เซตวิภัชนัยใน Γ-ริงใกล้ไม่เปลี่ยนหมู่
On Fuzzy Sets in nonassociative Γ-Near Rings

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บทคัดย่อ

วัตถุประสงค์ของบทความนี้คือการแนะนำแนวความคิดของริงย่อย Γ-nLA ไอดีลทางซ้ายวิภัชนัยและไอดีลทางขวาวิภัชนัยในริง Γ-nLA และเพื่อศึกษาริงย่อย Γ-nLA ไอดีลทางซ้ายวิภัชนัยและไอดีลทางขวาวิภัชนัยในริง Γ-nLA และลักษณะเฉพาะบางประการของไอดีลทางซ้ายวิภัชนัยและไอดีลทางขวาวิภัชนัยในริง Γ-nLA ที่เกี่ยวข้องกับไอดีลทางซ้าย (ขวา) ระดับ

คำสำคัญ : Γ-nLA-ริง ไอดีล ระดับ ไอดีลทางซ้าย

Abstract

The purpose of this paper is to introduce the notion of Γ-nLA-subring, fuzzy left and fuzzy right ideals in Γ-nLA-rings, and to study Γ-nLA-subring, fuzzy left and fuzzy right ideals in Γ-nLA-rings. Some characterizations of fuzzy left (right) ideals of a Γ-nLA-ring N are related to level left (right) ideals.

Keywords : Γ-nLA-ring; ideal; level; Γ-nLA-subring

1. Introduction

Let N be a nonempty set. A **fuzzy subset** of f is, by definition, an arbitrary mapping $f: N \to [0,1]$, where $[0,1]$ is the usual interval of real numbers. Lotfi A. Zadeh [10] in 1965 introduced the notion of a fuzzy set to describe vagueness mathematically in its very abstractness and to solve such problems he gave a certain grade of membership to each member of a given set. This in fact laid the basic of fuzzy set theory. In [1], Booth studied and gave a note on Γ-near rings. The notion of fuzzy coset was introduced by Sathyanarayana [7]. In [3], Jun et al. considered the fuzzification of left (resp. right) ideals of Γ-near rings, and investigated the related properties. Jun [2] introduced the notion of fuzzy left (resp. right) ideals, normal fuzzy ideals and fuzzy maximal ideals of Γ-Near-rings, and studied some of their properties.

In 1981, the notion of Γ-semigroups was introduced by Sen [8]. A groupoid S is called a Γ**-left almost semigroup (simply a** Γ**-LA-semigroup**) if it satisfies the **left invertive** law:

$$(a\gamma b)\alpha c = (c\gamma b)\alpha a,$$

for all $a,b,c \in S$ and $\gamma, \alpha \in \Gamma$ as in Zadeh [9]. This structure is also known as a Γ-Abel-Grassmann's groupoid (Γ-LA-semigroup). In this paper, we are going to investigate some interesting properties of recently discovered classes, namely Γ-LA-semigroup always satisfies the Γ**-medial** law:

$$(a\gamma b)\alpha(c\beta d) = (d\gamma c)\alpha(b\beta a),$$

for all $a,b,c,d \in S$ and $\gamma,\alpha,\beta \in \Gamma,$ see Satyanarayana [6], while a Γ-LA-semigroup S with left identity e always satisfies Γ-paramedial law:

$$(a\gamma b)\alpha(c\beta d) = (a\gamma c)\alpha(b\beta d),$$

for all $a,b,c,d \in S$ and $\gamma,\alpha,\beta \in \Gamma.$ It was much later when Kamran [4] in 1987 succeeded in defining a non-associative group which they called an LA-group and can be equally manipulated with as a subtractive group. The introduction of a left almost group (simply an LA-group) is an offshoot of an LA-semigroup. An LA-group is a non-associative structure with interesting properties. Furthermore, in this paper we characterize the fuzzy left (right) ideals of a Γ-nLA-ring related to level left (right) ideals.

2. Preliminaries

In this section, we refer to [4, 5, 9], for some elementary aspects and quote a few definitions, and essential examples to step up this study. For more details, we refer to the papers in the references.

Definition 2.1 [5] A groupoid S is called a **left almost semigroup** (simply an **LA-semigroup**) if it satisfies the left invertive law:

$$(ab)c = (cb)a$$

for all $a,b,c \in S.$

Definition 2.2. [9] Let S and Γ be nonempty sets. Then S is said to be a Γ-**left almost semigroup** (or simply a Γ-**LA-semigroup**), if there exists a mapping

$$S \times \Gamma \times S \to S$$

(the image of (a,γ,b) is $a\gamma b$) satisfies the identity

$$(a\gamma b)\alpha c = (c\gamma b)\alpha a,$$

for all $a,b,c \in S$ and $\gamma,\alpha \in \Gamma.$

Example 2.3. [9] Let $\Gamma = \{1,2,3\}.$ Define a mapping $□ \times \Gamma \times □ \to □$ by $a\gamma b = b - \gamma - a$ for all $a,b \in □$ and $\gamma \in \Gamma$ where "$-$" is a usual subtraction of integers. Then □ is a Γ-LA-semigroup.

Definition 2.4. [4] An LA-semigroup G with the binary operation "$\cdot$" is said to be a **left almost group** (simply an **LA-group**) if the following conditions are satisfied:

(i) There exists an element $e \in G$ such that $ea = a,$ for all $a \in G.$

(ii) For $a \in G,$ there exists $a^{-1} \in G$ such that $a^{-1}a = e$ and $aa^{-1} = e.$

Definition 2.5. Let $(N,+)$ be an LA-group and Γ be a nonempty set. Then N is said to be a Γ-**near left almost ring** (or simply a Γ-**nLA-ring**), if there exists a mapping $N \times \Gamma \times N \to N$ (the image of (x,γ,y) is $x\gamma y$) satisfying the following conditions;

(1). $x\gamma(y+z) = x\gamma y + x\gamma z;$

(2). $(x\gamma y)\alpha z = (z\gamma y)\alpha x,$

for all $x,y,z \in N$ and $\gamma,\alpha \in \Gamma.$

Example 2.6. Let N be an arbitrary nLA-ring and Γ any nonempty set. Define a mapping $N \times \Gamma \times N \to N$ by $x\gamma y = xy$ for all $x, y \in N$ and $\gamma \in \Gamma$. It is easy to see that N is a Γ-nLA-ring.

Example 2.7. Let $\Gamma = \square$. Then $\square$ is a Γ-nLA-ring by defining the binary operations as; for $x, y \in \square$, $x \oplus y = y - x$ and

$$x\gamma y = \begin{cases} 0 & ; x = 0 \text{ or } y = 0 \\ y\gamma^{-1}x^{-1} & ; \text{otherwise.} \end{cases}$$

By choosing $x = 2, y = 3,$ we show that

$$(2 \cdot 3) \cdot 6 = 6 \cdot \frac{1}{3} \cdot \frac{1}{2} = 1$$

but $(6 \cdot 3) \cdot 2 = 2 \cdot \frac{1}{3} \cdot \frac{1}{6} = \frac{1}{9}$. Hence $\square$ is not an nLA-ring.

Let N be a Γ-nLA-ring. If S is a nonempty subset of N and S is itself a Γ-nLA-ring under the binary operation induced by $N,$ then S is called a **Γ-nLA-subring** of $N.$ A Γ-LA-subring I of N is called a **left ideal** of N if $N\Gamma I \subseteq I$ and I is called a **right ideal** of N if for all $m, n \in N, \gamma \in \Gamma$ and $i \in I$ such that

$$(i + m)\gamma n - m\gamma n \in I$$

and I is called an **ideal** of N if I is both a left and a right ideal of $N.$

A function f from N to the unit interval $[0,1]$ is a **fuzzy subset** of N [9]. The Γ-nLA-ring N itself is a fuzzy subset of N such that $N(x) = 1$ for all $x \in N,$ denoted by $N.$ Let f and g be two fuzzy subsets of $N.$ Then the inclusion relation $f \subseteq g$ is defined by $f(x) \leq g(x),$ for all $x \in N.$ The intersection of f and $g,$ $f \cap g$ and $f \cup g$ are fuzzy subsets of N defined by

$$(f \cap g)(x) = min\{f(x), g(x)\},$$
$$(f \cup g)(x) = max\{f(x), g(x)\}$$

for all $x \in N.$ More generally, if $\{f_\alpha : \alpha \in \beta\}$ is a family of fuzzy subsets of $N,$ then $\bigcap_{\alpha \in \beta} f_\alpha$ and $\bigcup_{\alpha \in \beta} f_\alpha$ are defined as follows:

$$\left(\bigcap_{\alpha \in \beta} f_\alpha\right)(x) = \bigcap_{\alpha \in \beta} f_\alpha(x) = inf\{f_\alpha(x) : \alpha \in \beta\},$$

$$\left(\bigcup_{\alpha \in \beta} f_\alpha\right)(x) = \bigcup_{\alpha \in \beta} f_\alpha(x) = sup\{f_\alpha(x) : \alpha \in \beta\}$$

and will be the intersection and union of the family $\{f_\alpha : \alpha \in \beta\}$ of fuzzy subsets of $N.$ The **product** $f\Gamma g$ is defined as follows;

$$(f\Gamma g)(x) = \begin{cases} \bigcup_{x = y\gamma z} min\{f(y), g(z)\} & ; \text{for some } y, z \in N, \gamma \in \Gamma, \text{such that } x = yz \\ 0 & ; \text{otherwise} \end{cases}$$

Definition 2.8. A fuzzy subset f of N is called a **fuzzy Γ-nLA-subring** of N if

$$f(x-y)\geq min\{f(x),f(y)\}$$

and

$$f(x\gamma y)\geq min\{f(x),f(y)\},$$

for all $x,y\in N$ and $\gamma\in\Gamma.$ A fuzzy Γ-nLA-subring f of a Γ-nLA-ring N is called a **fuzzy left ideal** of N if $f(x\gamma y)\geq f(y)$ for all $x,y\in N$ and $\gamma\in\Gamma.$ A **fuzzy right ideal** of N is a fuzzy Γ-nLA-subring f of N such that

$$f((x+y)\gamma z-y\gamma z)\geq f(x),$$

for all $x,y,z\in N$ and $\gamma\in\Gamma.$ A **fuzzy ideal** of N is a fuzzy Γ-nLA-subring f of N such that $f(x\gamma y)\geq f(y)$ and

$$f((x+y)\gamma z-y\gamma z)\geq f(x),$$

for all $x,y,z\in N$ and $\gamma\in\Gamma.$

3. Fuzzy Sets

The results of the following theorems seem to play an important role to study fuzzy ideals in Γ-nLA-rings; these facts will be used frequently and normally we shall make no reference to this Lemmas.

Lemma 3.1. Let N be a Γ-nLA-ring. If f,g,h are fuzzy subsets of $N,$ then

$$(f\Gamma g)\Gamma h=(h\Gamma g)\Gamma f.$$

Proof. Assume that f,g,h are fuzzy subsets of $N.$ Let $x\in N.$ Then

$$(f\Gamma g)\Gamma h(x)=\bigcup_{x=y\gamma z} min\{(f\Gamma g)(y),h(z)\}$$

$$=\bigcup_{x=y\gamma z} min\left\{\bigcup_{y=a\alpha b} min\{f(a),g(b)\},h(z)\right\}$$

$$=\bigcup_{x=(a\alpha b)\gamma z} min\{min\{f(a),g(b)\},h(z)\}$$

$$=\bigcup_{x=(z\alpha b)\gamma a} min\{min\{h(z),g(b)\},f(a)\}$$

$$\leq\bigcup_{x=(z\alpha b)\gamma a} min\left\{\bigcup_{z\alpha b=c\beta d} min\{h(c),g(d)\},f(a)\right\}$$

$$=\bigcup_{x=(z\alpha b)\gamma a} min\{(h\Gamma g)(z\alpha b),f(a)\}$$

$$=\bigcup_{x=w\gamma a} min\{(h\Gamma g)(w),f(a)\}$$

$$=(h\Gamma g)\Gamma f(x).$$

Thus $(f\Gamma g)\Gamma h\subseteq(h\Gamma g)\Gamma f.$ Similarly,

$$(h\Gamma g)\Gamma f\subseteq(f\Gamma g)\Gamma h$$

and hence $(f\Gamma g)\Gamma h=(h\Gamma g)\Gamma f.$

If N is a Γ-nLA-ring and $F(N)$ is the collection of all fuzzy subsets of $N,$ then $(F(N),\Gamma)$ is an LA-semigroup.

Lemma 3.2. Let N be a Γ-nLA-ring with left identity. If f,g,h are fuzzy subsets of $N,$ then $f\Gamma(g\Gamma h)=g\Gamma(f\Gamma h).$

Proof. Assume that f,g,h are fuzzy subsets of $N.$ Let $x\in N.$ Then

$(f\Gamma(g\Gamma h))(x)$

$$=\bigcup_{x=y\gamma z} min\{f(y),g\Gamma h(z)\}$$

$$=\bigcup_{x=y\gamma z} min\left\{f(y),\bigcup_{z=a\alpha b} min\{g(a),h(b)\}\right\}$$

$$= \bigcup_{x=y\gamma(a\alpha b)} min\{f(y), min\{g(a), h(b)\}\}$$

$$= \bigcup_{x=a\gamma(y\alpha b)} min\{g(a), min\{f(y), h(b)\}\}$$

$$\leq \bigcup_{x=a\gamma(y\alpha b)} min\left\{g(a), \bigcup_{y\alpha b=c\beta d} min\{f(c), h(d)\}\right\}$$

$$= \bigcup_{x=a\gamma(y\alpha b)} min\{g(a), f\Gamma h(y\alpha b)\}$$

$$= \bigcup_{x=a\gamma w} min\{g(a), f\Gamma h(w)\}$$

$$= (g\Gamma(f\Gamma h))(x).$$

Thus $f\Gamma(g\Gamma h) \subseteq g\Gamma(f\Gamma h)$. Similarly,

$$g\Gamma(f\Gamma h) \subseteq f\Gamma(g\Gamma h)$$

and hence $f\Gamma(g\Gamma h) = g\Gamma(f\Gamma h)$.

Theorem 3.3. Let f be a fuzzy subset of a Γ-nLA-ring N. Then f is a fuzzy Γ-nLA-subring of N if and only if $f\Gamma f \subseteq f$ and $f(x-y) \geq min\{f(x), f(y)\}$ for all $x, y \in N$.

Proof. $(\Rightarrow)$ Suppose that f is a fuzzy Γ-nLA-subring of N. Let $x \in N$ and $\gamma \in \Gamma$. If $f\Gamma f(x) = 0$, then $f\Gamma f \subseteq f$. Otherwise

$$f\Gamma f(x) = \bigcup_{x=y\gamma z} min\{f(y), f(z)\}$$

$$\leq \bigcup_{x=y\gamma z} f(y\gamma z)$$

$$= f(x).$$

Thus $f\Gamma f \subseteq f$.

$(\Leftarrow)$ Assume that $f\Gamma f \subseteq f$ and

$$f(x-y) \geq min\{f(x), f(y)\},$$

for all $x, y \in N$. Let $x, y \in N$ and $\gamma \in \Gamma$. Then

$$f(x\gamma y) \geq f\Gamma f(x\gamma y)$$

$$= \bigcup_{x\gamma y=a\alpha b} min\{f(a), f(b)\}$$

$$\geq min\{f(x), f(y)\}.$$

This implies that f is a fuzzy Γ-nLA-subring of N.

Theorem 3.4. Let f be a fuzzy Γ-nLA-subring of a Γ-nLA-ring N. Then f is a fuzzy left ideal of N if and only if $N\Gamma f \subseteq f$.

Proof. $(\Rightarrow)$ Suppose that f is a fuzzy left ideal of N. Let $x \in N$ and $\gamma \in \Gamma$. If $N\Gamma f(x) = 0 \leq f(x)$, then $N\Gamma f \subseteq f$. Otherwise

$$N\Gamma f(x)$$

$$= \bigcup_{x=y\gamma z} min\{N(y), f(z)\}$$

$$= \bigcup_{x=y\gamma z} min\{1, f(z)\}$$

$$= \bigcup_{x=y\gamma z} f(z)$$

$$\leq \bigcup_{x=y\gamma z} f(y\gamma z)$$

$$= f(x).$$

Thus $N\Gamma f \subseteq f$.

$(\Leftarrow)$ Suppose that $N\Gamma f \subseteq f$. Let $x \in N$ and $\gamma \in \Gamma$. Then

$$f(x\gamma y) \geq N\Gamma f(x\gamma y)$$

$$= \bigcup_{x\gamma y=a\alpha b} min\{N(a), f(b)\}$$

$$\geq min\{N(a), f(y)\}$$

$$= min\{1, f(y)\}$$

$$= f(y).$$

This implies that f is a fuzzy left ideal of N.

Theorem 3.5. Let N be a Γ-nLA-ring with left identity N. Then $N\Gamma N = N$.
Proof. Let $x \in N$. Then

$$\begin{aligned} N\Gamma N(x) &= \bigcup_{x=y\gamma z} min\{N(y), N(z)\} \\ &= \bigcup_{x=y\gamma z} min\{1,1\} \\ &= 1 \\ &= N(x). \end{aligned}$$

This implies that $N\Gamma N = N$.

Theorem 3.6. Let I be a nonempty subset of a Γ-nLA-ring N and $f_I : N \to [0,1]$ be a fuzzy subset of N such that

$$f_I(x) = \begin{cases} 1 & ; x \in I \\ 0 & ; \text{otherwise.} \end{cases}$$

Then I is a Γ-nLA-subring of N if and only if f_I is a fuzzy Γ-nLA-subring of N.
Proof. $(\Rightarrow)$ Suppose that I is a Γ-nLA-subring of N. Let $x, y \in N$ and $\gamma \in \Gamma$. If $x \notin I$ or $y \notin I$, then $f_I(x) = 0$ or $f_I(y) = 0$ so that

$$f_I(x\gamma y) \geq 0 = min\{f_I(x), f_I(y)\}$$

and

$$f_I(x-y) \geq 0 = min\{f_I(x), f_I(y)\}.$$

If $x, y \in I$, then $f_I(x) = 1$ and $f_I(y) = 1$ so that

$$f_I(x\gamma y) = 1 = min\{f_I(x), f_I(y)\}$$

and

$$f_I(x-y) = 1 = min\{f_I(x), f_I(y)\}.$$

Therefore f_I is a fuzzy Γ-nLA-subring of N.

$(\Leftarrow)$ Assume that f_I is a fuzzy Γ-nLA-subring of N. Let $x, y \in I$ and $\gamma \in \Gamma$. Since

$$\begin{aligned} f_I(x\gamma y) &\geq min\{f_I(x), f_I(y)\} \\ &= \{1,1\} \\ &= 1 \end{aligned}$$

and

$$\begin{aligned} f_I(x-y) &\geq min\{f_I(x), f_I(y)\} \\ &= \{1,1\} \\ &= 1 \end{aligned}$$

this implies $f_I(x\gamma y) = 1$ and $f_I(x-y) = 1$. By the assumption, $x\gamma y, x-y \in I$. Hence I is a Γ-nLA-subring of N.

Theorem 3.7. Let I be a nonempty subset of a Γ-nLA-ring N and $f_I : N \to [0,1]$ be a fuzzy subset of N such that

$$f_I(x) = \begin{cases} 1 & ; x \in I \\ 0 & ; \text{otherwise.} \end{cases}$$

Then I is a left ideal of N if and only if f_I is a fuzzy left ideal of N.
Proof. $(\Rightarrow)$ Suppose I is a left ideal of N. By Theorem 3.6, we get f_I is a fuzzy Γ-nLA-subring of N. Let $x, y \in N$ and $\gamma \in \Gamma$. If $y \notin I$, then $f_I(y) = 0$ so that $f_I(x\gamma y) \geq 0 = f_I(y)$. If $y \in I$, then $x\gamma y \in I$ since I is a left ideal of N. This implies that $f_I(x\gamma y) = 1 = f_I(y)$. Thus f_I is a fuzzy left ideal of N.

($\Leftarrow$) Assume that f_I is a fuzzy left ideal of N. By Theorem 3.6, we get I is a Γ-nLA-subring of N. Let $r \in N, \gamma \in \Gamma$ and $x \in I$. Since $f_I(r\gamma x) \geq f_I(x) = 1$ and $f_I(r\gamma x) \in [0,1]$,
we get $f_I(r\gamma x) = 1$. This implies that $r\gamma x \in I$ and hence I is a left ideal of N.

Theorem 3.8. Let I be a nonempty subset of a Γ-nLA-ring N and $f_I : N \to [0,1]$ be a fuzzy subset of N such that

$$f_I(x) = \begin{cases} 1 & ; x \in I \\ 0 & ; \text{otherwise.} \end{cases}$$

Then I is a right ideal of N if and only if f_I is a fuzzy right ideal of N.

Proof. ($\Rightarrow$) Suppose I is a right ideal of N. By Theorem 3.6, we get f_I is a fuzzy Γ-nLA-subring of N. Let $x, y, z \in N$ and $\gamma \in \Gamma$. If $x \notin I$, then $f_I(x) = 0$ so that

$$f_I((x+y)\gamma z - y\gamma z) \geq 0 = f_I(x).$$

If $x \in I$, then $(x+y)\gamma z - y\gamma z \in I$ since I is a right ideal of N. This implies that

$$f_I((x+y)\gamma z - y\gamma z) = 1 = f_I(x).$$

Thus f_I is a fuzzy right ideal of N.

($\Leftarrow$) Assume that f_I is a fuzzy right ideal of N. By Theorem 3.6, we get I is a Γ-nLA-subring of N. Let $y, z \in N, \gamma \in \Gamma$ and $x \in I$. Since $f_I((x+y)\gamma z - y\gamma z) \geq f_I(x) = 1$ and

$$f_I((x+y)\gamma z - y\gamma z) \in [0,1],$$

we get $f_I((x+y)\gamma z - y\gamma z) = 1$. This implies that $(x+y)\gamma z - y\gamma z \in I$ and hence I is a right ideal of N..

Theorem 3.9. Let I be a nonempty subset of a Γ-nLA-ring N and $f_I : N \to [0,1]$ be a fuzzy subset of N such that

$$f_I(x) = \begin{cases} 1 & ; x \in I \\ 0 & ; \text{otherwise.} \end{cases}$$

Then I is an ideal of N if and only if f_I is a fuzzy ideal of N.

Proof. It is straightforward by Theorem 3.7 and Theorem 3.8.

Theorem 3.10. Let I be a nonempty subset of a Γ-nLA-ring $N, m \in (0,1]$ and f_I be a fuzzy set of N such that

$$f_I(x) = \begin{cases} m & ; x \in I \\ 0 & ; \text{otherwise.} \end{cases}$$

Then the following properties hold.

(1). I is a Γ-nLA-subring of N if and only if f_I is a fuzzy Γ-nLA-subring of N.

(2). I is a left ideal of N if and only if f_I is a fuzzy left ideal of N.

(3). I is a right ideal of N if and only if f_I is a fuzzy right ideal of N.

(4). I is an ideal of N if and only if f_I is a fuzzy ideal of N.

Proof. It is straightforward by Theorem 3.9.

Definition 3.11. Let f be a fuzzy subset of a Γ-nLA-ring N and $t \in (0,1]$. Then the set

$$U(f,t) := \{x \in N : f(x) \geq t\}$$

is called the **level set** of f.

Lemma 3.12. Let f be a fuzzy subset of a Γ-nLA-ring N. Then f is a fuzzy Γ-nLA-subring of N if and only if $U(f,t) \neq \varnothing$ is a Γ-nLA-subring of N, for all $t \in (0,1]$.

Proof. It is straightforward by Theorem 3.10.

Theorem 3.13. Let f be a fuzzy subset of a Γ-nLA-ring N. Then f is a fuzzy left ideal (right ideal, ideal) of N if and only if $U(f,t) \neq \varnothing$ is a left ideal (right ideal, ideal) of N, for all $t \in (0,1]$.

Proof. It is straightforward by Theorem 3.10.

Lemma 3.14. Let f be a fuzzy Γ-nLA-subring of a Γ-nLA-ring N. Then $f(0) \geq f(x)$, for all $x \in N$.

Proof. Let $x \in N$. Since f is a fuzzy Γ-nLA-subring of N, we get

$$\begin{aligned} f(0) &= f(x-x) \\ &\geq min\{f(x), f(x)\} \\ &= f(x). \end{aligned}$$

This implies that $f(0) \geq f(x)$, for all $x \in N$.

Theorem 3.15. Let f be a fuzzy subset of a Γ-nLA-ring N. If f is a fuzzy Γ-nLA-subring of N, then $N_f = \{x \in N : f(x) = 0\}$ is a Γ-nLA-subring of N.

Proof. Assume that f is a fuzzy Γ-nLA-subring of N. Let $x, y \in N_f$ and $\gamma \in \Gamma$. Then

$$f(x) = f(0)$$

and $f(y) = f(0)$ so that

$$\begin{aligned} f(x-y) &\geq min\{f(x), f(y)\} \\ &= min\{f(0), f(0)\} \\ &= f(0) \end{aligned}$$

and

$$\begin{aligned} f(x\gamma y) &\geq min\{f(x), f(y)\} \\ &= min\{f(0), f(0)\} \\ &= f(0). \end{aligned}$$

By Lemma 3.14, we get $f(x-y) = f(0)$ and $f(x\gamma y) = f(0)$ which implies that

$$x-y, x\gamma y \in N_f.$$

Hence N_f is a Γ-nLA-subring of N.

Theorem 3.16. Let f be a fuzzy subset of a Γ-nLA-ring N. If f is a fuzzy left ideal of N, then $N_f = \{x \in N : f(x) = 0\}$ is a left ideal of N.

Proof. Assume that f is a fuzzy left ideal of N. By Theorem 3.15, we get N_f is a Γ-nLA-subring of N. Let $r \in N, \gamma \in \Gamma$ and $x \in N_f$. Then

$$f(x) = f(0)$$

so that $f(r\gamma x) \geq f(x) = f(0)$. By Lemma 3.14 and $f(r\gamma x) \in [0,1]$, we get $f(r\gamma x) = f(0)$ which implies that $r\gamma x \in N_f$. Hence N_f is a left ideal of N.

Theorem 3.17. Let f be a fuzzy subset of a Γ-nLA-ring N. If f is a fuzzy right ideal of N, then $N_f = \{x \in N : f(x) = 0\}$ is a right ideal of N.

Proof. Assume that f is a fuzzy right ideal of N. By Theorem 3.15, we get N_f is a Γ-nLA-subring of N. Let $y,z\in N,\gamma\in\Gamma$ and $x\in N_f$. Then $f(x)=f(0)$ so that

$$f((x+y)\gamma z-y\gamma z)\geq f(x)=f(0).$$

By Lemma 3.14, we get

$$f((x+y)\gamma z-y\gamma z)=f(0)$$

which implies that $(x+y)\gamma z-y\gamma z\in N_f$. Hence N_f is a right ideal of N.

Theorem 3.18. Let f be a fuzzy subset of a Γ-nLA-ring N. If f is a fuzzy ideal of N, then $N_f=\{x\in N: f(x)=0\}$ is an ideal of N.

Proof. It is straightforward by Theorem 3.16 and Theorem 3.17.

4. Conclusions and Discussion

Many new classes of fuzzy subsets in Γ-nLA-rings have been discovered recently. All these have attracted researchers of the field to investigate these newly discovered classes in detail. This article investigates the Γ-nLA-subring, fuzzy left and fuzzy right ideals in Γ-nLA-rings. Some characterizations of fuzzy left (right) ideals of a Γ-nLA-ring N. related to level left (right) ideals.

5. References

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