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MAINTENANCE JOB SCHEDULING: A MULTI-CRITERIA APPROACH UNDER STOCHASTIC-FUZZY UNCERTAINTY

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ABSTRACT

A multi-criteria maintenance job scheduling model that minimises equipment and personnel idle times, and lateness of jobs under stochastic-fuzzy uncertainties is presented using a weighted integer linear programming. Job parameters were specified by fuzzy numbers and modelled using triangular membership function representations. The centre of gravity (COG) deffuzification scheme was used within a finite interval to obtain fuzzy variables. The fuzzy variables were then randomised using the instantaneous probability characteristics of arrival time, processing time and due time of the job specified by probability mass function (PMF). This was used to determine the stochastic measures. The stochastic-fuzzy data then became the model input. The mathematical model constrained by the available equipment, manpower and job availability times within the planning horizon was tested with a 15-job, 24-hour problem with declared equipment and manpower availability levels. The results, analyses and illustrations were used to demonstrate the feasibility of the model.

KEYWORDS: maintenance scheduling, fuzzy, stochastic, multi-criteria, stochastic-fuzzy arrival time, processing time, due date, defuzzification, weighting

1. INTRODUCTION

A survey of the operations and management of today's industries shows that maintenance activities contribute immensely to the success of industrial concerns. Therefore, good maintenance policy can increase availability of equipment by trading off between planned and unplanned downtime, which can cause major disruptions in manufacturing processes [1]. Thus, in order to maximise profit, maintenance tasks needs to be scheduled carefully and comprehensively, and must be performed according to prescribed procedures and standards. Hence, the maintenance unit can be viewed as the backbone of an industrial organisation. Managing the maintenance subsystem becomes an arduous task due to its complexity and overall effect on a company's resources and growth.

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These problems are multi-faceted, requiring a multi-objective formulation and solution [2-4]. However, when such multi-objectives have unit parity, a multi-criteria formulation may be used. Documentation of maintenance scheduling has utilized stochastic approach [5, 6], Fuzzy elements [7-11], and Genetic algorithm and fuzzy [12-14]. Sadly, the case of Stochastic fuzzy appears ignored in the literature. This paper presents an approach to maintenance job scheduling under the combination of stochastic-fuzzy uncertainty and multi-criteria objective formulation. The technique takes cognisance of the fact that decisions are made in the presence of multiple and possibly conflicting criteria, and the imprecision in scheduling parameters due to both human and machine resource factors [15]. As a result, classical approaches within deterministic scheduling theory, relying on precise data might not be suitable for representation of uncertain scenarios [16]. Consequently, the fuzzy scheduling models and algorithms have been extended to the stochastic-fuzzy case. This technique applies fuzzy and stochastic principles to capture imprecision in the job data. The stochastic-fuzzy parameters then become the basic input of the proposed model. The model draws together fuzzy and stochastic scheduling, and multi-criteria principles with the aim of successfully tackling difficult, uncertain and dynamic real-world scheduling problems [17]. Scheduling maintenance can be viewed as a problem with availability constraint. Lee and Liman [18] studied the availability problem and showed the tight error bound for the shortest processing time (SPT) heuristic. Lee and Lin [19] assumed that the processing time was deterministic while machine breakdown was a random process with certain distributions. Qi *et al.* [20] considered the scheduling problem that minimises total completion time. Liao and Chen [21] addressed the problem of minimising maximum tardiness by assuming several maintenance intervals. Kobbacy *et al*. [22] presented an heuristic approach for implementing the semi-parametric proportional hazard model (PHM) to schedule the next preventive maintenance interval based on the equipment's full condition history. The model only addresses preventive maintenance and not multi-criteria scenarios. Olorunniwo and Izuchukwu [23] applied the concept of maintenance improvement factors to generate preventive and overhaul maintenance schedules. Ashayeri *et al*. [24] report on simultaneous planning maintenance and production in a process industry environment using a mixed integer linear programming model. However, the deterministic model discussed does not afford insight into the nature of deterioration of machines. Duffuaa and Al-Sultan [25] proposed a stochastic formulation of the maintenance-scheduling problem as an extension of Robert and Escudero's work. This again did not address the multi-criteria proposition. Ogunwolu *et al.* [26] proposed a multi-criteria objective formulation to address maintenance job-shop scheduling problem using deterministic integer linear programming model in which job arrival, processing time and due-date are precisely known. This approach adequately captured a typical real life maintenance scenario except that job parameters were not considered under uncertain scenarios. The work by Ogunwolu *et al.* [26] informed the method proposed in this paper.

2. METHODOLOGY

The maintenance job schedule proposed in this paper was modelled as a weighted multi-criteria integer linear programming problem with three weighted criteria. The proposed criteria are based on minimisation of equipment and personnel idle times and delay in scheduling. The three criteria correspond to the respective terms of the objective function in the mathematical model presented in the next sub-section. The three criteria have time-parity and hence, integrated to a single objective problem with weighted value assigned to each criterion. The weights at an instance of optimisation sum up to one. The problem space is constrained by uncertain job arrival (due routine maintenance) times, imprecision in job type and uncertain commencement of maintenance as well as the need for the required personnel and equipment at any point within the schedule time horizon. Various data inputs are modelled under stochastic-fuzzy conditions in other to capture the effect of comprehensive uncertainty in the model. The schedule time horizon is described at one-hour intervals. Job arrival and maintenance duration are defined within a finite range modelled by triangular membership functions represented by a triplet, and assumed to have discrete time probabilities specified by probability mass function.

2.1 Problem Formulation

A multi-criteria formulation, which was drawn from a real-life case, was derived as a constrained 0-1 integer linear mathematical programming problem, stated as follows:

Objective:

Criteria 1: Minimisation of Equipment Idle Time (E I T)

Criteria 2: Minimisation of Manpower Idle Time (M I T)

Criteria 3: Minimising of Lateness of Job (L J)

Note: These objectives have time-unit parity and are complementary in the maintenance-scheduling problem

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Mathematical Programming Model

$$\text{Min } w_1 \sum_l \sum_k \left[TE_{lk(l)} - \sum_i \sum_j x_{ij} \tilde{\tilde{t}}\, n^e_{ilk(l)} \right] + w_2 \sum_p \left[TM_P - \sum_i \sum_j x_{ij} \tilde{\tilde{t}}_i\, n^M_{iP} \right]$$

$$+ w_3 \sum_i \sum_j x_{ij} \left\lfloor 2j - \tilde{\tilde{a}}_i - \tilde{\tilde{d}}_i \right\rfloor \qquad (1)$$

Subject to:

(C01) $\sum_i x_{ij} \left(j - \tilde{\tilde{a}}_i - 1 \right) \geq 0 \qquad \forall\ j$ (2)

(C02) $\sum_j x_{ij} \leq 1 \qquad \forall\ i$ (3)

(C03) $\sum_i x_{ij} \geq 0 \qquad \forall\ j$ (4)

(C04) $\sum_r x_{ir}\ n^e_{ilk(l)} \leq E_{lk(l)} \qquad \forall\ i, l, k$ (5)

(C05) $\sum_r x_{ir}\ n^m_{ip} \leq M_p \qquad \forall\ i, p$ (6)

(C06) $x_{ij} = \begin{cases} 1 & f \text{ job is schedule at time } j \\ 0 & \text{otherwise} \end{cases}$ (7)

(C07) $j \leq T$ (8)

(C08) $\tilde{\tilde{a}}_i + 2 < r \leq j - 1$ (9)

Where $\tilde{\tilde{a}}_i$, $\tilde{\tilde{t}}_i$ and $\tilde{\tilde{d}}_i$ are randomised-fuzzy input data for processing time, arrival time and due date for job *i* respectively

Problem Constraints

Constraint (1) stipulates the earliest time a job can be scheduled. Constraint (2) restricts the model to schedule a job once or not schedule at all. Constraint (3) allows none or more than one job to be scheduled at any discrete time points. Constraint (4) stipulates that the maximum number of individual equipment group in use at any time j cannot exceed the maximum available. Constraint

(5) stipulates that the maximum number engaged of an individual personnel group at any time j cannot exceed the maximum available. Constraint (6) is the binary integer decision available for determining the schedule time points that optimises the objective function. Constraint (7) limits all schedule time points to the stipulated time horizon T. Constraint (8) specifies a range of value of discreet time points based on maintenance durations of individual jobs during which relevant equipment and manpower are kept in use.

2.2 Fuzzy Input Model

In this paper uncertain job parameter are modelled by triangular membership functions represented by a triplet [a, b, c]. An example fuzzy processing time $\bar{t}_{ij}$, specification is shown below.

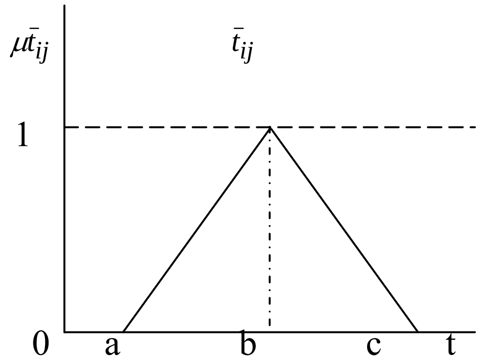

Figure 1 Fuzzy Time Specification

The wider the support of the membership function, the higher the uncertainty. The fuzzy parameter obtained at a membership grade of $\alpha \in [0, 1]$ is called the alpha-cut of the set. For the fuzzy set, $\bar{t}_{ij}$, specified by the triplet [a, b, c] as shown in Figure 1, the centre of gravity defuzzification, as reported in [26] is given as,

$$\bar{T}_i = \left[c - [\,(c - b)\,(c - a)]^{1/2}\right] \tag{10}$$

Where a, b, and c are lower bound, mean membership value or a modal point and upper bound respectively [27]. Defuzzification was done for different α – level at discrete time points to determine fuzzy arrival time $\bar{a}_i$, fuzzy processing time $\bar{t}_i$ and fuzzy due time $\bar{d}_i$. The equation (10) was applied to the deterministic data in Table 1 to determine the scheduling input data for fuzzy arrival time, fuzzy processing time and fuzzy due-time of the job. These fuzzy model parameters were derived as doublets $\lfloor a_{ie}, a_{i\ell} \rfloor$, $\lfloor t_{ie}, t_{i\ell} \rfloor$ and $\lfloor d_{ie}, d_{i\ell} \rfloor$, each doublet representing the earliest and the latest fuzzy time measures respectively over different alpha levels, for each job i.

2.3 Stochastic-Fuzzy Input Model

In order to obtain the stochastic-fuzzy input data ($\tilde{\bar{a}}_i, \tilde{\bar{t}}_i$, $\tilde{\bar{d}}_i$) for the model at any time point falling under a fuzzy interval, each of the three fuzzy job time-parameters is randomised to obtain a stochastic measure for it. Hence if

$\overline{a}_{ie}$ = beginning of the fuzzy arrival time point obtained for any job i.

$\overline{a}_{il}$ = end of the fuzzy arrival time point

p'_{ik} = the probability at instance of the time point j within the spread of $[a_l - a_e]$

$$\overline{\tilde{\tilde{A}}}_{ij} = \overline{a}_e + (\overline{a}_e - \overline{a}_l)\sum_{k=a_e}^{j} P_{ik} \quad (11)$$

where $\overline{\tilde{\tilde{A}}}_{ij}$ represents the stochastic-fuzzy input parameter for arrival time.

Similar equations to derive the Stochastic-Fuzzy input for processing time, and due-time for each job i, and every time point j. are given by,

$$\overline{\tilde{\tilde{T}}}_{ij} = \bar{t}_{ie} + (\bar{t}_{ie} - \bar{t}_{il})\sum_{k=t_e}^{j} P_{ik} \quad (12)$$

$$\overline{\tilde{\tilde{D}}}_{ij} = \overline{d}_{ie} + (\overline{d}_{ie} - \overline{d}_{il})\sum_{k=d_e}^{j} P_{ik} \quad (13)$$

2.4 Test Problem

The multi-criteria model under Fuzzy condition presents a unique approach to job scheduling problem. As exemplified in the previous discussion, the aim is to deal with breakdown and safe guard installation from deterioration and sustain productivity by quelling unscheduled maintenance and taking care of uncertainties. The maintenance-scheduling model was tested with 15 jobs (routine and corrective maintenance jobs) within a 24 –hour time horizon. It is assumed that 8 working hours correspond to a day’s job. Fuzzy concept was used to capture the imprecision in a real world scenario. Table 1 describes the Fuzzy time specifications for arrival time, maximum available equipment (electrical and mechanical tools) and manpower against the number of each group of equipments and manpower needed for each of the jobs. Three types of electrical tools and two mechanical tools are used. Two types of maintenance jobs, routine and corrective maintenance jobs, were scheduled. While jobs 4, 6, 9 and 12 are routine jobs whose arrival, maintenance processing and due-times are known precisely, the time specifications for other jobs are taken to be fuzzy each them being depicted by fuzzy triplet [a, b, c] as illustrated in Section 2.2. Tables 2, 3 and 4 exhibit probabilistic mass functional (PMF) values for arrival, maintenance duration and due times for each the jobs arrayed for scheduling. Table exhibits the arrival probabilistic mass functional values at individual time points in the schedule-horizon ranging from 1 to 30 hours. Again as in the specification of fuzzy times the routine jobs 4, 6, 9 and 12 have definite pmf value of 1 at specific single arrival times while other jobs have spreads of pmf values summing up to 1 for each job over a range of schedule time period. In Table 3, the maintenance duration are specified with probabilistic mass functional values for a possible range of durations (1 to 8 hours) for each job. In Table 4, pmf values are specified for due times of each maintenance job as explained for Table 2 but for maintenance time points 10 to 37 (in hours).

Table 1 Fuzzy Time Distribution and Job Equipment/Manpower Needs and Availability

								Group			Equipment					Manpower		
											Mechanical			Electrical				
								Type			1	2	3	1	2	1	2	3
								Maximum No			6	5	5	5	7	8	7	8
Job Type	Job No	Fuzzy Time Specifications									Number Required							
		Arrival time			Processing time			Due-time										
	i	*a*	*b*	*c*	*a*	*B*	*c*	*a*	*b*	*C*								
Routine	1		15			2			22		1	2	2	1	2	3	4	3
Corrective	2	3	9	11	1	3	6	13	16	21	2	2	1	1	2	4	3	4
Corrective	3	8	13	20	5	6	8	22	26	33	2	2	2	1	2	4	3	3
Routine	4		8			2			11		1	1	2	1	2	2	3	2
Corrective	5	2	6	13	1	2	4	14	15	22	2	1	2	2	1	3	2	2
Routine	6		3			3			10		2	2	1	1	2	4	3	4
Corrective	7	1	3	8	2	4	7	10	12	17	2	1	2	1	2	3	3	4
Corrective	8	18	20	26	1	2	5	27	30	31	1	2	2	2	2	3	2	2
Routine	9		12			3			19		2	2	1	1	2	3	3	3
Corrective	10	2	5	9	1	2	4	12	15	21	1	2	2	1	1	2	2	3
Corrective	11	7	18	23	1	3	6	24	29	31	1	2	2	2	2	3	4	2
Routine	12		21			4			24		2	1	2	1	2	4	3	4
Corrective	13	17	20	25	3	4	6	27	30	34	2	2	2	2	1	4	2	4
Corrective	14	23	27	30	1	2	4	33	36	38	1	2	2	2	1	3	2	4
Corrective	15	21	26	29	2	4	5	32	36	38	1	2	2	1	2	2	2	3

Table 2 Stochastic Time Specifications for Maintenance duration

Job		Possible Duration (hours)/ Probability Mass Functional values						
I	1	2	3	4	5	6	7	8
1		1						
2	0.2	0.1	0.1	0.2	0.3	0.1		
3					0.1	0.4	0.3	0.2
4		1						
5	0.3	0.1	0.2	0.4				
6			1					
7		0.1	0.15	0.3	0.25	0.1	0.1	
8	0.3	0.2	0.1	0.15	0.25			
9			1					
10	0.15	0.25	0.4	0.2				
11	0.1	0.3	0.15	0.25	0.05	0.15		
12				1				
13			0.2	0.4	0.15	0.25		
14	0.2	0.2	0.2	0.4				
15		0.1	0.4	0.3	0.2			

Table 4 Stochastic Time Specifications for Job Due Times

Job		Arrival Time Point (hour)/ Probability Mass Functional values												
i	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
1													1		
2				0.05	0.15	0.15	0.1	0.1	0.2	0.1	0.1	0.05			
3													0.05	0.2	0.05
4		1													
5					0.1	0.06	0.04	0.15	0.2	0.1	0.15	0.15	0.05		
6	1														
7	0.05	0.1	0.15	0.15	0.1	0.1	0.2	0.15							
8															
9										1					
10			0.05	0.05	0.2	0.05	0.1	0.05	0.15	0.1	0.1	0.15			
11															0.15
12															1
13															
14															
15															
Job	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**
i															
1															
2															
3	0.25	0.05	0.2	0.1	0.1										
4															
5															
6															
7															
8		0.15	0.2	0.05	0.15	0.2	0.25								
9															
10															
11	015	0.05	0.2	0.2	0.15	0.1									
12															
13						0.2	0.1	0.15	0.2	0.15	0.05	0.15			
14									0.2	0.15	0.1	0.2	0.15	0.2	
15							0.15	0.1	0.15	0.4	0.1	0.1			

2.5 Method of Solution

The test problem was run with the integer linear programming module of Qualitative Systems for Business Plus (QSB^+) software for various combinations of criteria constituting a main model consisting of all three criteria and four other variants made up as specified here under.

Variant 1: Criteria 1 and 3
Variant 2: Criteria 2 and 3
Variant 3: Criterion 3 alone
Variant 4: Criterion 2 alone

2.6 Methods of Analysis

A number of comparative measures were used for comparing results obtained for fuzzy, stochastic and stochastic-fuzzy input values of maintenance job arrival, processing and due times [3]. The basis of the comparison was to underscore the importance of taking account of a comprehensive account of fuzzy and stochastic uncertainties in model parameters. These have direct bearing on production planning.

1. The number of jobs scheduled: The model by constraint 2 allows a job to be scheduled or unscheduled within the given time-horizon. The measure of the number of jobs scheduled or unscheduled is a measure of satisfactoriness of any model variant under a form of uncertainty or combinations of uncertainties.

2. The number of uncompleted jobs: There are possibilities of jobs scheduled, which cannot be completed within a particular time-horizon. The number of jobs yet to be completed together with the time duration left for completion can also serve as bases of comparison of variants of the model under any form of uncertainty or a combination of uncertainties.

3.Equipment Utilization Indices: Hourly Equipment Utilization Index (HEUI) for each variant of the model under different forms or combinations of uncertainties can be defined as, $\lambda_{hj}^{e} = \sum_{k}\sum_{l}\left[\frac{\sum_{i}(x_{ij})n_{lk(l)}^{e}}{E_{lk(l)}}\right]$, for each j and $x_{ij} = 1$

Thus, the total equipment utilization index for each variant h within the maintenance scheduling time horizon T is,

$$\eta_{h}^{e} = \frac{1}{T}\sum_{j=1}^{T}\lambda_{hj}^{e}$$

4. Manpower Utilization Indices: Hourly Manpower Utilization Index (HMUI) for each variant under different forms or combinations of uncertainties is defined similar to (HEUI) as $\lambda_{hj}^{m} = \sum\left[\frac{\sum_{i}(x_{ij})n_{p}^{m}}{M_{p}}\right]$ for each j and x_{ij}. The Total Manpower Utilization Index for a variant h is also defined as, $\eta_{h}^{m} = \frac{1}{T}\sum_{j=1}^{T}\lambda_{hj}^{m}$

5. Number of branch and bound iterations required: The QSB+ solution procedure used zero integer tolerance and newest branching schemes. The number of iterations for the model and its variant under different forms or combinations of uncertainties is also a level ground for their comparison.

3. RESULTS

The summary schedules obtained from running the model and its variants under Fuzzy, Stochastic and Stochastic-Fuzzy uncertainties with Integer Linear Programming module of Quantitative Systems for Business Plus (QSB^{+}) software are illustrated in the tables below. Tables 5-7 show the schedules obtained for different variants of the model for the cases of fuzzy, stochastic and stochastic-fuzzy inputs values respectively.

Deductions from Optimal Schedules

The optimal scheduling trends are determined using horizontal blocks on a time-line with granularity of an hour. Tables 8-10 give summaries of deductions from optimal schedules. In the ensuing tables the schedules for fuzzy (F), stochastic (S) and stochastic-fuzzy (SF) scheduling results are compared and contrasted using the five measures of comparison earlier explained in Section 2.6.

4. DISCUSSION

Real world problems involve making decisions in the presence of conflicting criteria and under various possible uncertainties as in the case considered in this project. The analysis of the test problem exhibited in the last section is quite informative.

On the Optimal Objective Values: The Stochastic-Fuzzy version under the combination of the three criteria used (Main Model) performed comparative as good as for the solely stochastic or fuzzy versions. In some particular cases Stochastic-Fuzzy performed better in terms of the average daily optimal objective attained. This is particularly true for model variants 2 and 4.

On the Number of Iterations: The model and its variants perform better under Stochastic-Fuzzy uncertainty than under solely Fuzzy uncertainty and marginally inferior under solely stochastic uncertainty in terms of the number of iterations used to obtain optimal solutions.

On the number of Jobs Scheduled and Remaining Durations of Uncompleted Jobs: In virtually all the model variants and under all forms of uncertainties analysed, all the jobs were scheduled, thus a level plane ground to compare the performances both under the various combinations of criteria and uncertainties is the use of the number of hours (durations) remaining for uncompleted jobs. In this case too (as visibly demonstrated in the tables in the last section), all variants of the model perform comparatively better under Stochastic-Fuzzy uncertainty than under solely stochastic or fuzzy uncertainties (particularly better than under stochastic uncertainty).

On Equipment Utilization Indices: Equipment and Manpower utilization indices appear better as performance measures under stochastic uncertainty than under Stochastic-Fuzzy but in practical terms the situation under the latter tends to overuse. Values obtained for these performance measures under Stochastic-Fuzzy uncertainty are more realistic than the stochastic case under various combinations of uncertainty and particularly better in more respects than under Fuzzy uncertainty. In all, the performance of the Stochastic-Fuzzy version in all the variants of the model is comparatively as good or better (in some respects) than those for solely stochastic or fuzzy versions. This underscores the importance of taking cognisance of the comprehensive combinations of these forms of uncertainty in decision models as the problem on hand is.

Table 8 Comparison by Optimal Objective(s) Values and No of Iterations used

Model Variant	Uncertainty	Performance Measures for: Day 1 (Hour 1 – 8)		Day 2 (Hour 9 – 16)		Day 3 (Hour 17 – 24)		Average Opt. Objective Val.	Average No of Iterations
		Optimal Objective Value *	No of Iterations	Optimal Objective Value *	No of Iterations	Optimal Objective Value *	No of Iterations		
Main	F	-45.26	241	4.24	1	-16.37	3	61.36	134.0
	S	-46.15	509	-6.26	5	-24.13	51	23.93	60.3
	SF	-52.02	239	1.81	1	-34.05	11	35.88	86.3
1	F	-16.38	167	132.06	4	75.53	5	21.38	163.7
	S	26	371	102.95	6	55.12	25	-1.08	82.3
	SF	-27.38	166	128.28	4	-29.1	11	41.61	71.0
2	F	2.8	253	65.80	3	39.04	3	14.70	129.0
	S	-9.44	459	56.71	5	16.87	27	31.67	71.0
	SF	-6.04	233	64.29	3	-61.5	11	-49.90	51.7
3	F	1.44	209	77.78	1	45.61	3	-41.76	192.3
	S	-26.62	355	53.32	5	17.39	27	-55.38	63.0
	SF	-8.17	199	74.66	3	28.51	11	95	213
4	F	-69.3	135	-27.54	1	-52.85	19	-149.69	155
	S	-58.56	519	-22.08	5	-44.64	53	-125.28	577
	SF	-75	177	-29.63	1	-61.5	11	-166.13	189

**Negative objective values arise from the negative objective values obtained from Criterion 3*

Table 9 Comparison by Number of Scheduled Jobs (NSJ) and Remaining Duration of Uncompleted Jobs (RDUJ)

Model Variant	Uncertainty Type	Performance Measures for: Day 1 (Hour 1 – 8)		Day 2 (Hour 9 – 16)		Day 3 (Hour 17 – 24)		Average NSJ	Average RDUJ
		NSJ	RDUJ	NSJ	RDUJ	NSJ	RDUJ		
Main	F	6	7	4	2	5	4	5.00	4.33
	S	5	9	5	2	5	8	5.00	6.33
	SF	6	7	4	2	5	4	5.00	4.33
1	F	6	7	4	2	5	5	5.00	4.67
	S	5	11	5	2	5	8	5.00	7.00
	SF	6	9	4	2	5	4	5.00	5.00
2	F	6	9	4	2	5	4	5.00	5.00
	S	5	9	4	2	5	8	4.67	6.33
	SF	6	7	4	4	5	4	5.00	5.00
3	F	6	7	4	2	5	4	5.00	4.33
	S	5	9	5	2	5	8	5.00	6.33
	SF	6	7	4	4	5	4	5.00	5.00
4	F	6	7	3	2	5	4	4.67	4.33
	S	5	9	5	2	5	8	5.00	6.33
	SF	6	7	4	2	5	4	5.00	4.33

Key: S – Stochastic, F – Fuzzy, SF – Stochastic Fuzzy

Table 10 Comparison by Equipment and Manpower Utilization Indices

Model Variant	Uncertainty Type	Performance Measures for: Day 1 (Hour 1 – 8) Equip. Utilization Index (1)	Day 1 (Hour 1 – 8) Manpower Utilization Index (2)	Day 2 (Hour 9 – 16) Equip. Utiliz. Index (3)	Day 2 (Hour 9 – 16) M/Power Utiliz. Index (4)	Day 3 (Hour 17 – 24) Equip. Utiliz. Index (5)	Day 3 (Hour 17 – 24) M/Power Utiliz. Index (6)	Average Daily Equip. Index	Average Daily M/Power Index
Main	F	0.468	0.842	0.568	0.625	0.546	0.678	0.527	0.715
	S	0.455	0.865	0.715	0.829	0.580	0.828	0.583	0.847
	SF	0.324	0.567	0.616	.704	0.414	0.692	0.451	0.654
1	F	0.323	0.567	0.615	0.704	0.396	0.645	0.445	0.639
	S	0.590	0.859	0.667	0.829	0.580	0.828	0.612	0.839
	SF	0.400	0.720	0.544	.625	0.414	0.678	0.453	0.674
2	F	0.324	0.567	0.616	0.704	0.414	0.693	0.451	0.655
	S	0.600	0.670	0.72	0.77	0.780	0.890	0.700	0.767
	SF	0.396	0.700	0.56	.571	0.476	0.800	0.477	0.690
3	F	0.324	0.567	0.616	0.704	0.414	0.693	0.451	0.655
	S	0.600	0.670	0.72	0.77	0.780	0.890	0.700	0.777
	SF	0.324	0.567	0.616	0.704	0.414	0.692	0.450	0.654
4	F	0.42	0.770	0.63	0.56	0.640	0.600	0.563	0.643
	S	0.430	0.760	0.58	0.704	0.550	0.697	0.520	0.720
	SF	0.396	0.683	0.471	0.542	0.378	0.650	0.415	0.665

5. CONCLUSION

The multi-criteria approach to maintenance job scheduling under stochastic-fuzzy uncertainty is demonstrated in this work. The model formulation explicitly represents typical real world maintenance jobs planning and scheduling problem with due consideration given to available resources and cost of achieving the desired goal and uncertainty of job data captured comprehensively. The model results obtained as analysed shows comparatively good or better results under both multi-criteria and Stochastic-Fuzzy uncertainties than under solely Stochastic or Fuzzy uncertainty. Thus, the work provides a toolkit for managers in manufacturing in the cost effective scheduling implementation activities.

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