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Determining and modeling terminal velocity of fruits in water

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Terminal velocity of fruits in water is a means of hydro-sorting of them. In this study, the terminal velocity of fruits in water was theoretically formulated and then determined experimentally using water column. Some effective characters of two varieties of apple and Hayward kiwi fruit on terminal velocity were determined using standard methods. The effective of fore fruit characteristics on terminal velocity was investigated. The best models for terminal velocity of studied fruits were obtained using SPSS, 13, software. It was concluded that on online sorting systems; terminal velocity has potential to remove poor quality fruit from fruits.

Key words: terminal velocity, kiwi, apple, hydro-sorting

Introduction

As world markets for fruit and produce become more sophisticated and technology continues to provide product quality measurement, there is a corresponding market pull for produce with higher, or at least specified, quality levels. While fruit graders that employ near-infrared technologies are becoming more prevalent, they are expensive, and, perhaps more importantly, the calibrations and maintenance they require tend to remain outside the skills of pack house staff (Jordan and Clark, 2004). Density is a good indicator of fruit dry matter (Richardson *et al*., 1997; Jordan *et al*., 2000) thus becomes an interesting tool for fruit quality sorting because of its inherently lower cost and simpler operation. Some products (e.g., citrus, blueberries, and tomatoes) have also been sorted by flotation techniques for quality or defects (Perry and Perkins, 1968; Gutterman, 1976; Patzlaff, 1980).

Terminal velocity of fruits is a maximum velocity that each fruit can reach in specific medium. According to Jordan and Clerk (2004), an approach

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to fruit sorting is to use the terminal velocity of fruit moving in a fluid that has a density above or below the target density. Fruit with different terminal velocities will reach different depths after flowing a fixed distance in a flume and may be separated by suitably placed dividers. This approach could use water as a sorting medium, which provides huge advantages in terms of the resulting low corrosion and disposal difficulties, and the fact that it does not need any density adjustment. Additionally, this approach allows purely mechanical setting of the separation threshold by adjusting the divider positions and does not require changing the fluid density itself.

The authors embarked on a study to test terminal velocity of kiwi and apple in water column to determine if there was potential for terminal velocity methods in sorting industry. In particular, fruit size and density over the random ranges expected for studied fruits variety was investigated. In this study, fruit rising and dropping from the top of a water column whose density is lower than that of the fruit, was considered.

Notations

D	Fruit diameter, mm	F_b	Bouncy force, N
A_p	Projected area, cm^2	a	Acceleration, m/s^2
V	Volume, mm^3	v	Velocity, m/s
m	Mass of fruit, g	V_t	Terminal velocity, m/s
ρ_f	Fruit density, kgm^{-3}	T_y	Rising time, s
S_h	Shape factor of fruits	A	Constant factor
S_i	Size of fruits, mm	B	Constant factor
g	Gravitycal acceleration, m/s^2	b	Constant factor
ρ_w	Water density, kgm^{-3}	c	Constant factor
μ_w	Static viscosity of water	d	Constant factor
F_{tot}	Total force exerted to fruits, N	E	Constant factor
F_d	Drag force, N	e	Constant factor
C_D	Drag coefficient	k	Constant factor
N_R	Reynolds number		

Material and methods

Consider an apple of mass m, volume V, diameter D, and density ρ_f (=m/V), rising in water with density ρ_w ($\rho_f < \mu_w$) such that the largest cross-sectional area of fruit (A) is perpendicular to the direction of motion. The forces acting on it is a gravitational force (F_w) downward, a buoyancy force (F_b) upward, and a drag force (F_d) opposite to motion. The combination of these forces are accelerated the fruit at a rate (a) proportional to its mass (Crowe *et al.*, 2001):

$$F_{tot} = ma = F_w + F_d - F_b$$

$$ma = mg + 0.5\rho_w v^2 C_D A - \rho_w V g \qquad (1)$$

where *v* is the fruit velocity. Dividing equation 1 by $m = V\rho_f$, gives:

$$a = g\left(1 - \frac{\rho_w}{\rho_f}\right) + 0.5\rho_w v^2 C_D A/\left(V\rho_f\right) \qquad (2)$$

In equation 2; CD, drag coefficient, is a function of the velocity of the fruit and can be modeled well at low velocity using Stokes' law (Crowe *et al*., 2001). Thus:

$$C_D = \frac{24}{N_R} \quad \text{For} \quad N_R < 1 \qquad (3)$$

$$N_R = \frac{vD\rho_w}{\mu_w} \qquad (4)$$

then
$$C_D = \frac{24\mu_w}{vD\rho_w} \qquad (5)$$

where N_R is the dimensionless Reynolds number, μ_w is the static viscosity of the water, also a function of temperature, (Crowe *et al*., 2001) and D is the fruit diameter. Replacing in equation 2; equation 6 will result:

$$a = g\left(1 - \frac{\rho_w}{\rho_f}\right) + 12v\mu_w A/\left(VD\rho_f\right) \qquad (6)$$

For a spherical object, A/V can be computed directly as a function of the diameter, but kiwi fruit is more hyper-ellipsoidal than spherical. According to Jordan and Clerk (2004), by separating A/V into two parts: a dimensionless shape factor (Sh), and a pure size (size) Thus:

$$\frac{A}{V} = \frac{S_h}{size} = \left[A\big/V^{\frac{2}{3}}\right]\Big/\left[V^{\frac{1}{3}}\right] \qquad (7)$$

and by knowing that diameter is equal to equation 8: 3

$$D = e\left(\frac{6V}{\pi}\right)^{\frac{1}{3}} \qquad (8)$$

Replacing equations 7 and 8 into equation 6, acceleration becomes as equation 9:

$$a = kg\left(\frac{\pi}{6}\right)^{\frac{1}{3}}\left(1-\frac{\rho_w}{\rho_f}\right)+\left(\frac{12\mu_w v S_h}{V^{\frac{2}{3}}\rho_f}\right) \qquad (9)$$

where e and k are constant factors. Values for the fruit volume, density, projected area and fruit shape were then investigated using standard methods. When a particle rises through liquid at rest, its maximum dropping velocity (terminal velocity) is reached when the apparent weight of the particle due to gravitational forces equals the drag and buoyancy forces. Then, setting acceleration to zero in equation 9, the terminal velocity (v_t) of the fruit:

$$v_t = k\left(\frac{g\left(\frac{\pi}{6}\right)^{\frac{1}{3}}}{12\mu_w}\right)\left(\left(\rho_w-\rho_f\right)\frac{V^{\frac{2}{3}}}{S_h}\right) \qquad (10)$$

Terminal velocity proven theory for dropping state in $N_R<1$ condition is a new theory developed by authors and named as KHAT 1 Theory.

With considering $N_R>1$ for higher velocity and then simulate (Mohsenin, 1986):

$$C_D = \frac{const}{N_R^n} \qquad (11)$$

And also $N_R = \frac{vD\rho_w}{\mu_w}$, CD becomes::

$$C_D = \frac{K_4\mu_w^n}{v^n D^n \rho_w^n} \qquad (12)$$

With replacing A/V, D and CD from equation 7, 8 and 12 into equation 2, acceleration becomes:

$$a = g\left(1-\frac{\rho_w}{\rho_f}\right)+K_6\left(\frac{\mu_w^n\rho_w^{(1-n)}v^{(2-n)}S_h}{V^{\left(\frac{n+1}{3}\right)}\rho_f}\right) \qquad (13)$$

Then, setting acceleration to zero in equation 13, the terminal velocity (*vt*) of the fruit considering NR>1:

$$v_t = K_7 \frac{(\rho_w - \rho_f)^{\left(\frac{1}{2-n}\right)} V^{\left(\frac{n+1}{3(2-n)}\right)}}{\mu_w^{\left(\frac{n}{2-n}\right)} \rho_w^{\left(\frac{1-n}{2-n}\right)} S_h^{\left(\frac{1}{2-n}\right)}} \qquad (14)$$

Terminal velocity proven theory for dropping state in NR>1 condition is a new theory developed by authors and named as KHAT 2 Theory. Both KHAT 1 and KHAT 2 Theories show that the terminal velocity is proportion to the difference between fruit and water densities, volume, and shape factor of fruits.

Consider a kiwi of mass m, volume V, diameter D, and density ρ_f (=m/V), dropping in water with density ρ_w ($\rho_f < \mu_w$) and equation 11 for $N_R<1$ condition, the terminal velocity becomes as:

$$v_t = K_2 \left(\frac{g\left(\frac{6}{\pi}\right)^{\frac{1}{3}}}{12\mu_w} \right) \left((\rho_f - \rho_w) \frac{V^{\frac{2}{3}}}{S_h} \right) \qquad (15)$$

And the terminal velocity of kiwi fruit for higher velocity, NR>1, becomes as:

$$v_t = K_7 \frac{(\rho_f - \rho_w)^{\left(\frac{1}{2-n}\right)} V^{\left(\frac{n+1}{3(2-n)}\right)}}{\mu_w^{\left(\frac{n}{2-n}\right)} \rho_w^{\left(\frac{1-n}{2-n}\right)} S_h^{\left(\frac{1}{2-n}\right)}} \qquad (16)$$

The 30 Hayward kiwi fruits, 44 Delbarstival and 50 Redspar apples were transferred to the laboratory polyethylene bags to reduce water loss during transport. Fruits were then kept in cold storage at 4 ºC. All of the experiments were carried out at a room temperature, in the Biophysical and Biological laboratory in university of Tehran, Karaj, Iran.

Fruit mass was determined with an electronic balance of 0.1 g sensitively. Volume and fruit density were determined by the water displacement method (Mohsenin, 1986). Projected area of the specimens was determined from pictures of the fruits taken by Area Measurement System-Delta Tengland.

A glued Plexiglas column was constructed, height =1200 mm and cross-section=350x350 mm^2. This column was optimal, fruit diameter approximately 20% of column diameter, (Vanoni, 1975). The column was filled with tap water to a height of about 1100 mm (Fig. 1).

Each fruit was placed flat (i.e., with their largest two dimensions oriented horizontally) on the top of the column, and then released. In order to determine terminal velocity of fruits a digital camera, JVC (770) showed in Fig. 1, recorded the moving of fruits with 25 frames per second from releasing point (height =1100 mm) to the bottom of water column, simultaneously. Each fruit was tested three or four times. To calculate the dropping time of fruits from 40 cm depth to bottom of column (70 cm) the video to frame software were used to change video film to images. This method was used for two varieties of apples but apples were placed flat at the bottom of water column and rising velocity of them was considered because the apple density was lower than that of water. There was neglected 40 cm from start of motion because of time needed to reach terminal velocity (Jordan and clerk, 2004), and then dropping and rising terminal velocity of kiwi and apple fruits was calculated, respectively, knowing the fact that each picture takes 0.04 s and using following formula:

$$V_t = \frac{70 \times 10^{-2}}{(0.04 \times N)} \quad (17)$$

That N is the number of pictures from 40 cm to 110 cm after releasing point of water column. Determined data were considered for modeling terminal velocity using SPSS, 13, Software.

Fig. 1. Water column and camera setting to the side.

Results and discussion

The mean of difference between the fruit and water densities, volume of fruits and fruit shape factor and Reynolds number for Hayward kiwi fruit, Redspar and Delbarstival apples were shown in Table 1. The Reynolds number (NR) for studied fruits was bigger than unit. Hence, Equation 14 (KHAT 2 theory) for modeling terminal velocity of apples and Equation 16 for modeling terminal velocity of kiwi fruits was considered. With considering ρ_w and w as constant, equation 14 and 16 would be as follows:

$$v_t = K_8 \frac{(\rho_w - \rho_f)^{\left(\frac{1}{2-n}\right)} V^{\left(\frac{n+1}{3(2-n)}\right)}}{S_h^{\left(\frac{1}{2-n}\right)}} \qquad (18)$$

$$v_t = K_8 \frac{(\rho_f - \rho_w)^{\left(\frac{1}{2-n}\right)} V^{\left(\frac{n+1}{3(2-n)}\right)}}{S_h^{\left(\frac{1}{2-n}\right)}} \qquad (19)$$

where K_8 is constant. This can be generalized to equation 20 for apples and equation 21 for kiwi as:

$$V_t = A(\rho_w - \rho_f)^b V^c S_h^{-d} + E \qquad (20)$$

$$V_t = A(\rho_f - \rho_w)^b V^c S_h^{-d} + E \qquad (21)$$

That the parameters A, b, c, d and E take appropriate values. Parameter E was added to prevent errors. These models were optimized by adjusting various combinations of these five parameters to fit the models to maximize coefficient of determination. A number of models were tested, and the results are summarized as follows:

The best models for terminal velocity of Hayward kiwi fruits, Redspar and Delbarstival apples were obtained as 21, 22 and 23 models, respectively:

$$V_t = 0.025(\rho_f - \rho_w)^{0.316} V^{0.074} S_h^{-0.412} \qquad R^2=0.88 \quad (22)$$

$$V_t = 2.472(\rho_w - \rho_f)^{0.037} V^{0.044} S_h^{-0.146} - 3.156 \qquad R^2=0.72 \quad (23)$$

$$V_t = 0.572(\rho_w - \rho_f)^{0.102} V^{0.972} S_h^{-0.071} - 1.053 \qquad R^2=0.63 \quad (24)$$

Models 21, 22 and 23 with 0.88, 0.72 and 0.63, respectively, as coefficient of determination showed that KHAT 2 theory can predicted terminal velocity of fruits dropping or rising in water. The coefficient of determination for studied fruits was not equal because water is a medium that fruit in that has 6 degrees of freedom, 3 freedom degrees due to motion in X, Y and Z axis and 3 freedom degrees due to rotation in mentioned axis.

By eliminating shape factor in these models, models 24, 25 and 26 for terminal velocity of Hayward kiwi fruits, Redspar and Delbarstival apples, respectively, that there was not significant reduction in R2 value:

$$V_t = 0.028(\rho_f - \rho_w)^{0.278} V^{0.184} \quad R^2=0.87 \quad (25)$$

$$V_t = 3.127(\rho_w - \rho_f)^{0.03} V^{0.032} - 3.884 \quad R^2=0.68 \quad (26)$$

$$V_t = 0.649(\rho_w - \rho_f)^{0.079} V^{0.092} - 1.122 \quad R^2=0.63 \quad (27)$$

Above models with acceptable R2, showed that shape factor of fruits had negligible effect on their terminal velocities. A little more positive power of differences between water and fruit densities than that of volume in model 24 showed more effectiveness of differences between water and fruit densities than that of volume on terminal velocity of Hayward kiwi fruits and also such comparison in models 25 and 26 showed reverse result that the effectiveness of volume was more than that of differences between water and fruit densities on terminal velocity of apples.

Results showed that the most effective characters of studied fruits on their terminal velocities were volume and differences between water and fruit densities. It is concluded the basis of that fruits with approximately constant volume can be sorted on their densities.

Table. 1 The mean of difference between the fruit and water densities, volume power and shape of studied fruits.

	Hayward kiwi	Redspar apple	Delbarstival apple
$\|\rho_w - \rho_f\|$, Kg/m^3	167	166.85	172.09
V, cm^3	35.35	274.91	143.19
S_h	0.47	1.42	1.43
V_t, m/s	0.22	0.47	0.42
N_R	946.43	4066	2401

Conclusions

In this study, the terminal velocity of Hayward kiwi, Redspar and Delbarstival apples in water were theoretically formulated and then determined experimentally using water column. The best model of terminal velocity of studied fruits was modeled as function of fruit and water density, volume and fruit shape factor. Differences between fruit and water densities and volume of fruits were found as the most effective character on their terminal velocity but shape factor had the lowest effect on that.

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References

Crowe, C. T., Elger, D.F. and Roberson, J.A. (2001). Engineering Fluid Dynamics. 7th ed. New York, N.Y, John Wiley and Sons.

Gutterman, R.P. (1976). Mass flow density sorter for fruit and vegetables. In Quality Detection in Foods, 211-213. J. J. Gaffney, compiler. ASAE publication 1-76. St. Joseph, Mich.: ASAE.

Jordan, R.B. and Clark, C.J. (2004). Sorting of kiwifruit for quality using drop velocity in water. ASAE, 47 (6), 1991–1998.

Jordan, R.B., Walton, E.F., Klages, K.U. and Seelye, R.J. (2000). Postharvest fruit density as an indicator of dry matter and ripened soluble solids of kiwifruit. Postharvest Biological Technology. 20 (2), 163-173. 221

Mohsenin, N.N. (1986). Physical Properties of Plant and Animal Materials, second ed. Gordon and Breach Science Publishers, New York. 223

Patzlaff, A.W. (1980). Hydrodynamic blueberry sorting. U.S. Patent, No. 4225424.

Perry, R.L., Perkins, R.M., 1968. Separators for frost damaged oranges. Citrograph. 53 (8), 304- 312.

Richardson, A.C., McAneney, K.J. and Dawson, T.E. (1997). Carbohydrate dynamics in kiwifruit. Journal of Horticultural Science. 72 (5), 907-917.

Vanoni, V.A. (1975). Sedimentation Engineering. ASCE Manual 54. New York, N.Y. ASCE.

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