varepsilon)=F(\varepsilon)\cap G(\varepsilon)$. This is denoted by $(F,A)\cap(G,B)=(H,C)$.

Definition 3 [2]. The union of two soft sets (F,A) and (G,B) over X is the soft set (H,C), where $C = A\cup B$, $\forall \varepsilon \in C$ and

$$H(\varepsilon)=\begin{cases} F(\varepsilon), \text{if } \varepsilon \in A-B \\ G(\varepsilon), \text{if } \varepsilon \in B-A \\ F(\varepsilon)\cup G(\varepsilon), \varepsilon \in A\cap B \end{cases}$$

This is denoted by $(F,A)\cup(G,B)=(H,C)$.

Definition 4 [2]. Let (F,A) and (G,B) be two soft sets over X. Then (F,A) is called a soft subset of (G,B), denoted by $(F,A)\subset(G,B)$, if

(1) $A\subset B$,

(2) $F(\varepsilon)\subset G(\varepsilon)$ for each $\varepsilon\in A$.

Definition 5 [2]. A soft set (F,E) over X is said to be an absolute soft set denoted by $\tilde{X}$ if for all $\varepsilon\in E$, $F(\varepsilon)=X$.

Definition 6 [2]. A soft set (F,E) over X is said to be a null soft set denoted by Φ if for all $\varepsilon\in E$, $F(\varepsilon)=\emptyset$.

Definition 7 [7]. The relative complement of a soft set (F,E) is denoted by $(F,E)'$ and is defined by $(F,E)'=(F',E)$ where $F': E\to P(X)$ is a mapping given by $F'(\alpha)=X-F(\alpha)$ for all $\alpha\in E$.

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Definition 8 [7]**.** Let τ be a collection of soft sets over X. Then τ is said to be a soft topology on X if

(1) $\Phi, \tilde{X}$ belong to τ,
(2) the union of any number of soft sets in τ belongs to τ,
(3) the intersection of any two soft sets in τ belongs to τ.
The triplet (X,τ,E) is called a soft topological space over X.

Definition 9 [7]**.** Let (X,τ,E) be a soft topological space over X. Then the members of τ are said to be soft open sets in X.

Definition 10 [7]**.** Let (X,τ,E) be a soft topological space. A soft set (F,E) over X is said to be a soft closed set if its relative complement $(F,E)'$ belongs to τ.

Definition 11 [13]**.** Let (F,E) be a soft set over X and $x \in X$. The soft set (F,E) is called a soft point, denoted by (x_e,E) if for the element $e \in E$, $F(e)=\{x\}$ and $F(e')=\emptyset$ for all $e' \in E-\{e\}$. In this case we say that (x_e,E) is a soft point of a soft set (F,E).

INVERSE SYSTEM OF SOFT TOPOLOGICAL SPACES

Soft topological spaces and soft continuous mappings between them form a category and this category is an extension of the category of topological spaces. We denote this category by $STop$.

Definition 12. Any functor $F:\wedge^{op} \to STop$ is called an inverse system in $STop$ and the limit of F is called the inverse limit of F.

Now let us examine Definition 12. Let

$$(\underline{X},\underline{E})=\left(\{(X_{\alpha,}\tau_\alpha,E_\alpha)\}_{\alpha\in\wedge},\{(p_\alpha^{\alpha'},q_\alpha^{\alpha'}):(X_{\alpha'},\tau_{\alpha'},E_{\alpha'})\to(X_\alpha,\tau_\alpha,E_\alpha)\}_{\alpha\prec\alpha'}\right) \tag{1}$$

be an inverse system of soft topological spaces. The inverse system (1) consists of the following two systems of sets

$$\left(\{X_\alpha\}_{\alpha\in\wedge},\{p_\alpha^{\alpha'}:X_{\alpha'}\to X_\alpha\}_{\alpha\prec\alpha'}\right) \tag{2}$$

$$\left(\{E_\alpha\}_{\alpha\in\wedge},\{q_\alpha^{\alpha'}:E_{\alpha'}\to E_\alpha\}_{\alpha\prec\alpha'}\right). \tag{3}$$

Let $X=\underleftarrow{\lim}X_\alpha$ and $E=\underleftarrow{\lim}E_\alpha$ be two inverse limits of (2) and (3) respectively. Let $\left(\prod X_\alpha,\prod\tau_\alpha,\prod E_\alpha\right)$ be a product of soft topological spaces.

If $\left(F,\prod E_\alpha\right)\in\prod\tau_\alpha$, then we define the following soft set

$$F\Big|_{\underset{\alpha\in\wedge}{\underleftarrow{\lim}}E_\alpha}:\underset{\alpha\in\wedge}{\underleftarrow{\lim}}E_\alpha\to\prod_{\alpha\in\wedge}X_\alpha$$

such that $F\Big|_{\underset{\alpha\in\wedge}{\underleftarrow{\lim}}E_\alpha}(\{e_\alpha\})=F(\{e_\alpha\})\cap\underset{\alpha\in\wedge}{\underleftarrow{\lim}}X_\alpha$ for each $\{e_\alpha\}\in\underset{\alpha\in\wedge}{\underleftarrow{\lim}}E_\alpha$. We denote the soft set by $\left(F\Big|_{\underleftarrow{\lim}X_\alpha},\underleftarrow{\lim}E_\alpha\right)$. It is clear that the family $\tau=\left\{\left(F\Big|_{\underleftarrow{\lim}X_\alpha},\underleftarrow{\lim}E_\alpha\right)\Big|\left(F,\prod E_\alpha\right)\in\prod\tau_\alpha\right\}$ is a soft topology on (X,E) and (X,τ,E) is a soft topological space. Let

$$\pi_\alpha:\underleftarrow{\lim}X_\alpha\to X_\alpha,q_\alpha:\underleftarrow{\lim}E_\alpha\to E_\alpha$$

be two projection mappings for each $\alpha\in\wedge$. Then

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$$(\pi_\alpha, q_\alpha):(X,\tau,E)\to(X_\alpha,\tau_\alpha,E_\alpha)$$

is a soft continuous mapping.

Theorem 1. Let $(\underline{X},\underline{E})$ be an inverse system of soft topological spaces in (1). Then every inverse system has a limit and this limit is $(\underleftarrow{\lim}X_\alpha,\tau,\underleftarrow{\lim}E_\alpha)$ and unique.

Proof. Let (Y,τ',E') be a soft topological space and $\{(\varphi_\alpha,\psi_\alpha):(Y,\tau',E')\to(X_\alpha,\tau_\alpha,E_\alpha)\}_{\alpha\in\wedge}$ be soft continuous mappings such that Diagram 1 is commutative:

$$\begin{tikzcd}
(Y,\tau',E') \arrow[r, "(\varphi_\alpha,\psi_\alpha)"] \arrow[d, "(\varphi_{\alpha'},\psi_{\alpha'})"'] & (X_\alpha,\tau_\alpha,E_\alpha) \\
(X_{\alpha'},\tau_{\alpha'},E_{\alpha'}) \arrow[ur, "(p_\alpha^{\alpha'},q_\alpha^{\alpha'})"'] &
\end{tikzcd}$$

Diagram 1

Let us define $\chi:E'\to E$ and $f:Y\to X$ by $\chi(e')=\{\psi_\alpha(e')\}, f(y_{e'})=\{\varphi_\alpha(y)_{\psi_\alpha(e')}\}$ for each $e'\in E'$. In this case since Diagram 1 is commutative, we have $\chi(e')\in\underleftarrow{\lim}E_\alpha$ and $f(y_{e'})\in(X,\tau,E)$. Hence $(f,\chi):(Y,\tau',E')\to(X,\tau,E)$ is a soft continuous mapping.

Indeed, for each $y_{e'}\in(Y,\tau',E')$ we have

$$(\varphi_\alpha,\psi_\alpha)(y_{e'})=(\varphi_\alpha(y))_{\psi_\alpha(e')},$$

which implies that

$$(\pi_\alpha,q_\alpha)\circ(f,\chi)(y_{e'})=(\pi_\alpha,q_\alpha)(f(y))_{\chi(e')}=(\pi_\alpha,q_\alpha)\left(\left\{(\varphi_\alpha(y))_{\psi_\alpha(e')}\right\}\right)=(\varphi_\alpha(y))_{\psi_\alpha(e')}.$$

So the mapping (f,χ) is unique and the condition $(\varphi_\alpha,\psi_\alpha)=(\pi_\alpha,q_\alpha)\circ(f,\chi)$ is satisfied. ∎

Definition 13. Let

$$(\underline{Y},\underline{E'})=\left(\{(Y_\beta,\tau'_\beta,E'_\beta)\}_{\beta\in\wedge'},\{(r_\beta^{\beta'},\chi_\beta^{\beta'}):(Y_{\beta'},\tau'_{\beta'},E'_{\beta'})\to(Y_\beta,\tau'_\beta,E'_\beta)\}_{\beta\prec\beta'}\right) \tag{4}$$

be an inverse system of soft topological spaces and let $\varphi:\wedge'\to\wedge$ be an isotone mapping and

$$(f_\beta,g_\beta):(X_{\varphi(\beta)},\tau_{\varphi(\beta)},E_{\varphi(\beta)})\to(Y_\beta,\tau'_\beta,E'_\beta)$$

be the soft continuous mappings for all $\beta\in\wedge'$. If the following equality $(r_\beta^{\beta'},\chi_\beta^{\beta'})\circ(f_{\beta'},g_{\beta'})=(f_\beta,g_\beta)\circ(p_{\varphi(\beta)}^{\varphi(\beta')},q_{\varphi(\beta)}^{\varphi(\beta')})$ is satisfied for all $\beta'\succ\beta$, then the family $\left(\varphi:\wedge'\to\wedge,\{(f_\beta,g_\beta)\}_{\beta\in\wedge'}\right)$ is said to be a morphism from (1) to (4).

It is clear that the inverse systems of soft topological spaces and their morphisms construct a category. The category is denoted by $Inv(STop)$.

If $\left(\varphi:\wedge'\to\wedge,\{(f_\beta,g_\beta):(X_{\varphi(\beta)},\tau_{\varphi(\beta)},E_{\varphi(\beta)})\to(Y_\beta,\tau'_\beta,E'_\beta)\}_{\beta\in\wedge'}\right)$ is a morphism of inverse systems, then the family $\left(\varphi:\wedge'\to\wedge,\{g_\beta:E_{\varphi(\beta)}\to E'_\beta\}_{\beta\in\wedge'}\right)$ is a morphism from the inverse system of sets $\left(\{E_\alpha\}_{\alpha\in\wedge},\{q_\alpha^{\alpha'}:E_{\alpha'}\to E_\alpha\}_{\alpha\prec\alpha'}\right)$ to the inverse system of sets $\left(\{E'_\beta\}_{\beta\in\wedge'},\{\chi_\beta^{\beta'}:E'_{\beta'}\to E'_\beta\}_{\beta\prec\beta'}\right)$. Similarly, $\left(\varphi:\wedge'\to\wedge,\{f_\beta:X_{\varphi(\beta)}\to Y_\beta\}_{\beta\in\wedge'}\right)$ is a morphism from the inverse system of sets $\left(\{X_\alpha\}_{\alpha\in\wedge},\{p_\alpha^{\alpha'}:X_{\alpha'}\to X_\alpha\}_{\alpha\prec\alpha'}\right)$ to the inverse system of sets $\left(\{Y_\beta\}_{\beta\in\wedge'},\{r_\beta^{\beta'}:Y_{\beta'}\to Y_\beta\}_{\beta\prec\beta'}\right)$. Let us denote the limits of these morphisms with $\underleftarrow{\lim} f_\beta$ and $\underleftarrow{\lim} g_\beta$ respectively. In this case,

$$\left(\underleftarrow{\lim} f_\beta,\underleftarrow{\lim} g_\beta\right):\underleftarrow{\lim}\left(X_\alpha,\tau_\alpha,E_\alpha\right)\to\underleftarrow{\lim}\left(Y_\beta,\tau'_\beta,E'_\beta\right)$$

is a soft continuous mapping of soft topological spaces.

Theorem 2. The corresponding

$$\left\{\left(X_\alpha,\tau_\alpha,E_\alpha\right)\right\}_{\alpha\in\wedge}\mapsto\underleftarrow{\lim}\left(X_\alpha,\tau_\alpha,E_\alpha\right),\left(\varphi:\wedge'\to\wedge,\left\{\left(f_\beta,g_\beta\right)\right\}_{\beta\in\wedge'}\right)\mapsto\left(\underleftarrow{\lim} f_\beta,\underleftarrow{\lim} g_\beta\right)$$

is a functor from $Inv(STop)$ to $STop$.

Proof. The proof is straightforward.■

Theorem 3. Let $(\underline{X},\underline{E})$ be an inverse system in (1). Then the following family

$$\mathrm{B}=\left\{\left(\pi_\alpha,q_\alpha\right)^{-1}\left(F_\alpha,E_\alpha\right)\middle|\alpha\in\wedge,\left(F_\alpha,E_\alpha\right)\in\tau_\alpha\right\}$$

is a base of topology of $(X,\tau,E)=\underleftarrow{\lim}\left(X_\alpha,\tau_\alpha,E_\alpha\right)$.

Proof. Since $(\pi_\alpha,q_\alpha):(X,\tau,E)\to(X_\alpha,\tau_\alpha,E_\alpha)$ is a soft continuous mapping for all $\alpha\in\wedge$, the sets of B are soft open sets. It suffices to show that for every soft open $(G,E)\in(X,\tau,E)$ and every soft point $x_e\in(G,E)$, there exist $\alpha_0\in\wedge$ and $\left(F_{\alpha_0},E_{\alpha_0}\right)\in\tau_{\alpha_0}$ such that $x_e\in\left(\pi_{\alpha_0},q_{\alpha_0}\right)^{-1}\left(F_{\alpha_0},E_{\alpha_0}\right)\subset(G,E)$. Let (G,E) be an arbitrary soft open set in (X,τ,E) and $x_e=\left\{x_{e_\alpha}^\alpha\right\}\in(G,E)$. By the definition of soft subspace, there exists a soft open set $(H,E)\subset\prod\left(X_\alpha,\tau_\alpha,E_\alpha\right)$ such that $(G,E)=(X,\tau,E)\cap(H,E)$. Thus, we have $\alpha_1,\alpha_2,...,\alpha_k\in\wedge$ and $\left(F_{\alpha_i},E_{\alpha_i}\right)\in\tau_{\alpha_i}$ such that

$$x_e\in\left(p_{\alpha_1},q_{\alpha_1}\right)^{-1}\left(F_{\alpha_1},E_{\alpha_1}\right)\cap...\cap\left(p_{\alpha_k},q_{\alpha_k}\right)^{-1}\left(F_{\alpha_k},E_{\alpha_k}\right)\subset(H,E).$$

Since $\wedge$ is a directed set, there exists a $\alpha_0\in\wedge$ such that $\alpha_0\succ\alpha_1,...,\alpha_0\succ\alpha_k$. Also, since $\left(p_{\alpha_i}^{\alpha_0},q_{\alpha_i}^{\alpha_0}\right):\left(X_{\alpha_0},\tau_{\alpha_0},E_{\alpha_0}\right)\to\left(X_{\alpha_i},\tau_{\alpha_i},E_{\alpha_i}\right)$ are soft continuous mappings for all $1\le i\le k$, $\bigcap_{i=1}^{k}\left(p_{\alpha_i}^{\alpha_0},q_{\alpha_i}^{\alpha_0}\right)^{-1}\left(F_{\alpha_i},E_{\alpha_i}\right)=\left(F_{\alpha_0},E_{\alpha_0}\right)$ is a soft open set in $\left(X_{\alpha_0},\tau_{\alpha_0},E_{\alpha_0}\right)$. Since $\left(p_{\alpha_i}^{\alpha_0},q_{\alpha_i}^{\alpha_0}\right)\left(x_{e_{\alpha_0}}^{\alpha_0}\right)=x_{e_{\alpha_i}}^{\alpha_i}$, $x_{e_{\alpha_0}}^{\alpha_0}\in\left(F_{\alpha_0},E_{\alpha_0}\right)$ and $x_e\in\left(\pi_{\alpha_0},q_{\alpha_0}\right)^{-1}\left(F_{\alpha_0},E_{\alpha_0}\right)$, we have

$$\begin{aligned}\left(\pi_{\alpha_0},q_{\alpha_0}\right)^{-1}\left(p_{\alpha_i}^{\alpha_0},q_{\alpha_i}^{\alpha_0}\right)^{-1}\left(F_{\alpha_i},E_{\alpha_i}\right)&=\left(\pi_{\alpha_i},q_{\alpha_i}\right)^{-1}\left(F_{\alpha_i},E_{\alpha_i}\right)\\&=\prod\left(X_\alpha,\tau_\alpha,E_\alpha\right)\cap\left(p_{\alpha_i},q_{\alpha_i}\right)^{-1}\left(F_{\alpha_i},E_{\alpha_i}\right).\end{aligned}$$

Consequently, we obtain

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$$x_e \in \left(\pi_{\alpha_0}, q_{\alpha_0}\right)^{-1}\left(F_{\alpha_0}, E_{\alpha_0}\right) = \left(\pi_{\alpha_0}, q_{\alpha_0}\right)^{-1}\left(\bigcap_{i=1}^{k}\left(p_{\alpha_i}^{\alpha_0}, q_{\alpha_i}^{\alpha_0}\right)^{-1}\left(F_{\alpha_i}, E_{\alpha_i}\right)\right)$$
$$= \bigcap_{i=1}^{k}\left(\prod\left(X_\alpha, \tau_\alpha, E_\alpha\right) \cap \left(p_{\alpha_i}, q_{\alpha_i}\right)^{-1}\left(F_{\alpha_i}, E_{\alpha_i}\right)\right)$$
$$= \prod\left(X_\alpha, \tau_\alpha, E_\alpha\right) \cap \left(\bigcap_{i=1}^{k}\left(p_{\alpha_i}, q_{\alpha_i}\right)^{-1}\left(F_{\alpha_i}, E_{\alpha_i}\right)\right) \subset \prod\left(X_\alpha, \tau_\alpha, E_\alpha\right) \cap (H, E) = (G, E).$$
■

Theorem 4. Let $\left(\underline{X}, \underline{E}\right)$ be an inverse system in (1). Then

a) If $\left(p_\alpha^{\alpha'}, q_\alpha^{\alpha'}\right) : \left(X_{\alpha'}, \tau_{\alpha'}, E_{\alpha'}\right) \to \left(X_\alpha, \tau_\alpha, E_\alpha\right)$ is an injective mapping for all $\alpha \in \wedge$, $\left(\pi_\alpha, q_\alpha\right) : (X, \tau, E) \to \left(X_\alpha, \tau_\alpha, E_\alpha\right)$ is an injective soft mapping.

b) If $\left(p_\alpha^{\alpha'}, q_\alpha^{\alpha'}\right) : \left(X_{\alpha'}, \tau_{\alpha'}, E_{\alpha'}\right) \to \left(X_\alpha, \tau_\alpha, E_\alpha\right)$ is a bijective mapping for all $\alpha \in \wedge$, $\left(\pi_\alpha, q_\alpha\right) : (X, \tau, E) \to \left(X_\alpha, \tau_\alpha, E_\alpha\right)$ is a bijective soft mapping.

Proof. a) For the soft points $x_e = \left\{x_{e_\alpha}^\alpha\right\} \neq y_{e'} = \left\{y_{e'_\alpha}^\alpha\right\} \in (X, \tau, E)$, let $\left(\pi_{\alpha_1}, q_{\alpha_1}\right)(x_e) = \left(\pi_{\alpha_1}, q_{\alpha_1}\right)(y_{e'})$. For $\alpha' \succ \alpha_1$, since $\left(p_{\alpha_1}^{\alpha'}, q_{\alpha_1}^{\alpha'}\right) : \left(X_{\alpha'}, \tau_{\alpha'}, E_{\alpha'}\right) \to \left(X_{\alpha_1}, \tau_{\alpha_1}, E_{\alpha_1}\right)$ is an injective mapping and $\left(p_{\alpha_1}^{\alpha'}, q_{\alpha_1}^{\alpha'}\right)\left(x_{e_{\alpha'}}^{\alpha'}\right) = x_{e_{\alpha_1}}^{\alpha_1} = y_{e'_{\alpha_1}}^{\alpha_1} = \left(p_{\alpha_1}^{\alpha'}, q_{\alpha_1}^{\alpha'}\right)\left(y_{e'_{\alpha'}}^{\alpha'}\right)$, we have $x_{e_{\alpha'}}^{\alpha'} = y_{e'_{\alpha'}}^{\alpha'}$. For an arbitrary $\alpha \in \wedge$, since $\wedge$ is a directed set, there exists $\alpha'' \in \wedge$ such that $\alpha'' \succ \alpha, \alpha'' \succ \alpha_1$ for all $\alpha, \alpha_1 \in \wedge$. Then $x_{e_{\alpha_1}}^{\alpha_1} = y_{e'_{\alpha_1}}^{\alpha_1}$ implies that $x_{e_{\alpha''}}^{\alpha''} = y_{e'_{\alpha''}}^{\alpha''}$. Hence

$$x_{e_\alpha}^\alpha = \left(p_\alpha^{\alpha''}, q_\alpha^{\alpha''}\right)\left(x_{e_{\alpha''}}^{\alpha''}\right) = \left(p_\alpha^{\alpha''}, q_\alpha^{\alpha''}\right)\left(y_{e'_{\alpha''}}^{\alpha''}\right) = y_{e'_\alpha}^\alpha$$

is satisfied, i.e. $x_e = y_{e'}$.

b) Now we want to show that the soft mapping $\left(\pi_{\alpha_1}, q_{\alpha_1}\right) : (X, \tau, E) \to \left(X_{\alpha_1}, \tau_{\alpha_1}, E_{\alpha_1}\right)$ is a surjective soft mapping. Let $x_{e_{\alpha_1}}^{\alpha_1} \in \left(X_{\alpha_1}, \tau_{\alpha_1}, E_{\alpha_1}\right)$ be an arbitrary soft point. For $\alpha' \succ \alpha_1$, we take $x_{e_{\alpha'}}^{\alpha'} = \left(p_{\alpha_1}^{\alpha'}, q_{\alpha_1}^{\alpha'}\right)^{-1}\left(x_{e_{\alpha_1}}^{\alpha_1}\right)$. Then there exists $\alpha' \in \wedge$ such that $\alpha' \succ \alpha$ and $\alpha' \succ \alpha_1$ for all $\alpha \in \wedge$. Hence we obtain soft point $x_{e_\alpha}^\alpha = \left(p_\alpha^{\alpha'}, q_\alpha^{\alpha'}\right)\left(x_{e_{\alpha'}}^{\alpha'}\right) = \left(p_\alpha^{\alpha'}, q_\alpha^{\alpha'}\right)\left(p_{\alpha_1}^{\alpha'}, q_{\alpha_1}^{\alpha'}\right)^{-1}\left(x_{e_{\alpha_1}}^{\alpha_1}\right)$. Now we want to show that $x_e = \left\{x_{e_\alpha}^\alpha\right\} \in (X, \tau, E)$. We choose $\alpha', \tilde{\alpha}'$ such that $\alpha' \succ \alpha, \alpha' \succ \alpha_1$ and $\tilde{\alpha}' \succ \tilde{\alpha}, \tilde{\alpha}' \succ \alpha_1$ for all $\tilde{\alpha} \succ \alpha$. Thus,

$$x_{e_\alpha}^\alpha = \left(p_\alpha^{\alpha'}, q_\alpha^{\alpha'}\right)\left(x_{e_{\alpha'}}^{\alpha'}\right) = \left(p_\alpha^{\alpha'}, q_\alpha^{\alpha'}\right)\left(\left(p_{\alpha_1}^{\alpha'}, q_{\alpha_1}^{\alpha'}\right)^{-1}\left(x_{e_{\alpha_1}}^{\alpha_1}\right)\right),$$
$$x_{e_{\tilde{\alpha}}}^{\tilde{\alpha}} = \left(p_{\tilde{\alpha}}^{\tilde{\alpha}'}, q_{\tilde{\alpha}}^{\tilde{\alpha}'}\right)\left(x_{e_{\tilde{\alpha}'}}^{\tilde{\alpha}'}\right) = \left(p_{\tilde{\alpha}}^{\tilde{\alpha}'}, q_{\tilde{\alpha}}^{\tilde{\alpha}'}\right)\left(\left(p_{\alpha_1}^{\tilde{\alpha}'}, q_{\alpha_1}^{\tilde{\alpha}'}\right)^{-1}\left(x_{e_{\alpha_1}}^{\alpha_1}\right)\right).$$

Now we take $\alpha'' \in \wedge$ such that $\alpha'' \succ \alpha'$, $\alpha'' \succ \tilde{\alpha}'$. Then

$$x_{e_{\alpha_1}}^{\alpha_1} = \left(p_{\alpha_1}^{\alpha''}, q_{\alpha_1}^{\alpha''}\right)\left(x_{e_{\alpha''}}^{\alpha''}\right) = \left(p_{\alpha_1}^{\alpha'}, q_{\alpha_1}^{\alpha'}\right)\left(p_{\alpha_1}^{\alpha''}, q_{\alpha_1}^{\alpha''}\right)\left(x_{e_{\alpha''}}^{\alpha''}\right)$$
$$= \left(p_{\alpha_1}^{\tilde{\alpha}'}, q_{\alpha_1}^{\tilde{\alpha}'}\right)\left(p_{\tilde{\alpha}'}^{\alpha''}, q_{\tilde{\alpha}'}^{\alpha''}\right)\left(x_{e_{\alpha''}}^{\alpha''}\right)$$

and

$$x_{e_{\alpha_1}}^{\alpha_1} = \left(p_{\alpha_1}^{\alpha'}, q_{\alpha_1}^{\alpha'}\right)\left(x_{e_{\alpha'}}^{\alpha'}\right) = \left(p_{\alpha_1}^{\tilde{\alpha}'}, q_{\alpha_1}^{\tilde{\alpha}'}\right)\left(x_{e_{\tilde{\alpha}'}}^{\tilde{\alpha}'}\right).$$

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Since the soft mappings $\left(p_{\alpha_1}^{\alpha'},q_{\alpha_1}^{\alpha'}\right)$, $\left(p_{\alpha_1}^{\tilde{\alpha}'},q_{\alpha_1}^{\tilde{\alpha}'}\right)$ are bijective mappings, $\left(p_{\alpha_1}^{\alpha''},q_{\alpha_1}^{\alpha''}\right)\left(x_{e_{\alpha''}}^{\alpha''}\right)=x_{e_{\alpha'}}^{\alpha'}$ and $\left(p_{\tilde{\alpha}'}^{\alpha''},q_{\tilde{\alpha}'}^{\alpha''}\right)\left(x_{e_{\alpha''}}^{\alpha''}\right)=\left(x_{e_{\tilde{\alpha}'}}^{\tilde{\alpha}'}\right)$ are obtained. Thus,

$$\left(p_{\alpha}^{\alpha''},q_{\alpha}^{\alpha''}\right)\left(x_{e_{\alpha''}}^{\alpha''}\right)=\left(p_{\alpha}^{\alpha'},q_{\alpha}^{\alpha'}\right)\left(p_{\alpha'}^{\alpha''},q_{\alpha'}^{\alpha''}\right)\left(x_{e_{\alpha''}}^{\alpha''}\right)=x_{e_{\alpha}}^{\alpha}\ ,$$

$$\left(p_{\tilde{\alpha}}^{\alpha''},q_{\tilde{\alpha}}^{\alpha''}\right)\left(x_{e_{\alpha''}}^{\alpha''}\right)=\left(p_{\tilde{\alpha}}^{\tilde{\alpha}'},q_{\tilde{\alpha}}^{\tilde{\alpha}'}\right)\left(p_{\tilde{\alpha}'}^{\alpha''},q_{\tilde{\alpha}'}^{\alpha''}\right)\left(x_{e_{\alpha''}}^{\alpha''}\right)=x_{e_{\tilde{\alpha}}}^{\tilde{\alpha}}.$$

Hence $\left(p_{\alpha}^{\tilde{\alpha}},q_{\alpha}^{\tilde{\alpha}}\right)\left(x_{e_{\tilde{\alpha}}}^{\tilde{\alpha}}\right)=\left(p_{\alpha}^{\alpha''},q_{\alpha}^{\alpha''}\right)\left(x_{e_{\alpha''}}^{\alpha''}\right)=x_{\alpha}$ is obtained. Thus, $x_e=\left\{x_{e_{\alpha}}^{\alpha}\right\}\in(X,\tau,E)$ and $\left(\pi_{\alpha_1},q_{\alpha_1}\right)(x_e)=x_{e_{\alpha_1}}^{\alpha_1}$.■

Corollary 1. Let $(\underline{X},\underline{E})$ be an inverse system in (1). If $\left(p_{\alpha}^{\alpha'},q_{\alpha}^{\alpha'}\right):(X_{\alpha'},\tau_{\alpha'},E_{\alpha'})\to(X_{\alpha},\tau_{\alpha},E_{\alpha})$ is a soft homeomorphism, then the soft mapping

$$(\pi_{\alpha},q_{\alpha}):(X,\tau,E)\to(X_{\alpha},\tau_{\alpha},E_{\alpha})$$

is a soft homeomorphism.

Proof. By Theorem 4, $(\pi_{\alpha},q_{\alpha})$ is a bijective and a soft continuous mapping. From Theorem 3, the family $\left\{(\pi_{\alpha},q_{\alpha})^{-1}(F_{\alpha},E_{\alpha}):\alpha\in\wedge,(F_{\alpha},E_{\alpha})\in\tau_{\alpha}\right\}$ is a base of soft topology of (X,τ,E). Then since $(\pi_{\alpha},q_{\alpha})\left((\pi_{\alpha},q_{\alpha})^{-1}(F_{\alpha},E_{\alpha})\right)=(F_{\alpha},E_{\alpha})$, $(\pi_{\alpha},q_{\alpha})$ is a soft open mapping, $(\pi_{\alpha},q_{\alpha})$ is a soft homeomorphism.■

Lemma 1. Let $(\underline{X},\underline{E})$ be an inverse system in (1). If $(\underline{X},\underline{E})$ is an inverse system of soft T_2-spaces, then (X,τ,E) is a soft closed set in $\prod_{\alpha\in\wedge}(X_{\alpha},\tau_{\alpha},E_{\alpha})$.

Proof. It is sufficient to show that $\left(\prod_{\alpha\in\wedge}(X_{\alpha},\tau_{\alpha},E_{\alpha})\Big|_{\lim\limits_{\leftarrow}\underline{E}}\right)\backslash(X,\tau,E)$ is a soft open set. For every $x_e=\left\{x_{e_{\alpha}}^{\alpha}\right\}\in\prod_{\alpha\in\wedge}(X_{\alpha},\tau_{\alpha},E_{\alpha})/(X,\tau,E)$, $x_e\notin(X,\tau,E)$. Then there exists $\alpha\prec\beta\in\wedge$ such that $\left(p_{\alpha}^{\beta'},q_{\alpha}^{\beta'}\right)\left(x_{e_{\beta'}}^{\beta'}\right)\neq x_{e_{\alpha}}^{\alpha}$ and $x_{e_{\alpha}}^{\alpha},\left(p_{\alpha}^{\beta},q_{\alpha}^{\beta}\right)\left(x_{e_{\beta}}^{\beta}\right)\in(X_{\alpha},\tau_{\alpha},E_{\alpha})$. Since $(X_{\alpha},\tau_{\alpha},E_{\alpha})$ is a soft T_2-space, there exist (F_{α},E_{α}), $(G_{\alpha},E_{\alpha})\in\tau_{\alpha}$ such that $x_{e_{\alpha}}^{\alpha}\in(F_{\alpha},E_{\alpha})$, $\left(p_{\alpha}^{\beta},q_{\alpha}^{\beta}\right)\left(x_{e_{\beta}}^{\beta}\right)\in(G_{\alpha},E_{\alpha})$ and $(F_{\alpha},E_{\alpha})\cap(G_{\alpha},E_{\alpha})=\Phi$. We take (F_{α},E_{α}) as $(X_{\alpha},\tau_{\alpha},E_{\alpha})$ and $\left(p_{\alpha}^{\beta},q_{\alpha}^{\beta}\right)^{-1}(G_{\alpha},E_{\alpha})$ as $(X_{\beta},\tau_{\beta},E_{\beta})$. Then the soft set $(F_{\alpha},E_{\alpha})\times\left(p_{\alpha}^{\beta},q_{\alpha}^{\beta}\right)^{-1}(G_{\alpha},E_{\alpha})\times\prod_{\gamma\neq\alpha,\beta}(X_{\gamma},\tau_{\gamma},E_{\gamma})$ is a soft neighborhood of soft point x_e and $(F_{\alpha},E_{\alpha})\times\left(p_{\alpha}^{\beta},q_{\alpha}^{\beta}\right)^{-1}(G_{\alpha},E_{\alpha})\times\prod_{\gamma\neq\alpha,\beta}(X_{\gamma},\tau_{\gamma},E_{\gamma})\cap(X,\tau,E)=\Phi$. Hence $x_e\in(F_{\alpha},E_{\alpha})\times\left(p_{\alpha}^{\beta},q_{\alpha}^{\beta}\right)^{-1}(G_{\alpha},E_{\alpha})\times\prod_{\gamma\neq\alpha,\beta}(X_{\gamma},\tau_{\gamma},E_{\gamma})\subset\left(\prod_{\alpha\in\wedge}(X_{\alpha},\tau_{\alpha},E_{\alpha})\Big|_{\lim\limits_{\leftarrow}\underline{E}}\right)$. This implies that $\left(\prod_{\alpha\in\wedge}(X_{\alpha},\tau_{\alpha},E_{\alpha})\Big|_{\lim\limits_{\leftarrow}\underline{E}}\right)\backslash(X,\tau,E)$ is a soft open set, i.e. (X,τ,E) is soft closed set.■

Theorem 5. Let $(\underline{X},\underline{E})$ be an inverse system of non-null soft compact soft T_2-spaces. Then (X,τ,E) is a non-null soft compact set.

Proof. Since $(X_\alpha,\tau_\alpha,E_\alpha)$ are soft compact spaces for all $\alpha\in\wedge$, then $\prod_{\alpha\in\wedge}(X_\alpha,\tau_\alpha,E_\alpha)$ is a soft compact space. In this case $\prod_{\alpha\in\wedge}(X_\alpha,\tau_\alpha,E_\alpha)\Big|_{\lim\limits_{\leftarrow}\underline{E}}$ is a soft compact space. From Lemma 1, (X,τ,E) is a soft closed set in $\prod_{\alpha\in\wedge}(X_\alpha,\tau_\alpha,E_\alpha)\Big|_{\lim\limits_{\leftarrow}\underline{E}}$. Since the soft closed set of a soft compact space is soft compact, (X,τ,E) is a soft compact space.

Now we want to show that (X,τ,E) is a non-null soft set. For all $\beta\in\wedge$,

$$\left(Y_\beta,\prod_\alpha E_\alpha\right)=\left\{\left\{x_{e_\alpha}^\alpha\right\}\in\prod_{\alpha\in\wedge}(X_\alpha,E_\alpha)\Big|\left(p_\gamma^\beta,q_\gamma^\beta\right)\left(x_{e_\beta}^\beta\right)=x_{e_\gamma}^\gamma,\forall\gamma\prec\beta\right\}$$

is a non-null soft set. Indeed, let $z_{e_\beta}^\beta\in\left(X_\beta,\tau_\beta,E_\beta\right)$ be any soft point and $z_{e_\gamma}^\gamma=\left(p_\gamma^\beta,q_\gamma^\beta\right)\left(z_{e_\beta}^\beta\right)$ for all $\gamma\prec\beta$. Let $z_{e_\sigma}^\sigma$ be any arbitrary soft point for $\sigma\in\wedge$. In this case $z_e=\left\{z_{e_\alpha}^\alpha\right\}\in\left(Y_\beta,\prod_\alpha E_\alpha\right)$. Similar to Lemma 1, we can show that $\left(Y_\beta,\prod_\alpha E_\alpha\right)$ is a soft closed set. It is clear that $\left(Y_\beta,\prod_\alpha E_\alpha\right)\subset\left(Y_\alpha,\prod_\alpha E_\alpha\right)$ for all $\alpha\prec\beta$. Then the family $\left(Y_\beta,\prod_\alpha E_\alpha\right)$ is a centralised soft closed set family in $\prod_{\alpha\in\wedge}(X_\alpha,\tau_\alpha,E_\alpha)$. Since $\prod_{\alpha\in\wedge}(X_\alpha,\tau_\alpha,E_\alpha)$ is a soft compact space, $\bigcap_{\beta\in\wedge}\left(Y_\beta,\prod_\alpha E_\alpha\right)\neq\Phi$. Thus, the intersection of soft points in $\bigcap_{\beta\in\wedge}\left(Y_\beta,\prod_\alpha E_\alpha\right)$ is (X,τ,E), and this implies that $(X,\tau,E)\neq\Phi$. ■

Theorem 6. Let $(\underline{X},\underline{E})=\left(\left\{(X_\alpha,\tau_\alpha,E_\alpha)\right\}_{\alpha\in\wedge},\left\{p_\alpha^{\alpha'},q_\alpha^{\alpha'}\right\}_{\alpha\prec\alpha'}\right)$ and $(\underline{Y},\underline{E'})=\left(\left\{(Y_\beta,\tau'_\beta,E'_\beta)\right\}_{\beta\in\wedge'},\left\{r_\beta^{\beta'},\gamma_\beta^{\beta'}\right\}_{\beta\prec\beta'}\right)$ be two inverse systems of soft topological spaces, and $(\underline{f},\underline{g})=\left\{\varphi:\wedge'\to\wedge,\left\{(f_\beta,g_\beta):\left(X_{\varphi(\beta)},\tau_{\varphi(\beta)},E_{\varphi(\beta)}\right)\to\left(Y_\beta,\tau'_\beta,E'_\beta\right)\right\}_{\beta\in\wedge'}\right\}$ be a soft mapping of these spaces. If $(f_\beta,g_\beta):\left(X_{\varphi(\beta)},\tau_{\varphi(\beta)},E_{\varphi(\beta)}\right)\to\left(Y_\beta,\tau'_\beta,E'_\beta\right)$ are a soft homeomorphism for all $\beta\in\wedge'$, then $\underleftarrow{\lim}(\underline{f},\underline{g}):(X,\tau,E)\to(Y,\tau',E')$ is a soft homeomorphism.

Proof. Firstly, we want to show that $\underleftarrow{\lim}(\underline{f},\underline{g}):(X,\tau,E)\to(Y,\tau',E')$ is an injective soft mapping. Let $x_e=\left\{x_{e_\alpha}^\alpha\right\}$, $y_{\bar{e}}=\left\{y_{\bar{e}_\alpha}^\alpha\right\}\in(X,\tau,E)$ and $x_e\neq y_{\bar{e}}$. Then there exists $\alpha_0\in\wedge$ such that $x_{e_{\alpha_0}}^{\alpha_0}\neq y_{\bar{e}_{\alpha_0}}^{\alpha_0}$. We choose $\beta\in\wedge'$ such that $\varphi(\beta)\succ\alpha_0$. Since

$$\left(p_{\alpha_0}^{\varphi(\beta)},q_{\alpha_0}^{\varphi(\beta)}\right)\left(x_{\alpha_{\varphi(\beta)}}^{\varphi(\beta)}\right)=x_{e_{\alpha_0}}^{\alpha_0}\neq y_{\bar{e}_{\alpha_0}}^{\alpha_0}=\left(p_{\alpha_0}^{\varphi(\beta)},q_{\alpha_0}^{\varphi(\beta)}\right)\left(y_{e'_{\varphi(\beta)}}^{\varphi(\beta)}\right),$$

then $x^{\varphi(\beta)}_{\alpha_{\varphi(\beta)}} \neq y^{\varphi(\beta)}_{e'_{\varphi(\beta)}}$. Also, since (f_β, g_β) is a soft injective mapping, we have $(f_\beta, g_\beta)\left(x^{\varphi(\beta)}_{\alpha_{\varphi(\beta)}}\right) \neq (f_\beta, g_\beta)\left(y^{\varphi(\beta)}_{e'_{\varphi(\beta)}}\right)$. This implies that $\underleftarrow{\lim}(\underline{f}, \underline{g}) : (X, \tau, E) \to (Y, \tau', E')$ is an injective soft mapping.

Now we want to show that $\underleftarrow{\lim}(\underline{f}, \underline{g}) : (X, \tau, E) \to (Y, \tau', E')$ is a surjective soft mapping. Let $y_{e'} = \left\{ y^{\beta}_{e'_\beta} \right\} \in (Y, \tau', E')$ be an arbitrary soft point. Since (f_β, g_β) is a surjective soft mapping, there exists a soft point $z^{\varphi(\beta)}_{\bar{e}_{\varphi(\beta)}}$ such that $(f_\beta, g_\beta)\left(z^{\varphi(\beta)}_{\bar{e}_{\varphi(\beta)}}\right) = y^{\beta}_{e'_\beta}$. We take $\beta \in \wedge'$ such that $\varphi(\beta) \succ \alpha$ for all $\alpha \in \wedge$, and $x^{\alpha}_{e_\alpha} = \left(p^{\varphi(\beta)}_{\alpha}, q^{\varphi(\beta)}_{\alpha}\right)\left(z^{\varphi(\beta)}_{\bar{e}_{\varphi(\beta)}}\right)$. The soft point $x^{\alpha}_{e_\alpha}$ is independent from $\beta \in \wedge'$, $x_e = \left\{ x^{\alpha}_{e_\alpha} \right\} \in (X, \tau, E)$ and $\underleftarrow{\lim}(\underline{f}, \underline{g})(x_e) = y_{e'}$.

Now let us show that $\underleftarrow{\lim}(\underline{f}, \underline{g}) : (X, \tau, E) \to (Y, \tau', E')$ is an open soft mapping. Since $(f_\beta, g_\beta) \circ \left(\pi_{\varphi(\beta)}, q_{\varphi(\beta)}\right) = (\pi_\beta, q_\beta) \circ \underleftarrow{\lim}(\underline{f}, \underline{g})$ for all $\beta \in \wedge'$, we have $\left(\pi_{\varphi(\beta)}, q_{\varphi(\beta)}\right)^{-1} \circ (f_\beta, g_\beta)^{-1} = \left(\underleftarrow{\lim}(\underline{f}, \underline{g})\right)^{-1} \circ (\pi_\beta, q_\beta)^{-1}$. Let $(f_\beta, g_\beta) \circ \left(F_{\varphi(\beta)}, E_{\varphi(\beta)}\right) = (G_\beta, E'_\beta)$. Since (f_β, g_β) is a soft homeomorphism, we obtain

$$
\begin{aligned}
\left(\pi_{\varphi(\beta)}, q_{\varphi(\beta)}\right)^{-1}\left(F_{\varphi(\beta)}, E_{\varphi(\beta)}\right) &= \left(\pi_{\varphi(\beta)}, q_{\varphi(\beta)}\right)^{-1}(f_\beta, g_\beta)^{-1}\left((f_\beta, g_\beta)\left(F_{\varphi(\beta)}, E_{\varphi(\beta)}\right)\right) \\
&= \left(\underleftarrow{\lim}(\underline{f}, \underline{g})\right)^{-1} \circ (\pi_\beta, q_\beta)^{-1}\left((f_\beta, g_\beta)\left(F_{\varphi(\beta)}, E_{\varphi(\beta)}\right)\right).
\end{aligned}
$$

Thus, we have

$$
\left(\underleftarrow{\lim}(\underline{f}, \underline{g})\right)\left[\left(\pi_{\varphi(\beta)}, q_{\varphi(\beta)}\right)^{-1}\left(F_{\varphi(\beta)}, E_{\varphi(\beta)}\right)\right] = \underleftarrow{\lim}(\underline{f}, \underline{g})\left[\left(\underleftarrow{\lim}(\underline{f}, \underline{g})\right)^{-1}\right](\pi_\beta, q_\beta)^{-1},
$$

$$
(f_\beta, g_\beta)\left(F_{\varphi(\beta)}, E_{\varphi(\beta)}\right) = (\pi_\beta, q_\beta)^{-1}(f_\beta, g_\beta)\left(F_{\varphi(\beta)}, E_{\varphi(\beta)}\right).
$$

Consequently, $(\pi_\beta, q_\beta)^{-1}(f_\beta, g_\beta)\left(F_{\varphi(\beta)}, E_{\varphi(\beta)}\right)$ is a soft open set.■

DIRECT SYSTEM OF SOFT TOPOLOGICAL SPACES

Let (X, τ, E) be a soft topological space, $\sim_1$ be an equivalence relation on X and $\sim_2$ be an equivalence relation on E. Let $p : X \to X/\sim_1$ and $q : E \to E/\sim_2$ be two canonical mappings. Then $(p, q) : (X, \tau, E) \to \left(X/\sim_1, E/\sim_2\right)$ is a soft mapping.

If we define a family $\tilde{\tau}$ such that $\left(F, E/\sim_2\right) \in \tilde{\tau} \Leftrightarrow (p, q)^{-1}\left(F, E/\sim_2\right) \in \tau$, then $\tilde{\tau}$ is a soft topology in $\left(X/\sim_1, E/\sim_2\right)$ and $(p, q) : (X, \tau, E) \to \left(X/\sim_1, \tilde{\tau}, E/\sim_2\right)$ is a soft continuous mapping.

Definition 14. Let (X, τ, E) be a soft topological space. The soft topological space $\left(X/\sim_1, \tilde{\tau}, E/\sim_2\right)$ is called a quotient space of (X, τ, E).

Example 1. Let $X = \{x_1, x_2, x_3, x_4, x_5\}$, $E = \{e_1, e_2, e_3\}$ and $\tau = \{(F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$, where $(F_1, E), (F_2, E), (F_3, E)$ and (F_4, E) are soft sets over X, defined as

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$$F_1(e_1)=\{x_1,x_2,x_3\},\ F_1(e_2)=\{x_1,x_2,x_3\},\ F_1(e_3)=\{x_4,x_5\},$$
$$F_2(e_1)=\{x_1,x_2\},\ F_2(e_2)=\{x_2,x_4,x_5\},\ F_2(e_3)=\{x_4,x_5\},$$
$$F_3(e_1)=\{x_1,x_2\},\ F_3(e_2)=\{x_2\},\ F_3(e_3)=\{x_4,x_5\},$$
$$F_4(e_1)=\{x_1,x_2,x_3\},\ F_4(e_2)=X,\ F_4(e_3)=\{x_4,x_5\}.$$

Then τ defines a soft topology on X and (X,τ,E) is a soft topological space over X. Let $\sim_1$ be an equivalence relation on X such that $x_1\sim_1 x_2\sim_1 x_3$ and $x_4\sim_1 x_5$. Let $\sim_2$ be an equivalence relation on E such that $e_1\sim_2 e_2$ and $e_3\sim_2 e_3$. It can be easily seen that $X/{\sim_1}=\{[x_1],[x_4]\}$ and $E/{\sim_2}=\{[e_1],[e_3]\}$. Then $\tilde{\tau}=\left\{\tilde{X}/{\sim_1},\tilde{\Phi},\left(G,E/{\sim_2}\right)\right\}$ is a soft topology, where $G([e_1])=[x_1]$ and $G([e_3])=[x_4]$.

Note that the equivalent class of every soft point $x_\alpha\in(X,\tau,E)$ is denoted by $[x]_{[\alpha]}$, where $[x]_{[\alpha]}$ is also a soft point.

Let (X,τ,E) and (Y,τ',E') be two soft topological spaces and $(f,g):(X,\tau,E)\to(Y,\tau',E')$ be a soft continuous mapping. Let $\sim_X$, $\sim_E$, $\sim_Y$, $\sim_{E'}$ be four equivalence relations on X, E, Y, E' respectively. Here, (f,g) preserves these relations. It is clear that

$$\left(\tilde{f},\tilde{g}\right):\left(X/{\sim_X},\tilde{\tau},E/{\sim_E}\right)\to\left(Y/{\sim_Y},\tilde{\tau}',E'/{\sim_{E'}}\right)$$

is a soft mapping and defined by $\left(\tilde{f},\tilde{g}\right)\left([x]_{[\alpha]}\right)=[f(x)]_{[g(\alpha)]}$ for all soft point $[x]_{[\alpha]}$.

Proposition 1. Let $(f,g):(X,\tau,E)\to(Y,\tau',E')$ be a soft continuous mapping. If (f,g) preserves equivalence relations, then $\left(\tilde{f},\tilde{g}\right):\left(X/{\sim_X},\tilde{\tau},E/{\sim_E}\right)\to\left(Y/{\sim_Y},\tilde{\tau}',E'/{\sim_{E'}}\right)$ is a soft continuous mapping and the condition $(p_2,q_2)\circ(f,g)=\left(\tilde{f},\tilde{g}\right)\circ(p_1,q_1)$ is satisfied, where (p_1,q_1) and (p_2,q_2) are soft canonical mappings.

Proof. The proof is straightforward.■

Let $\{(X_\alpha,\tau_\alpha,E_\alpha)\}_{\alpha\in\wedge}$ be a family of soft topological spaces such that $X_\alpha\cap X_{\alpha'}=\varnothing$ and $E_\alpha\cap E_{\alpha'}=\varnothing$ for all $\alpha\neq\alpha'\in\wedge$. Let $\tilde{X}$ be a union of soft points belonging to these soft topological spaces and $E=\bigcup_{\alpha\in\wedge}E_\alpha$. Then $\left(\tilde{X},E\right)$ is a family of soft sets.

Example 2. Let $X_1=\{x^1,x^2,x^3\}$, $E_1=\{e_1,e_2\}$ and $X_2=\{x^4,x^5\}$, $E_2=\{e_3,e_4\}$. Then we have $\tilde{X}=\{x^1_{e_1},x^1_{e_2},x^2_{e_1},x^2_{e_2},x^3_{e_1},x^3_{e_2},x^4_{e_3},x^4_{e_4},x^5_{e_3},x^5_{e_4}\}$. Let us define $X=X_1\cup X_2$ and $E=E_1\cup E_2$ and form the soft sets by using the parameter set E on X. It is clear that the soft points $x^1_{e_3},x^1_{e_5},x^4_{e_1}$ do not belong to $\tilde{X}$.

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Definition 15. Let $\{(X_\alpha,\tau_\alpha,E_\alpha)\}_{\alpha\in\wedge}$ be a family of soft topological spaces. The soft set $(F,E)\in\left(\tilde{X},E\right)$ is a soft open set if and only if $(F,E)\cap(X_\alpha,\tau_\alpha,E_\alpha)\in\tau_\alpha$ for all $\alpha\in\wedge$. Such open sets construct soft topology. Let us denote it by τ.

Definition 16. The soft topological space $\left(\tilde{X},\tau,E\right)$ is called a direct sum of soft topological spaces $\{(X_\alpha,\tau_\alpha,E_\alpha)\}_{\alpha\in\wedge}$, denoted by $\underset{\alpha\in\wedge}{\oplus}(X_\alpha,\tau_\alpha,E_\alpha)$.

It is clear that since $i_\alpha : X_\alpha \to X=\bigcup_{\alpha\in\wedge} X_\alpha$ and $j_\alpha : E_\alpha\to E=\bigcup_{\alpha\in\wedge}E_\alpha$ are embedding mappings for all $\alpha\in\wedge$, the soft mapping

$$(i_\alpha,j_\alpha):(X_\alpha,\tau_\alpha,E_\alpha)\to\left(\tilde{X},\tau,E\right)$$

is a soft continuous mapping.

Let $\{(X_\alpha,\tau_\alpha,E_\alpha)\}_{\alpha\in\wedge}$ and $\{(Y_\alpha,\tau'_\alpha,E'_\alpha)\}_{\alpha\in\wedge}$ be two families of pairwise disjoint, soft topological spaces with $\left(\tilde{X},\tau,E\right)=\underset{\alpha\in\wedge}{\oplus}(X_\alpha,\tau_\alpha,E_\alpha)$ and $\left(\tilde{Y},\tau',E'\right)=\underset{\alpha\in\wedge}{\oplus}(Y_\alpha,\tau'_\alpha,E'_\alpha)$. If $\{(f_\alpha,g_\alpha):(X_\alpha,\tau_\alpha,E_\alpha)\to(Y_\alpha,\tau'_\alpha,E'_\alpha)\}_{\alpha\in\wedge}$ is a family of soft continuous mappings, then we define the soft mapping

$$\underset{\alpha\in\wedge}{\oplus}(f_\alpha,g_\alpha):\underset{\alpha\in\wedge}{\oplus}(X_\alpha,\tau_\alpha,E_\alpha)\to\underset{\alpha\in\wedge}{\oplus}(Y_\alpha,\tau'_\alpha,E'_\alpha)$$

as follows: for all $x_e\in\tilde{X}$, there exists $\alpha_0\in\wedge$ such that $x_e\in(X_\alpha,\tau_\alpha,E_\alpha)$ and

$$\underset{\alpha\in\wedge}{\oplus}(f_\alpha,g_\alpha)(x_e)=\left(f_{\alpha_0}(x)\right)_{g_{\alpha_0}(e)} .$$

Lemma 2. Let $\{(f_\alpha,g_\alpha):(X_\alpha,\tau_\alpha,E_\alpha)\to(Y_\alpha,\tau'_\alpha,E'_\alpha)\}_{\alpha\in\wedge}$ be a family of soft continuous mappings. Then $\underset{\alpha\in\wedge}{\oplus}(f_\alpha,g_\alpha)$ is a soft continuous mapping and $\oplus:\prod STop\to\prod STop$ is a functor.

Proof. The proof is straightforvard.■

Let $(f_\alpha,g_\alpha):(X_\alpha,\tau_\alpha,E_\alpha)\to(Y,\tau',E')$ be a soft continuous mappings for all $\alpha\in\wedge$. Then we define the soft mapping

$$(f,g)=\nabla(f_\alpha,g_\alpha):\underset{\alpha\in\wedge}{\oplus}(X_\alpha,\tau_\alpha,E_\alpha)\to(Y,\tau',E')$$

as:

$$(f,g)(x_e)=\left(f_{x_0}(x)\right)_{g_{\alpha_0}(e)},$$

where $x_e\in\underset{\alpha\in\wedge}{\oplus}(X_\alpha,\tau_\alpha,E_\alpha)$, i.e. $x_e\in\left(X_{\alpha_0},\tau_{\alpha_0},E_{\alpha_0}\right)$ for $\alpha_0\in\wedge$.

Proposition 2. Let $(f,g):\underset{\alpha\in\wedge}{\oplus}(X_\alpha,\tau_\alpha,E_\alpha)\to(Y,\tau',E')$ be a soft mapping. Then $(f,g):\underset{\alpha\in\wedge}{\oplus}(X_\alpha,\tau_\alpha,E_\alpha)\to(Y,\tau',E')$ is a soft continuous mapping if and only if $(f,g)\circ(i_\alpha,j_\alpha):(X_\alpha,\tau_\alpha,E_\alpha)\to(Y,\tau',E')$ is a soft continuous mapping.

Proof. The proof is straightforward.■

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Definition 17. Any functor $D:\wedge \to STop$, where $\wedge$ is a directed poset (considered as a category), is called a direct system in $STop$. The limit of D is called the direct limit of D and denoted by $\varinjlim\left(\overline{X},\overline{E}\right)$.

Let

$$\left(\overline{X},\overline{E}\right)=\left(\left\{\left(X^{\alpha},\tau^{\alpha},E^{\alpha}\right)\right\}_{\alpha\in\wedge},\left\{\left(p_{\alpha}^{\alpha'},q_{\alpha}^{\alpha'}\right):\left(X^{\alpha},\tau^{\alpha},E^{\alpha}\right)\to\left(X^{\alpha'},\tau^{\alpha'},E^{\alpha'}\right)\right\}_{\alpha\prec\alpha'}\right) \tag{5}$$

be a direct system. By using the direct system (5), we obtain the following two direct systems of the sets:

$$\left(\left\{X^{\alpha}\right\}_{\alpha\in\wedge},\left\{p_{\alpha}^{\alpha'}:X^{\alpha}\to X^{\alpha'}\right\}_{\alpha\prec\alpha'}\right), \tag{6}$$

$$\left(\left\{E^{\alpha}\right\}_{\alpha\in\wedge},\left\{q_{\alpha}^{\alpha'}:E^{\alpha}\to E^{\alpha'}\right\}_{\alpha\prec\alpha'}\right). \tag{7}$$

Let us consider the direct limits of (6) and (7) by $X=\varinjlim X^{\alpha}$ and $E=\varinjlim E^{\alpha}$ respectively. The equivalence relation on soft topological sum $\left(X,\tau,E\right)=\oplus\left(X^{\alpha},\tau^{\alpha},E^{\alpha}\right)$ is given by $x_e\sim y_{e'}$ and there exists $\alpha''\succ\alpha,\alpha'$ such that

$$\left(p_{\alpha}^{\alpha''},q_{\alpha}^{\alpha''}\right)\left(x_e\right)=\left(p_{\alpha'}^{\alpha''},q_{\alpha'}^{\alpha''}\right)\left(y_{e'}\right),$$

where $x_e\in\left(X^{\alpha},\tau^{\alpha},E^{\alpha}\right)$ and $y_{e'}\in\left(X^{\alpha'},\tau^{\alpha'},E^{\alpha'}\right)$. Then we denote this direct limit by $\varinjlim\left(\overline{X},\overline{E}\right)=\left(X,\tau,E\right)/\sim$. It is clear that if $x_e\sim y_{e'}$, then $x\sim y$ and $e\sim e'$. Hence each soft point of $\varinjlim\left(\overline{X},\overline{E}\right)$ is denoted by $\left[x\right]_{[e]}$. Thus, $\varinjlim\left(\overline{X},\overline{E}\right)=\left(\varinjlim X^{\alpha},\tilde{\tau},\varinjlim E^{\alpha}\right)$ is a soft topological space.

Theorem 7. Every direct system in (5) has a limit in $STop$ and this limit is unique.

Proof. Let $\left(i^{\alpha},j^{\alpha}\right):\left(X^{\alpha},\tau^{\alpha},E^{\alpha}\right)\to\oplus\left(X^{\alpha},\tau^{\alpha},E^{\alpha}\right)$ be an embedding mapping, $\left(p,q\right):\oplus\left(X^{\alpha},\tau^{\alpha},E^{\alpha}\right)\to\varinjlim\left(\overline{X},\overline{E}\right)$ be a canonical mapping and $\left(\pi^{\alpha},q^{\alpha}\right)=\left(p,q\right)\circ\left(i^{\alpha},j^{\alpha}\right):\left(X^{\alpha},\tau^{\alpha},E^{\alpha}\right)\to\varinjlim\left(\overline{X},\overline{E}\right)$ for all $\alpha\in\wedge$. It is clear that for all $\alpha\prec\alpha',\left(\pi^{\alpha},q^{\alpha}\right)=\left(p_{\alpha}^{\alpha'},q_{\alpha}^{\alpha'}\right)\circ\left(\pi^{\alpha'},q^{\alpha'}\right)$ is satisfied. Now we must show that $\varinjlim\left(\overline{X},\overline{E}\right)$ is a direct limit. For this, it suffices to show that there exists a unique, soft continuous mapping $\left(f,g\right):\varinjlim\left(\overline{X},\overline{E}\right)\to\left(Y,\tau',E\right)$ which makes up the following commutative diagram (Diagram 2).

$$\begin{tikzcd}
\left(X^{\alpha},\tau^{\alpha},E^{\alpha}\right) \arrow[r, "\left(\varphi^{\alpha},\psi^{\alpha}\right)"] \arrow[d, "\left(\pi^{\alpha},q^{\alpha}\right)"'] & \left(Y,\tau',E'\right) \\
\varinjlim\left(\overline{X},\overline{E}\right) \arrow[ur, "\left(f,g\right)"'] &
\end{tikzcd}$$

Diagram 2

Let $\left\{\left(\varphi^{\alpha},\psi^{\alpha}\right)\right\}_{\alpha\in\wedge}$ be a family of soft continuous mappings which makes up the commutative Diagram 3 for each soft topological space $\left(Y,\tau',E'\right)$.

$$\begin{tikzcd}
\left(X^{\alpha},\tau^{\alpha},E^{\alpha}\right) \arrow[r, "\left(\varphi^{\alpha},\psi^{\alpha}\right)"] \arrow[d, "\left(p_{\alpha}^{\alpha'},q_{\alpha}^{\alpha'}\right)"'] & \left(Y,\tau',E'\right) \\
\left(X^{\alpha'},\tau^{\alpha'},E^{\alpha'}\right) \arrow[ur, "\left(\varphi^{\alpha'},\psi^{\alpha'}\right)"'] &
\end{tikzcd}$$

Diagram 3

Let $\left[x\right]_{[e]}$ belong to $\underrightarrow{\lim}\left(\overline{X},\overline{E}\right)$. Then there exists $\alpha\in\wedge$ and $x_{e_{\alpha}}^{\alpha}\in\left(X^{\alpha},\tau^{\alpha},E^{\alpha}\right)$ such that $\left(\pi^{\alpha},q^{\alpha}\right)\left(x_{e_{\alpha}}^{\alpha}\right)=\left[x\right]_{[e]}$. Hence we define the soft mapping

$$(f,g):\underrightarrow{\lim}\left(\overline{X},\overline{E}\right)\to\left(Y,\tau',E\right)$$

by the formula:

$$(f,g)\left(\left[x\right]_{[e]}\right)=\left(\varphi^{\alpha},\psi^{\alpha}\right)\left(x_{e_{\alpha}}^{\alpha}\right).$$

It is clear that $(f,g):\underrightarrow{\lim}\left(\overline{X},\overline{E}\right)\to\left(Y,\tau',E\right)$ is well-defined and Diagram 3 is commutative.■

Let

$$\left(\overline{X},\overline{E}\right)=\left(\left\{\left(X^{\alpha},\tau^{\alpha},E^{\alpha}\right)\right\}_{\alpha\in\wedge},\left\{\left(p_{\alpha}^{\alpha'},q_{\alpha}^{\alpha'}\right)\right\}_{\alpha\prec\alpha'}\right),$$

$$\left(\overline{Y},\overline{E'}\right)=\left(\left\{\left(Y^{\beta},\tau'^{\beta},E'^{\beta}\right)\right\}_{\beta\in\wedge'},\left\{\left(r_{\beta}^{\beta'},q_{\beta}^{\beta'}\right)\right\}_{\beta\prec\beta'}\right) \tag{8}$$

be two direct systems in *STop* , $\varphi:\wedge\to\wedge'$ be an isotone mapping for all $\alpha\in\wedge$, and $\left(f^{\alpha},g^{\alpha}\right):\left(X^{\alpha},\tau^{\alpha},E^{\alpha}\right)\to\left(Y^{\varphi(\alpha)},\tau'^{\varphi(\alpha)},E'^{\varphi(\alpha)}\right)$ be a soft continuous mapping.

Definition 18. Let $\left(\overline{X},\overline{E}\right)$ and $\left(\overline{Y},\overline{E'}\right)$ be two direct systems. If the equality $\left(r_{\varphi(\alpha)}^{\varphi(\alpha')},q_{\varphi(\alpha)}^{\varphi(\alpha')}\right)\circ\left(f^{\alpha},g^{\alpha}\right)=\left(f^{\alpha'},g^{\alpha'}\right)\circ\left(p_{\alpha}^{\alpha'},q_{\alpha}^{\alpha'}\right)$ is satisfied for all $\alpha\prec\alpha'$, then the family

$$\left(\overline{f},\overline{g}\right)=\left(\varphi:\wedge\to\wedge',\left\{(f^{\alpha},g^{\alpha})\right\}_{\alpha\in\wedge}\right) \tag{9}$$

is called a morphism from the direct system $\left(\overline{X},\overline{E}\right)$ to the direct system $\left(\overline{Y},\overline{E'}\right)$.

It is clear that the direct systems in *STop* and these morphisms between them construct a category denoted by $Dir\left(STop\right)$. Let $\left(\overline{f},\overline{g}\right)$ be a morphism of direct systems. Then

$$\oplus\left(f^{\alpha},g^{\alpha}\right):\oplus\left(X^{\alpha},\tau^{\alpha},E^{\alpha}\right)\to\oplus\left(Y^{\varphi(\alpha)},\tau'^{\varphi(\alpha)},E'^{\varphi(\alpha)}\right)$$

is a soft continuous mapping and preserves an equivalence relation on these direct limits. Then the mapping $\oplus\left(f^{\alpha},g^{\alpha}\right)$ defines a soft mapping of quotient spaces. We denote the mapping by

$$\underrightarrow{\lim}\left(\overline{f},\overline{g}\right):\underrightarrow{\lim}\left(\overline{X},\overline{E}\right)\to\underrightarrow{\lim}\left(\overline{Y},\overline{E'}\right).$$

Theorem 8. Let $Dir(STop)$ be the category of all direct systems in $STop$ and all mappings between them. Then

$$\underrightarrow{\lim} : Dir(STop) \to STop$$

is a functor.

Proof. The proof is straightforward.■

Theorem 9. Let $(\overline{X}, \overline{E})$ be a direct system of soft topological spaces in (5).

a) If each $\left(p_\alpha^{\alpha'}, q_\alpha^{\alpha'}\right) : \left(X^\alpha, \tau^\alpha, E^\alpha\right) \to \left(X^{\alpha'}, \tau^{\alpha'}, E^{\alpha'}\right)$ is an injective soft mapping, then $\left(\pi^\alpha, q^\alpha\right) : \left(X^\alpha, \tau^\alpha, E^\alpha\right) \to \underrightarrow{\lim}(\overline{X}, \overline{E})$ is also an injective soft mapping.

b) If each $\left(p_\alpha^{\alpha'}, q_\alpha^{\alpha'}\right) : \left(X^\alpha, \tau^\alpha, E^\alpha\right) \to \left(X^{\alpha'}, \tau^{\alpha'}, E^{\alpha'}\right)$ is a bijective soft mapping, then $\left(\pi^\alpha, q^\alpha\right) : \left(X^\alpha, \tau^\alpha, E^\alpha\right) \to \underrightarrow{\lim}(\overline{X}, \overline{E})$ is also a bijective soft mapping.

Proof. a) For each soft point $x_{e_\alpha}^\alpha, y_{e'_\alpha}^\alpha \in \left(X^\alpha, \tau^\alpha, E^\alpha\right)$, let $x_{e_\alpha}^\alpha \neq y_{e'_\alpha}^\alpha$ and $\left(\pi^\alpha, q^\alpha\right)\left(x_{e_\alpha}^\alpha\right) = \left(\pi^\alpha, q^\alpha\right)\left(y_{e'_\alpha}^\alpha\right)$. If $x_{e_\alpha}^\alpha \neq y_{e'_\alpha}^\alpha$, then $x^\alpha \neq y^\alpha$ or $e_\alpha \neq e'_\alpha$. Hence the soft points $x_{e_\alpha}^\alpha, y_{e'_\alpha}^\alpha$ are equivalent. Then there exists $\beta \in \wedge'$ such that $\left(p_\alpha^\beta, q_\alpha^\beta\right)\left(x_{e_\alpha}^\alpha\right) = \left(p_\alpha^\beta, q_\alpha^\beta\right)\left(y_{e'_\alpha}^\alpha\right)$. Since $\left(p_\alpha^\beta, q_\alpha^\beta\right)$ is an injective mapping, $x_{e_\alpha}^\alpha = y_{e'_\alpha}^\alpha$, i.e. $x_{e_\alpha}^\alpha = y_{e'_\alpha}^\alpha$ and $e_\alpha = e'_\alpha$. Thus, the soft mapping $\left(\pi^\alpha, q^\alpha\right)$ is injective soft mapping..

b) Let us show that $\left(\pi^\alpha, q^\alpha\right)$ is an surjective mapping. Let $[x]_{[e]} \in \underrightarrow{\lim}(\overline{X}, \overline{E})$ be an arbitrary soft point. Since the canonical mapping $(p,q) : \oplus\left(X^\alpha, \tau^\alpha, E^\alpha\right) \to \underrightarrow{\lim}(\overline{X}, \overline{E})$ is a surjective mapping, there exists a soft point $\tilde{x}_{\tilde{e}} \oplus \left(X^\alpha, \tau^\alpha, E^\alpha\right)$ such that $(p,q)\left(\tilde{x}_{\tilde{e}}\right) = [x]_{[e]}$. From the definition of the soft topological sum, we have $\tilde{x}_{\tilde{e}} = x_{e_{\alpha'}}^{\alpha'} \in \left(X^{\alpha'}, \tau^{\alpha'}, E^{\alpha'}\right)$. In this case $\left(\pi^{\alpha'}, q^{\alpha'}\right)\left(x_{e_{\alpha'}}^{\alpha'}\right) = [x]_{[e]}$ is satisfied. Let us choose $\alpha'' \in \wedge$ such that $\alpha'' \succ \alpha, \alpha'$. Since $\left(p_\alpha^{\alpha''}, q_\alpha^{\alpha''}\right) : \left(X^\alpha, \tau^\alpha, E^\alpha\right) \to \left(X^{\alpha''}, \tau^{\alpha''}, E^{\alpha''}\right)$ is a soft surjective mapping, there exists a soft point $x_{e_\alpha}^\alpha \in \left(X^\alpha, \tau^\alpha, E^\alpha\right)$ such that $\left(p_\alpha^{\alpha''}, q_\alpha^{\alpha''}\right)\left(x_{e_\alpha}^\alpha\right) = \left(p_\alpha^{\alpha'}, q_\alpha^{\alpha'}\right)\left(x_{e_{\alpha'}}^{\alpha'}\right)$ for soft point $\left(p_\alpha^{\alpha'}, q_\alpha^{\alpha'}\right)\left(x_{e_{\alpha'}}^{\alpha'}\right) \in \left(X^{\alpha''}, \tau^{\alpha''}, E^{\alpha''}\right)$. Hence the soft points $x_{e_\alpha}^\alpha, x_{e_{\alpha'}}^{\alpha'}$ are equivalent and $\left(\pi^\alpha, q^\alpha\right)\left(x_{e_\alpha}^\alpha\right) = \left(\pi^{\alpha'}, q^{\alpha'}\right)\left(x_{e_{\alpha'}}^{\alpha'}\right) = [x]_{[e]}$.■

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