RESEARCH ARTICLES

J. Sci. Soc. Thailand, 10 (1984) 21 - 35

GRAVITY OVERRIDE OF STEAM FRONT DISPLACING WATER IN POROUS MEDIA

B. PHONGDARA
Department of Physics, Prince of Songkla University, Haad Yai, Thailand

(Received 31 October 1983)

Abstract

The report describes laboratory results of gravity override of steam diplacing water in horizontal porous media. New formulae describing the slopes of interfaces have been derived for completely filled and partially filled cases. For the completely filled case, the present approach, which uses a simple, one dimensional flow equation, shows a better time-dependence fit than the existing theory. It is identical to the existing theory at later times. For the partially filled case, the present theory shows an excellent fit to experimental results.

Introduction

In his investigation of displacement of oil by water under segregated flow, Dietz[1] found conditions for stable and unstable displacement. The condition for stable displacement is that the angle between the fluids interface and the direction of flow should remain constant throughout the displacement. By assuming that there is no pressure drop across the interface, that the fluids are incompressible and that permeability is constant throughout the formation, Dietz formulated a critical flow rate below which the displacement is stable and above which it is unstable.

Van Lookeren[2] used the same idea as Dietz to study the shape of the growing steam zone when steam is injected into a reservoir to displace oil. According to van Lookeren's theory, the shape of the steam zone is mainly controlled by one group of parameters, which includes steam injection rate, pressure and effective formation permeability to steam. Condensation of steam is the main difference between van Lookeren's theory and Dietz's theory. Due to condensation, the interface between steam and liquid is more vertical when flow rate (steam injection rate) is higher.

Even though van Lookeren mentioned that heat loss to the upperburden and lowerburden promotes more gravity override, he did not incorporate heat loss into the equation.

Rhee,[3] and Rhee and Doscher[4] used van Lookeren's theory to calculate the shape of the steam-oil interface, taking into account the effect of heat loss to the upperburden and lowerburden by using Marx-Langenheim's theory [5].

Effects of condensation in fluid displacement were discussed thoroughly by Miller[6] as he studied the stability of the interface of steam displacing water when gravity override is omitted.

Theory

Van Lookeren Theory[2]

van Lookeren assumed that the flow potentials of each phase are constant in the plane perpendicular to the bedding plane and that the fluid velocity in each phase is constant if there is no heat loss. He the derived the equation of the slope of the interface (θ) between steam and oil for a regular steam consumption distribution in the area where steam-liquid interface is present (see Fig.1)

$$\tan(\theta) = -A_L(1 - M^*) \tag{1}$$

$$\text{where } A_L = \frac{\mu_1 q_1(X_b)}{(\rho_0 - \rho_1) g w K_1 \rho_1 b}$$

$$M^* = \frac{\mu_0 K_1 \rho_1 q_0(X_e)}{\mu_1 K_0 \rho_0 q_1(X_b)}$$

μ_1 = steam viscosity (N-s-m^{-2})
q_1 = steam flow rate (kg-s^{-1})
ρ_1 = steam density (kg-m^{3})
K_1 = steam permeability (darcy)
μ_0 = oil viscosity (N-s-m^{-2})
q_0 = oil flow rate (kg-s^{-1})
ρ_0 = oil density (kg-m^{-3})
K_0 = oil permeability (darcy)
g = gravitational acceleration
v = reservoir width (m)
w = reservoir height (m)
X_b and X_e are as in Fig.1

Eq.1 may be written as

$$\tan(\theta) = \frac{\mu_1 V_1}{K_1 g(\rho_0 - \rho_1)} - \frac{\mu_0 V_0}{K_0 g(\rho_0 - \rho_1)} \quad (2)$$

where V_1 = steam superficial velocity (m-s^{-1})
V_0 = oil superficial velocity (m-s^{-1})

Slope of the interface from Eq.2 decreases as the interface moves away from the injection end since $q_1(X_b)$ decreases due to heat loss to the cap rock and base rock.

Rhee, S.W. and Doscher, T.M.[4] used Marx-Langenheim[5] theory to add the effect of heat loss through the cap rock and base rock to van Lookeren theory.[2] Since the present work does not involve heat loss, details of Rhee and Doscher theory will not be discussed here.

Miller Theory[4]

Miller studied the stability of the interface between steam and water when steam displaces water. He assumed that (1) there is no heat loss through the base rock and cap rock, (2) no gravity override - this means the interface is vertical - and (3) there is no water saturation on the steam side. Miller then obtained these relationships (see Fig.2)

$$V = V_1 \frac{\rho_1(H_1 - H_{2\infty})}{\varepsilon\rho_1(H_1 - H_{2\infty}) + (1 - \varepsilon)\rho_s C_{ps}(T_1 - T_{2\infty})} \quad (3)$$

$$V_2 = V_1 \frac{\varepsilon\rho_1(H_1 - H_{2\infty}) + \frac{\rho_1}{\rho_2}(1 - \varepsilon)\rho_s C_{ps}(T_1 - T_1 - T_{2\infty})}{\varepsilon\rho_1(H_1 - H_{2\infty}) + (1 - \varepsilon)\rho_s C_{ps}(T_1 - T_{2\infty})} \quad (4)$$

where S_2 = water saturation
ε = reservoir porosity
V = velocity of the interface (m-s^{-1})
V_2 = water superficial velocity (m-s^{1})
T_1 = steam temperature (°C)
$T_{2\infty}$ = water temperature at large distances from the front (°C)
ρ_2 = water density (kg-m^{-3})
ρ_s = rock density (kg-m^{-3})
C_{ps} = rock specific heat (J-kg^{-1} °C^{-1})
H_1 = steam specific enthalpy (J-kg^{-1})
$H_{2\infty}$ = water specific enthalpy at large distances from the front (J-kg^{-1})

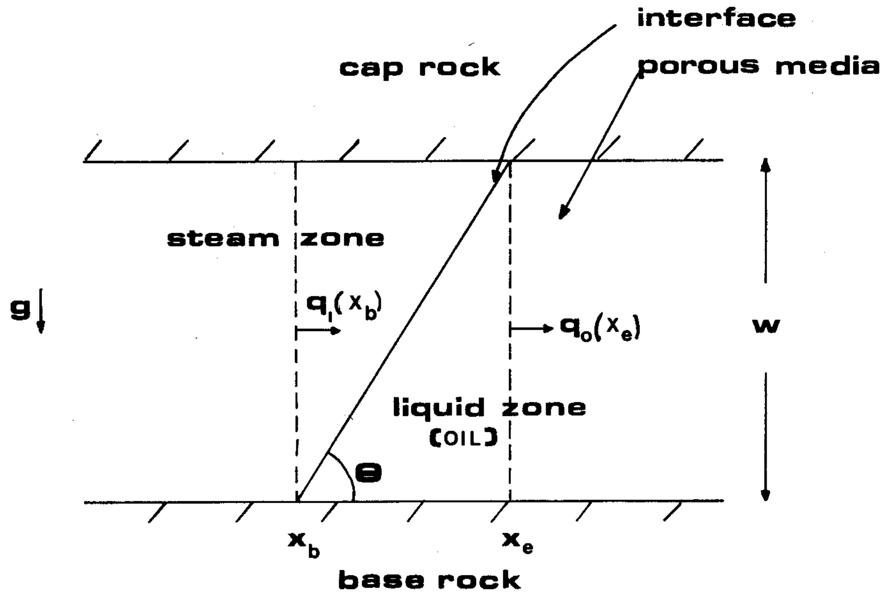

Figure 1 Diagram of steam displacing oil in a horizontal reservoir (van Lookeren).

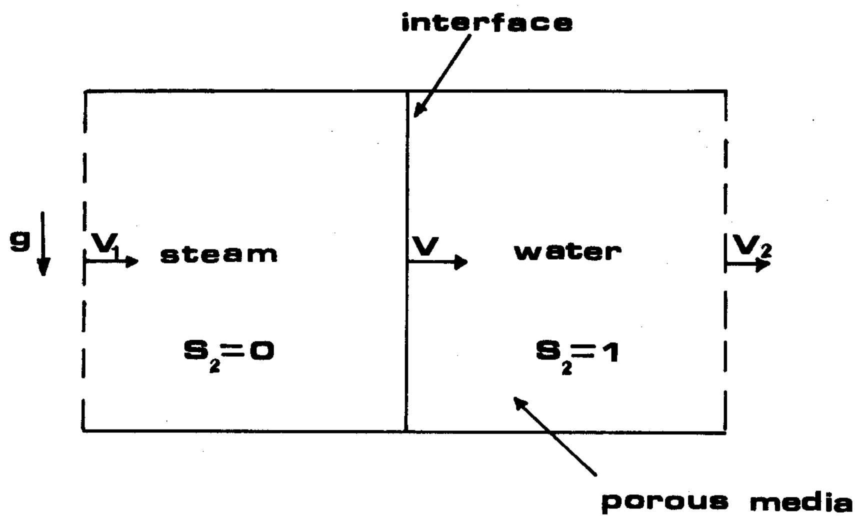

Figure 2 Diagram of steam displacing water with no gravity override (Miller).

In these experiments, V_2, water superficial velocity, was controlled. V and V_1 were calculated using Eqs.3 and 4. Some parameters and conditions were adjusted to fit the experimental set up.

The New Approach

Assumptions:

The results of the experiment show that it is a regular steam-consumption distribution case since the interface has the shape of a plane. Because the experiments in all cases take only a short time to run, it is assumed that the amount of condensed water on the steam side is negligible. This makes it a single phase flow: the water permeability and the steam permeability are then set to be equal. Since the rate of redistribution toward a acapillary gravitational equilibrium configuration within a normal column of fluids is hight relative to the rate at which the interface advances towards the water side, a one dimensional flow is then assumed [1,2]. Also for mathematical simplicity, a sharp interface is assumed. Steam pressure is assumed constant regardless of height due to a small steam density.

The new approach is divided into two cases. Case one is when the pressures at both ends of the reservoir are kept constant, which is the normal condition for a real reservoir. The second case is when the water output flow rate is kept constant (this results in an uncontrolled steam inflow rate). The experiment reported here was run according to the second case. It should be noticed that the new approach is based on steam displacing water in porous media.

Constant Pressure Case

Steam pressure at the inlet (Z = O) is kept constant at P_1 (independent of y). Water pressure at the outlet (Z = L) is also kept constant though its value depends on y due to the gravitational head of water (see Fig.3)

According to Darcy's law,[7] average steam superficial velocity along the dotted line is

$$V_1(y,t) = \frac{K}{\mu_1} \frac{P_1 - P_2(y,t)}{\eta(y,t)} \qquad (5)$$

and the average water superficial velocity is

$$V_2(y,t) = \frac{K}{\mu_2} \frac{P_2(y,t) - P_c - P_3(0) - \rho_2 g y}{L - \eta(y,t)} \qquad (6)$$

where P_c = capillary pressure across the interface
K = reservoir permeability
L = reservoir length
$\eta(y,t)$ = interface location

From Eqs.3,4,5,6 and from $\eta(y, 0) = \eta_0$, the following is obtained:

$$a\eta^2 + L\alpha_2\eta - (C + \frac{K}{\mu_2}(P_1 - P_c - P_3(0) + \rho_2 gy)t) = 0 \tag{7}$$

where
$$\begin{aligned} \alpha_1 &= V_1/V \\ \alpha_2 &= V_2/V \\ V &= \frac{\partial\eta^2}{\partial t} \\ C &= L\alpha_2\eta_0 + a\eta_0^2 \\ a &= \frac{\alpha_1}{2}\frac{\mu_1}{\mu_2} - \frac{\alpha_2}{2} \end{aligned} \tag{8}$$

For constant pressure case the value of n_0 is set as $n_0 = 0$, thus

$$V(y,t) = \frac{\partial\eta}{\partial t} = \frac{C_1 K}{(C_2^2 L^2 + 2KC_1(C_3 - C_2)t)^{1/2}} \tag{9}$$

where
$$\begin{aligned} C_1 &= P_1 - P_3(0) + \rho_2 gy \\ C_2 &= \alpha_2\mu_2 \\ C_3 &= \alpha_1\mu_1 \end{aligned} \tag{10}$$

Differentiating Eq.7 w.r.t.y, and then setting $n = Vt$,

$$\tan(\theta) = \frac{\partial y}{\partial\eta} = \frac{(2aVt + \alpha_2 L)\mu_2}{K\rho_2 gt} \tag{11}$$

At large t, Eq.11 becomes

$$\tan(\theta) \cong \frac{1}{K\rho_2 g}\left[V_1\mu_1 - V_2\mu_2\right] \tag{12}$$

It should be noted that van Lookeren's theory (Eq.2) gives the same results as Eq.12 if it is applied to the "steam displacing water" case and steam density (ρ_1) is ignored.

Constant Output Flow Rate Case

Since in this case water output flow rate is kept constant, the pressures at both ends of the cell are not fixed (from now on the word "cell" is used instead of "reservoir" to make it relevant to the cells used in the experiment). Instead of Eq.7, Eq.13 is obtained:

$$a\bar{\eta}^2 + L\alpha_2\eta - C = \frac{K}{\mu_2} b(t) + \frac{K}{\mu_2}\rho_2 gyt \qquad (13)$$

where $b(t) = \int (P_1 - P_c - P_3(0))dt$

It should be noted that b(t) is not known since P_1 and $P_3(0)$ are not fixed. In these experiments, the water output flow rate, Q, is kept constant. Therefore

$$Q = D\int_{-\frac{w}{2}}^{\frac{w}{2}} V_2 dy = \alpha_2 D \frac{\partial}{\partial t}\int_{-\frac{w}{2}}^{\frac{w}{2}} \eta(y,t)dy \qquad (14)$$

where D is the thickness of the cell.

Solving Eqs.13 and 14

$$(1 + U_G \bar{V})^{3/2} - (1 + U_G - \bar{V})^{3/2} = \{A\bar{V} + 3\sqrt{1 + U_G^o}\}\bar{V} \qquad (15)$$

where $U_G = \frac{4a}{(\alpha_2 L)^2}\left(\frac{K}{\mu_2} b(t) + L\alpha_2\eta_0 + a\eta_0^2\right)$

$$\bar{V}(t) = \frac{2aK\rho_2 gwt}{\mu_2(\alpha_2 L)^2} \qquad (16)$$

$$A = \frac{3QL\mu_2}{w^2K\rho_2 g}\alpha$$

$$U_G^0 = U_G(t = 0)$$

Once U_G is known, the location of the interface may be found from

$$\eta = \frac{\alpha_2 L}{2a}\left(-1 + \left(1 + U_G + \frac{2\overline{V}y}{w}\right)^{1/2}\right) \tag{17}$$

Values of U_G can be found by iteration technique once other parameters are known.

Partially Filled Case

This is the case when the cell is partially filled with water at the start of the experiment (see Fig.4). Experimental results show an insignificant effect on θ due to the presence of nitrogen.

Since the width of the cells used in the experiments is very small compared to their height and length, it is assumed to be a two dimensional system. Applying Darcy's law on an infinitesimal portion of water at A (see Fig.4) with the assumption that the water superficial velocity in y direction is small,

$$V_Z = -\frac{K}{\mu_2}\frac{P}{Z} \tag{18}$$

$$\frac{K}{\mu_2}\left(\frac{\partial P}{\partial y}\rho_2 g\right) \simeq 0 \tag{19}$$

where V_Z is water superficial velocity in Z direction.

$$\text{Since } dP = \frac{\partial P}{\partial Z}dZ + \frac{\partial P}{\partial y}dy \tag{20}$$

$$\text{Since } dP = -\frac{\mu_2 V_Z}{K}dZ + \rho_2 g dy \tag{21}$$

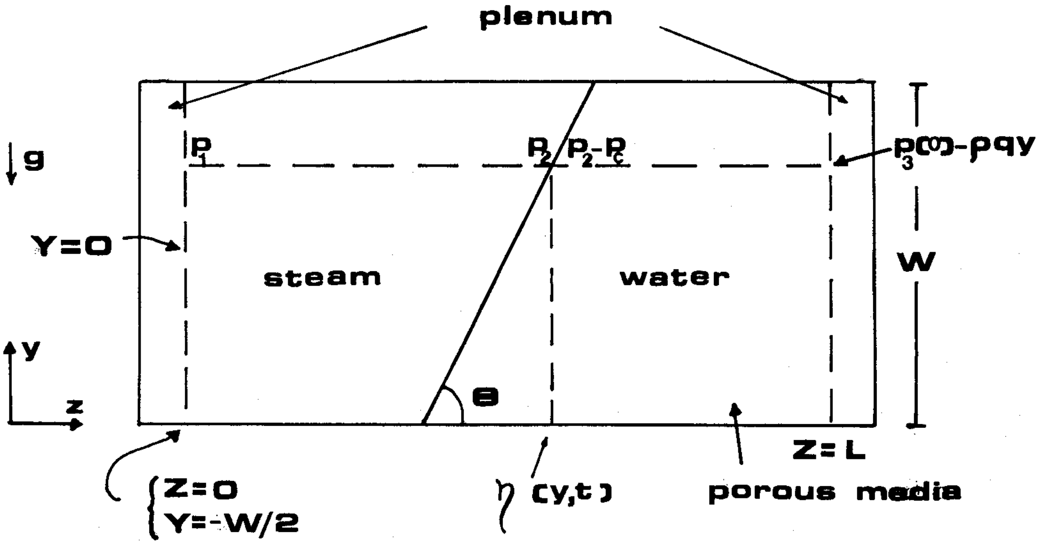

Figure 3 Diagram of steam displacing water in a cell with gravity override effect (filled case).

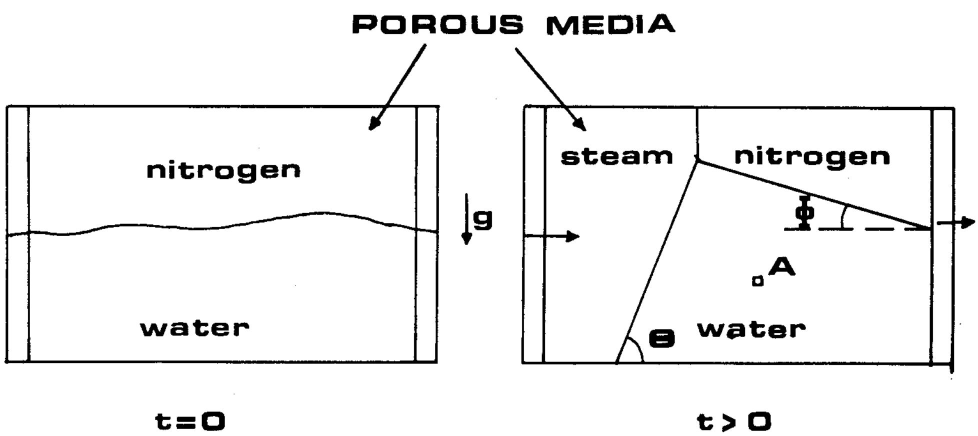

Figure 4 Diagram of steam displacing water in a cell with gravity override effect (partially filled case).

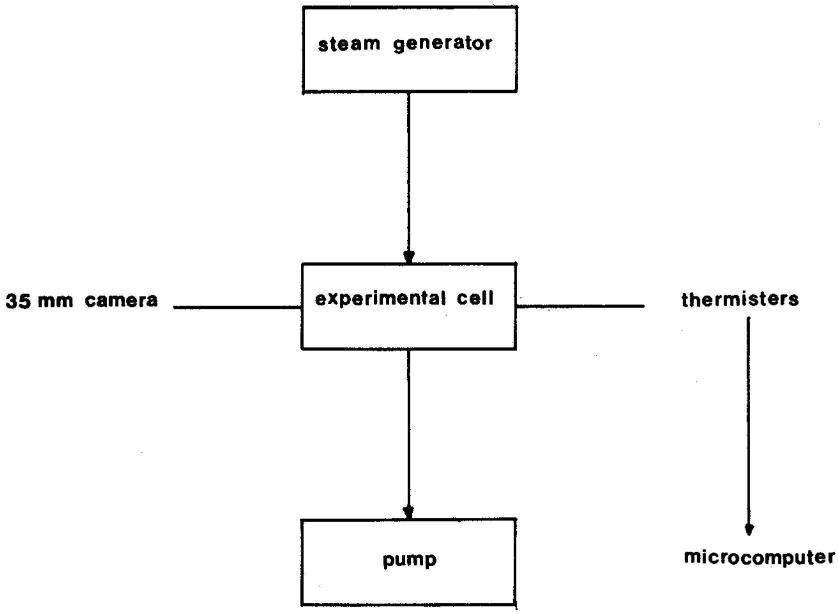

Figure 5 Diagram of experimental set-up.

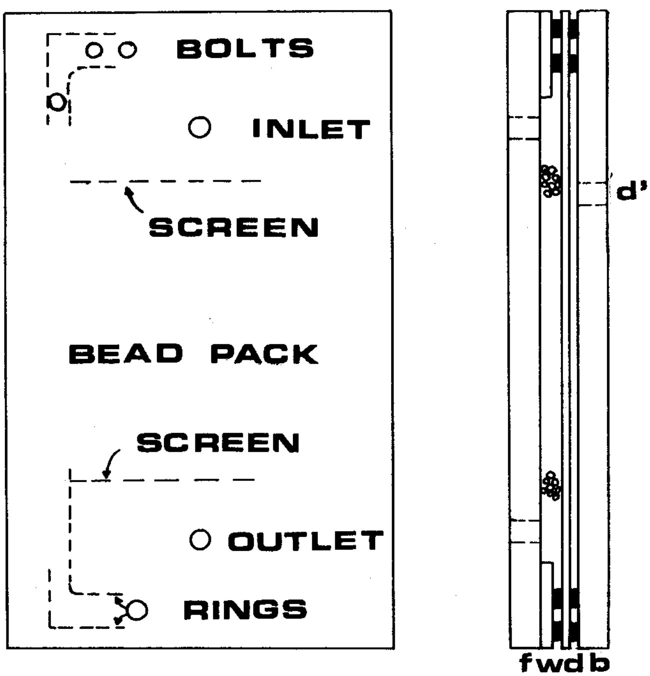

Figure 6 Diagram of experimental cell; f and b are 1/2 inch plexiglass; w is a 1/4 inch plexiglass spacer; d is a 1/16 inch plexiglass diaphragm. An overpressure of about 1 psi applied to the diaphragm through d'.

Applying Eq.21 along the interface between nitrogen and water,

$$\rho_2 g dy - \frac{\mu_2 V_Z dZ}{K} \simeq 0$$

$$\text{then} \quad \tan(\Phi) = \frac{dy}{dZ} \simeq -\frac{\mu_2 V_Z}{K\rho_2 g} \qquad (22)$$

Apparatus and Data Collection

A diagram of the experiment is shown in Fig.5. The outflow rate is controlled by setting the constant volume displacement pump (Fluid Metering, Inc.lab pump, Model RPD, 0 to 450 cm/min). Since the effect of gravity override can be observed at the interface, it is best to study first the case in which the interface moves at a constant velocity. By fixing the outflow, the velocity of the interface is maintained at a constant value. Thus, the inflow must increase just enough to supply the heat to the interface and also to supply the increasing heat loss through the walls of the cells. This process occurs automatically with our arrangement.

The Hele-Shaw cell is shown in Fig.6. Typically, the bead bed measures 40 cm × 20 cm × 0.64 cm. The beads are held in place by a 1/2 inch sheet of plexiglass, a screen (openings < 0.5 mm.) and a 1/16 inch plexiglass sheet. This sheet acts as a diaphragm and with a slight pressure (1 psi), keeps the beads from shifting as the inevitable bowing of the pelxiglass occurs. A type II plexiglass which is rated for use up to 200°F is used. Permeability of the cell is 240 darcy with the bead diameter of 500-700 um. For the cells with thermisters (for temperature measurement), the thermisters are enclosed in glass beads and placed on the inner surface of the front plexiglass plate with the wires passing through the front plate. The voltages from these termisters are converted to temperature. From these temperature measurements, it is found that the visual interface and the temperature front locate at about the same place. Results of temperature profile will not be presented here.

When gravity override effect is studied, the cell is placed on it's edge so that the 20 cm dimension is vertical. A 35 mm camera is placed firmly in front of the cell so that the full picture of the cell plus a clock can be taken. Locations of the interface and time are recorded on photographs at the rate of about 3 pictures/min (depending on the outflow rate).

Results

By fixing the outflow (water), the velocity of the interface is maintained at a constant value. This is true experimentally since the plot of the heated area against time is linear. An effective value of C_{ps} is used in Miller's theory (Eq.3,4) to obtain the relationships between V_1, V_2 and V. Steam permeability (K_1) is set equal to water permeability (K_2). Average value of water viscosity is also used in the calculations. Capillary pressure across the interface (P_C) is omitted.

Results from van Lookeren's theory (with $K_1 = K_2$ and $\mu_2 = 0.65$ cp) are also compared with experimental results. An HP.85 computer is used for calculating U_G (see "Theory"). The following are the parameters used in the calculations.

$$
\begin{aligned}
K_1 &= K_2 = 240 \text{ darcy} \\
\mu_1 &= 1.2 \times 10^{-5} \text{ N-s-m}^{-2} \\
\mu_2 &= 6.5 \times 10^{-4} \text{ N-s-m}^{-2} \\
p_1 &= 0.58 \text{ kg-m}^{-3} \\
p_2 &= 1000 \text{ kg-m}^{-3} \\
g &= 9.81 \text{ m-s}^{-2} \\
\varepsilon &= 0.36 \\
H_1 &= 2.71 \times 10^{6} \text{ J-kg}^{-1} \\
H_2 &= 1.07 \times 10^{5} \text{ J-kg}^{-1} \\
\rho_s &= 2.5 \times 10^{3} \text{ kg-m}^{-3} \text{ (density of glass bends)} \\
C_{ps}\text{eff} &= 4.33C_{ps} = 3625.1 \text{ J-kg}^{-1}\,{}^{\circ}\text{C}^{-1}
\end{aligned}
$$

Figs.7 and 8 show θ (see Fig.3) as a function of time at different output flow rates. The cell was initially completely filled with water for these two experiments. Both experimental results show a relatively constant θ at later times. van Lookeren's theory shows a better fit at a higher output flow rate. It should be noted that K_1 and μ_2 are adjusted values. Until these parameters are measured experimentally, one can not say much about van Lookeren's theory. However, the present theory shows a better time-dependence on θ than that of van Lookeren, since the latter shows only a constant value of θ.

Figs.9 and 10 show Φ (see fig.4) as a function of time at different output flow rates. The cell was initially filled with water and nitrogen as shown in Fig.4. The present theory shows a good fit with experimental results. There is a lack of data at the early times; this is because Φ cannot be read until a relatively steady state has been achieved.

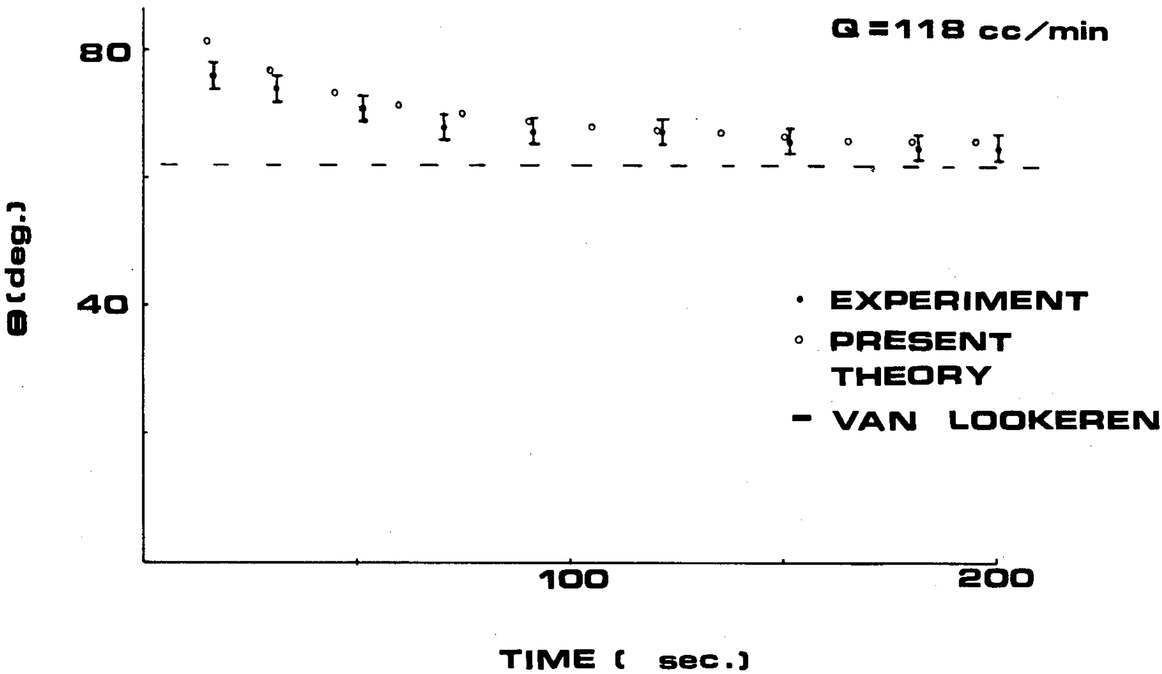

Figure 7 θ as a function of time at Q = 118 cc/min.

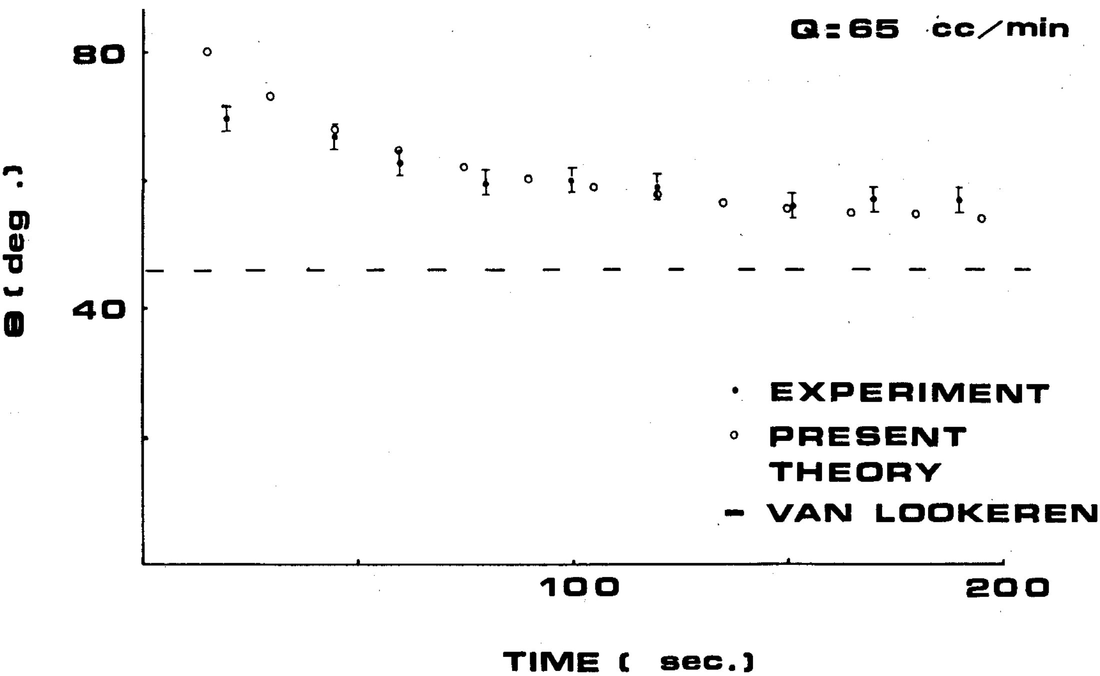

Figure 8 θ as a function of time at Q = 65 cc/min.

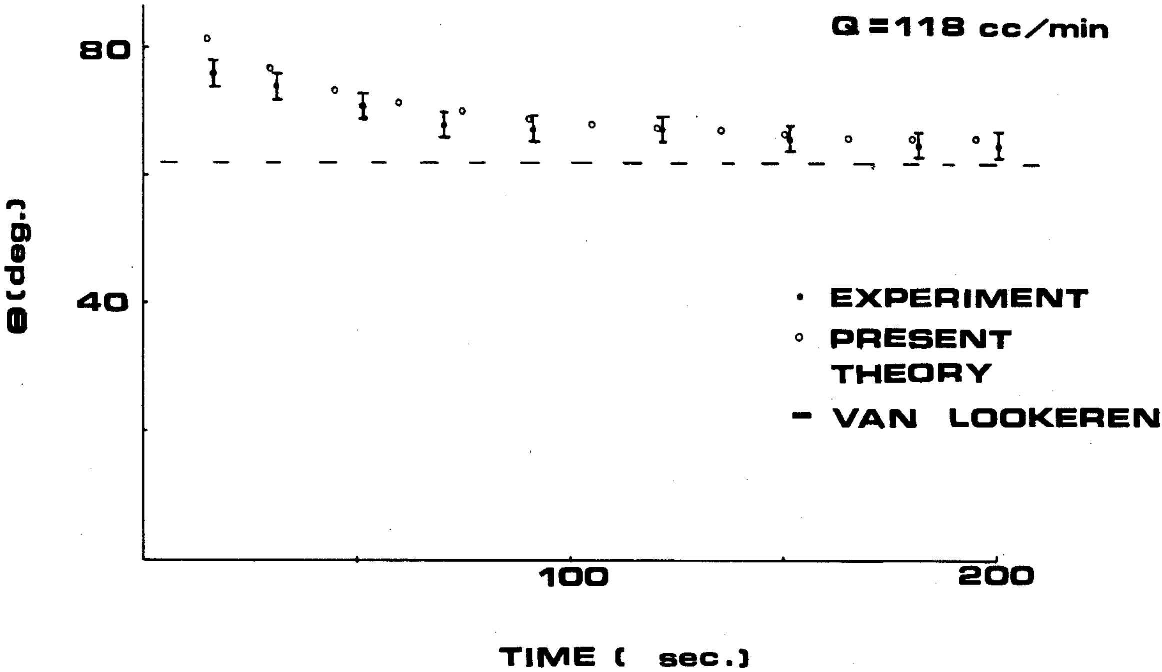

Figure 7 θ as a function of time at Q = 118 cc/min.

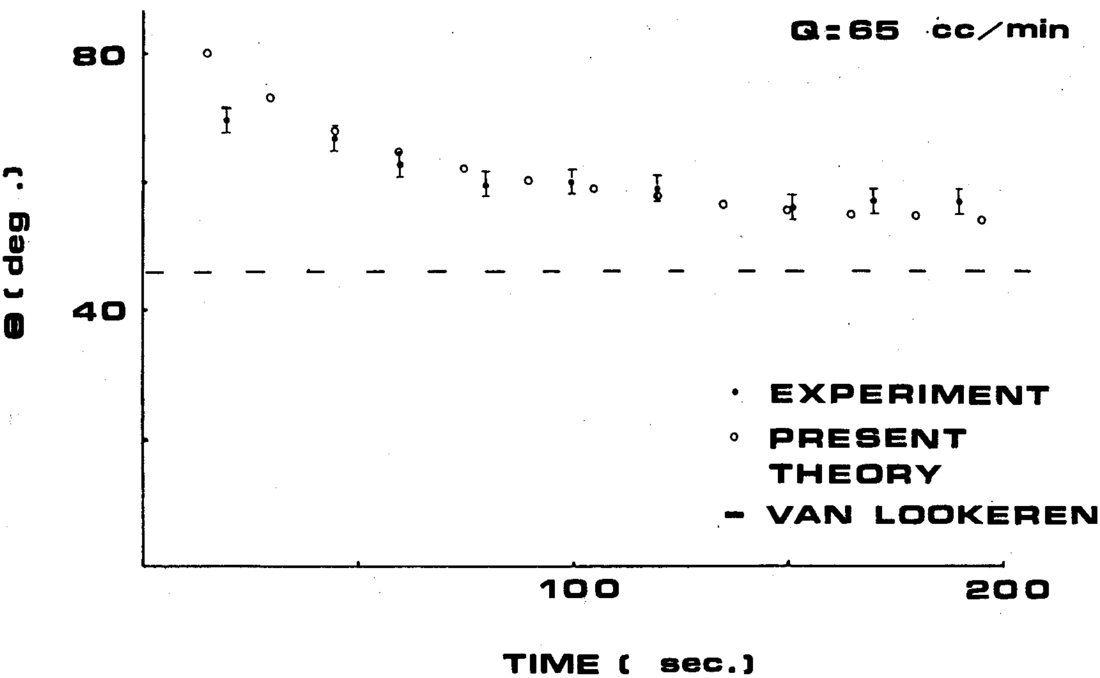

Figure 8 θ as a function of time at Q = 65 cc/min.

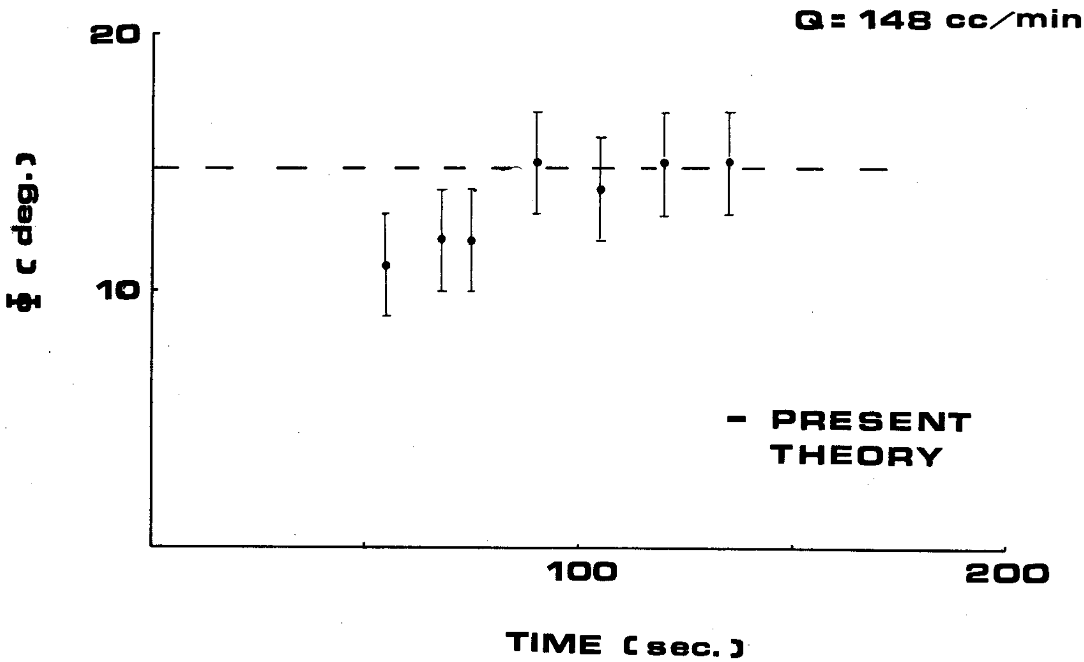

Figure 9 Φ as a function of time at Q = 148 cc/min.

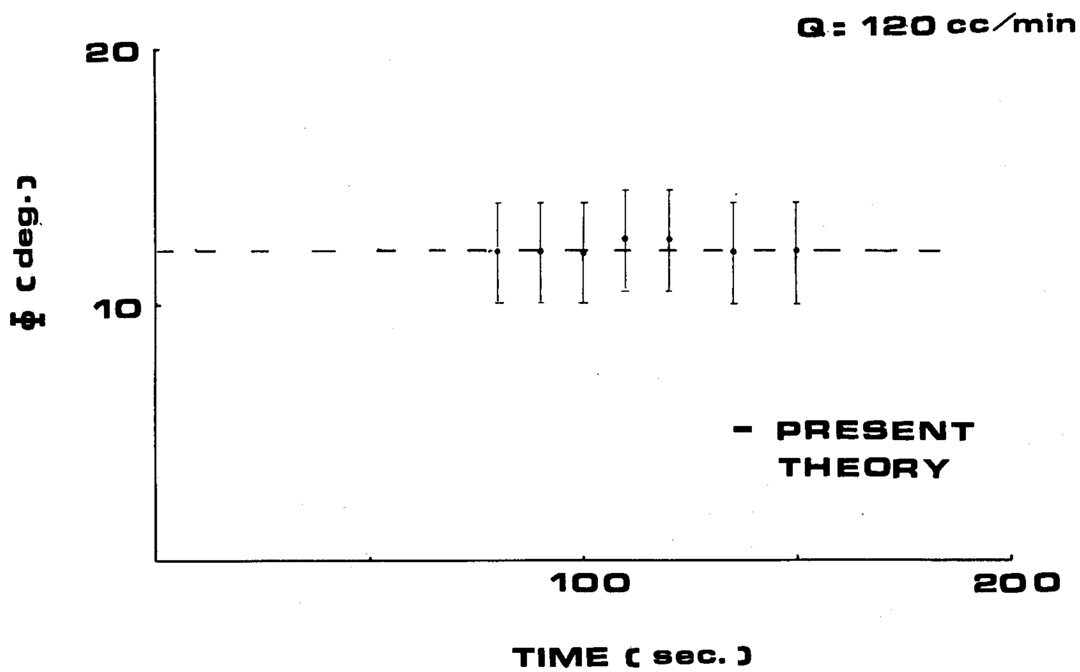

Figure 10 Φ as a function of time a Q = 120 cc/min.

Conclusion

The present theory shows a good fit with experimental results even though some unknown parameters need to be measured before a quantitative conclusion can be made. It shows a better time-dependence fit than van Lookeren's theory. The values of μ_2, K_2 and C_{ps} should be measured and the effects of permeability (K) should also be studied before the present theory might usefully be appliced.

For the calculation of Φ, the theory shows an excellent fit with experimental results. Values of V_2 and μ_2 in this theory should be measured directly.

Acknowledgements

The author wishes to thank his wife, Dr. D.M. Bachelet, for her support throughout this work. He also wishes to thank Dr. A. Geater for reading the manuscript and Mr. Prasit Intasuwanno for typing it. Part of this work was supported by a World Bank Scholarship through Prince of Songkla University.

References

1. Dietz, D.N. (1953) *Proc.Kon.Ned.Akad. Wetensch. B56,* 83
2. van Lookeren,J.(1977) *Annual Fall Meeting of SPE of AIME,* Oct 9-12, Denver, CO.
3. Rhee,S.W.(1979) *Ph.D Dissertation,* University of Southern California, Los Angeles.
4. Rhee,S.W. and Doscher,T.M.(August 1980) *Soc. Pet. Ang.J.,* 249-266.
5. Marx,J.W. and Langenheim,R.H.(1959) *Trans. AIME,* **216,** 312-315.
6. Miller,CA.(1975) *A.I.Ch.E. J.,* **21,** 474-479.
7. Scheidagger, A.E.(1957) *The Physics of Flow Through Porous Media,* Macmillan, New York.

บทคัดย่อ

บทความนี้กล่าวถึงผลของการทดลองในห้องปฏิบัติการเกี่ยวกับอิทธิพลของแรงดึงดูดของโลกต่อไอน้ำที่ใช้แทนที่น้ำในตัวกลางที่มีรูพรุนที่อยู่ในแนวระนาบ ทฤษฎีใหม่ที่กล่าวถึงความชันของรอยต่อในกรณีที่มีน้ำอยู่เต็มและกรณีที่มีน้ำอยู่เพียงบางส่วนได้ถูกสร้างขึ้น สำหรับกรณีที่มีน้ำอยู่เต็มทฤษฎีใหม่ซึ่งใช้ทฤษฎีการไหลทางเดียวแบบง่าย ๆ ได้ให้ผลซึ่งขึ้นกับเวลาได้ดีกว่าทฤษฎีซึ่งมีอยู่ก่อน ทฤษฎีใหม่นี้ให้ผลเหมือนทฤษฎีเก่าเมื่อเวลานานออกไป สำหรับกรณีที่มีน้ำอยู่เพียงบางส่วนทฤษฎีใหม่ให้ผลใกล้เคียงกับผลการทดลองมาก