

Stress Concentration Due to Shear Lag Effect in Simply Supported Steel Box Girders with Longitudinal Stiffeners

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ABSTRACT

The objectives of this research is to study about the shear lag effect and focus on stress concentration for steel box girder with longitudinal stiffeners. All of models in this research used the finite element method for studying the shear lag effect and the finite element mesh must be made with carefulness to assess stress concentration. The study examines the stress concentration in a flange due to the shear lag in a simply supported steel box girder by the three-dimensional finite element method using shell elements under two loading conditions of uniformly distributed load along girder length and concentrated load. Definitely, parametric study with respect to the geometry of steel box girder is used. The dependency of finite element mesh is carefully emphasized. It is also reported that the stress distributions in the flange are different from those of the elementary theory. Based on the results, empirical formulas are proposed to calculate stress concentration factors due to the shear lag effect.

1. INTRODUCTION

This study aims to investigate shear lag in simply supported steel box girder with stiffener under two loading conditions. The simply supported steel box girder is shown in Figure 1. In the elementary beam theory, the normal stress in the longitudinal direction produced by bending deformation is assumed to be proportional to the distance from neutral axis and uniform across the flange width. If the flange gets wider, this assumption becomes invalid and a phenomenon called shear lag will happen.

The geometric properties of box girders are shown in Figure 1 (c). The geometric properties are half flange width (B), span length (L), height of web (H), thickness of flange (t_f), thickness of web (t_w), cross sectional area of the stiffeners (A_s) and cross sectional area of flange (A_f).

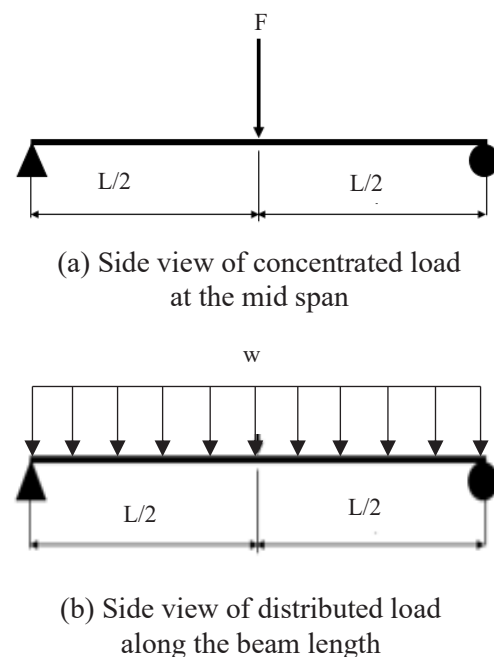
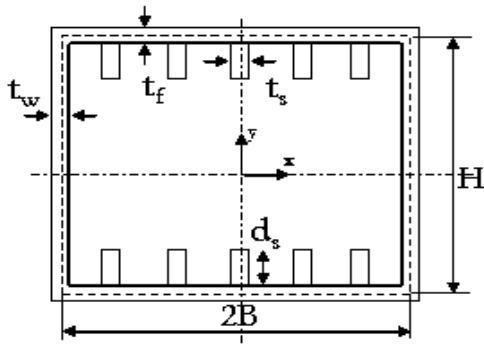


Figure 1 Structural geometry of box girder



(c) Cross section and geometric properties

Figure 1 (Cont.) Structural geometry of box girder

Empirical formulas for assessment of shear lag effect are shown in term of the effective width but this way cannot give the exact value of stress concentration. Moffatt and Dowling [3] give the meaning of the effective width is:

$$B_e = \frac{1}{\sigma_{\max}} \int_0^{2B} \sigma_y dx \quad (1)$$

Where B_e is the half effective width, and the numerator is the integration of the normal stress in the flange, σ_y , while the denominator is the actual maximum normal stress in the flange due to shear lag, σ_{\max} . Yamaguchi, T., et al. [6] are realized that the evaluation of the maximum stress by the effective width approach invites error by itself. So, in the present study, the empirical formulas are proposed for evaluating stress concentration factors obtained by the present finite element analysis instead of the effective width.

The AASHTO LRFD Bridge Design Specifications 2010 [4] is used for design steel beams. By varying the proportions of geometric properties of the steel girder, the linear FEA is performed. From the results of finite element method (FEM), the proposed formula for calculation stress concentration factor is proposed.

For finite element models, the structure is created by using 3D 4 nodes shell element. Due to symmetry, this study uses a quarter of steel box girder for analysis. In all analyses, finite element program, MARC 2016 [5], is used. The square elements are used in this study. The element

meshes are used to learn about the effect of finite element mesh on stress concentration and used to reduce incorrect value by multimesh extrapolation method for all steel box girders under two loading conditions.

2. Box girder to be analyzed

The stress concentration factors can be evaluated in steel box girder underneath concentrated load. Stress concentration factor, K_e is the ratio of the maximum normal stress in the flange due to the shear lag to that of the elementary beam theory [6]. Previous researches not have the way to calculate a precise stress distribution that have the shear lag. And many researchers do not give their loading condition exactly [6].

In finite element models, a unique load in the beam theory can be applied in various ways [6]. In this study, load conditions that make local effects in the flange are neglected. For the concentrated load, two loading models shown in Figure 2 are adopted: Load C-1 is a concentrated load at the middle of the web, Load C-2 is a uniformly distributed load along the height of the web, Load D-1 is a uniformly distributed load along the centerline of the web and Load D-2 is a uniformly distributed load along the beam axis and the web height of every cross section [6]. Tenchev [1] is one of few researchers that provided loading condition and he used Load C-2 and Load D-1 for his concentrated and distributed load [1].

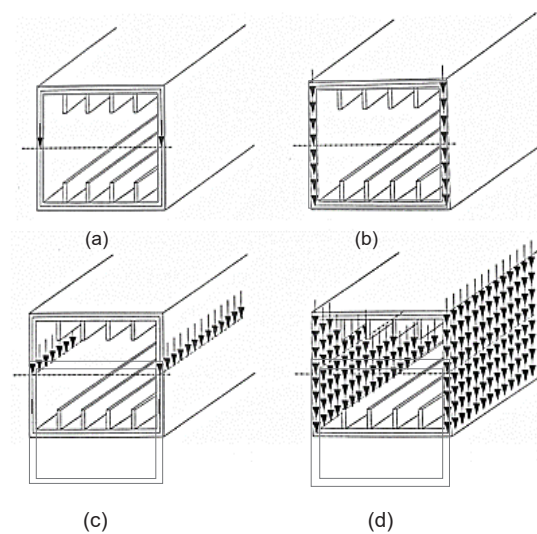


Figure 2 Concentrated load (a) Load C-1, (b) Load C-2; Distributed load (c) Load D-1, (d) Load D-2

3. STRESS EVALUATION

The finite element model is analyzed by finite element method, shell elements are used. The analysis results of finite element mesh are more powerful. The number of elements in Figure 3 are 4,288, 17,152, 68,608, and 274,432 for mesh A to mesh D, respectively.

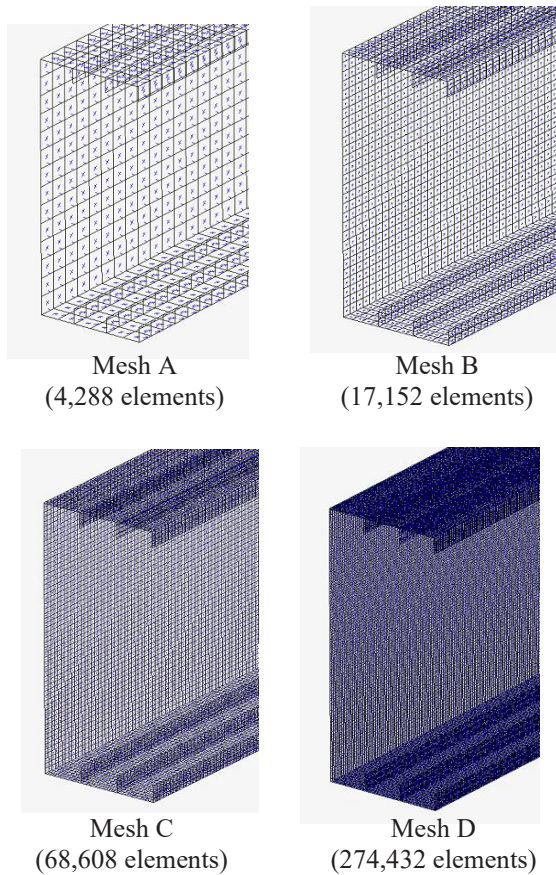


Figure 3 Finite element meshes

The previous study is to learn about the effect of element mesh. Figure 4 shows the normal stress distributions in upper flange at the mid span of steel box girder. In this Figure 4, σ_{FEM} is the normal stress from finite element analysis and σ_{beam} is the normal stress from the elementary beam theory and constant value. This is the result of box girder ($H/L = 0.05$, $B/H = 0.4$, $T_f/T_w = 1.3$, $A_s/A_f = 0.3$) under Load C-2 by four meshes, mesh A to mesh D.

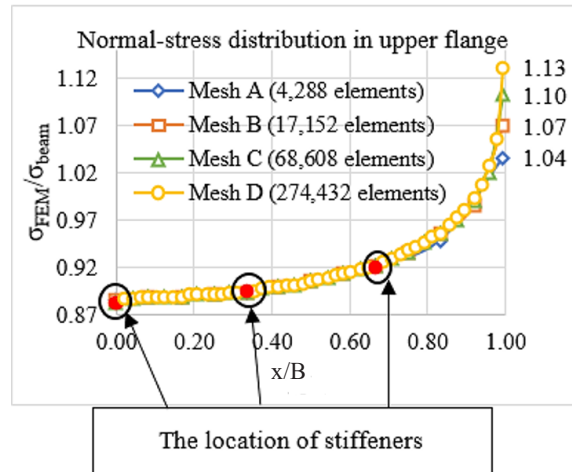


Figure 4 Normal stress distributions in the upper flange

Figure 5 shows the variation of the normal stress in the flange with respect to a representative element size Δ . It is observed that the four lines in the graph become almost straight for small Δ . The linear extrapolation shown by the dotted lines in the graph can be used to estimate the converged stress. "This extrapolation method is called the multimesh extrapolation method" by Cook et al. [2]. Importantly, the four lines in Figure 5 are almost straight, which is in accordance with the description of Cook et al. [2].

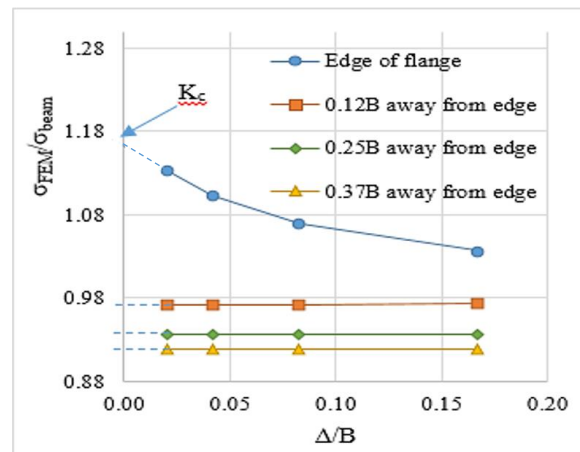


Figure 5 Variation of normal stress with respect to representative element size

4. PARAMETRIC STUDY

Stress concentration factor, K_c stands for the ratio of the maximum normal stress, $\sigma_{max,FEM}$, which is calculated by finite element analysis to the elementary beam theory stress, σ_{beam} . Figure 6 shows the normal stress distribution in the

upper flange at the mid span of simply supported beam with $B/H = 1.0$, $H/L = 0.2$ and $T_f/T_w = 2.5$ is presented in Figure 6. K_c increases with the increase of A_s/A_f . Cross sectional area of stiffener, A_s is equal to area of one stiffener multiplied by no. of stiffeners ($t_s \times d_s \times$ no. of stiffeners) and cross sectional area of flange, A_f is equal to two multiplied by area of flange ($2 \times (2B + t_w) \times t_f$).

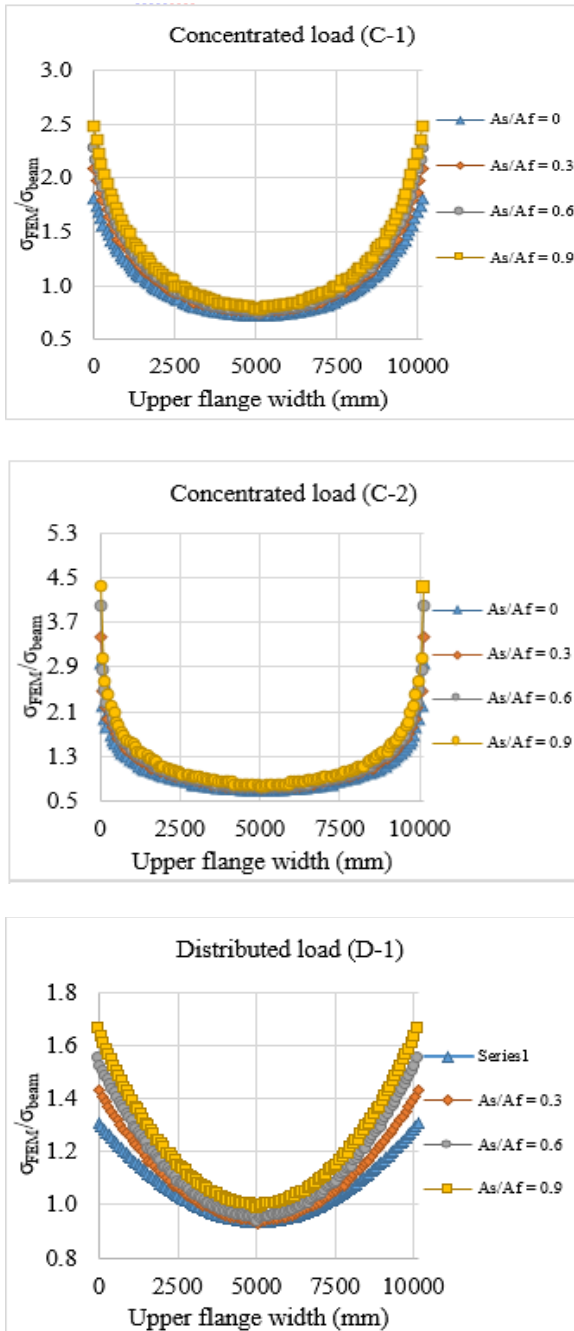


Figure 6 The normal stress distribution in the upper flange at the mid span of simply supported beam

The shear lag effect on simply supported box girder with longitudinal stiffeners depends on five factors:

- Type of loading
- Half flange width/height of web ratio of the girder (B/H)
- Height of web/span length ratio of the girder (H/L)
- Thickness of flange/web ratio of the girder (T_f/T_w)
- Cross sectional area of the stiffeners/area of the flange ratio of the girder (A_s/A_f)

5. FINITE ELEMENT ANALYSIS

For the FEM model, the structural is modeled by using three-dimensional 4-node shell elements. It is noted that due to symmetry only a quarter of the box girder is analyzed as shown in Figure 7. The elasticity modulus of box girder materials is $E = 2.06 \times 10^5$ MPa and Poisson's ratio is equal to 0.3. In finite element model of a quarter of the box girder, the general rule for a symmetry displacement condition is that the displacement vector component perpendicular to the plane is zero and the rotational vector components parallel to the plane are zero. For an anti-symmetry condition, the reverse conditions are applied (displacements in the plane are zero; the rotation normal to the plane is zero) [7].

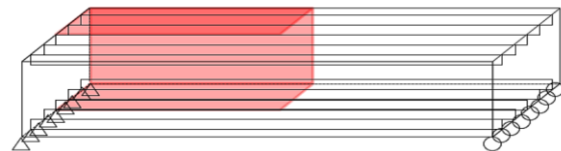


Figure 7 A quarter of box girder

6. Effect of geometric properties

This study uses AASHTO Standard for design initial dimension of steel box girder as shown in Figure 8. The initial proportions are $H/L = 0.05$, $B/H = 0.4$, $t_f/t_w = 1.3$, and $A_s/A_f = 0.3$. From the initial proportions can vary the proportion, and the following values are considered: $H/L = 0.05, 0.1, 0.15, 0.2$; $B/H = 0.4, 0.6, 0.8, 1.0$; $t_f/t_w = 1.3, 1.9, 2.5$. The longitudinal stiffeners in a beam are considered: $A_s/A_f = 0, 0.3, 0.6, 0.9$ in which A_s is total area of stiffeners on each flange and A_f is area of the flange. And the values of H , t_f , t_s , and d_s are fixed equal to 2,027 mm, 12 mm, 12 mm, and 120 mm respectively. The combination of all these values results in 192 models different from each other in geometry.

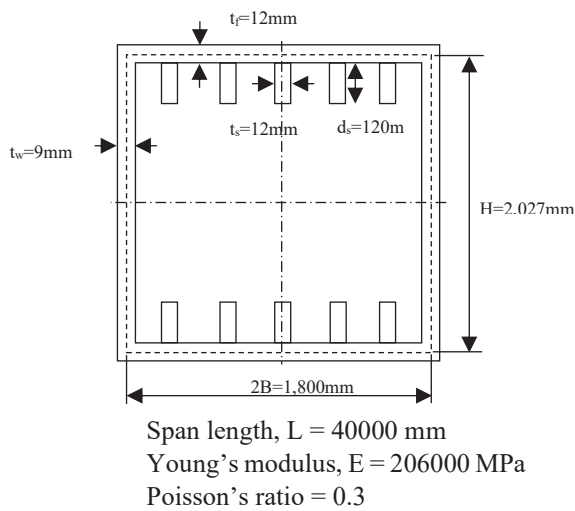


Figure 8 Cross section and geometric properties

Figure 9 shows K_c for the cross section of $A_s/A_f = 0.9$, $t_f/t_w = 1.3, 1.9, 2.5$ under Load C-1 and Load C-2. Load C-2 makes larger K_c than Load C-1 constantly. K_c value under Load D-1 and Load D-2 are not hardly different, but K_c value under Load D-1 is quite more than that of Load D-2. For distributed load, Load D-1 is used to study. The shear lag phenomenon associated with a wide flange, it is expected that K_c value becomes larger as H/L increase.

(a) $A_s/A_f = 0.9$, $t_f/t_w = 1.3$

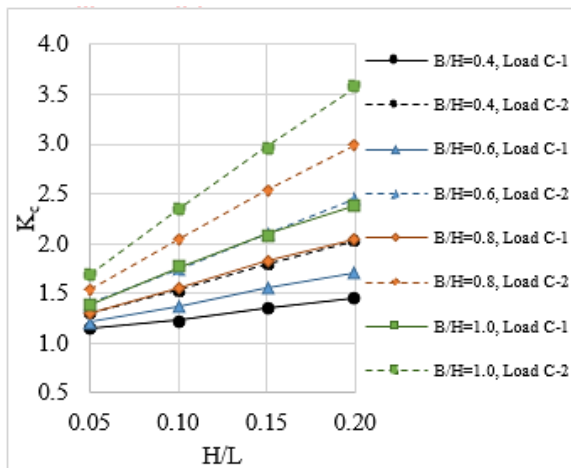
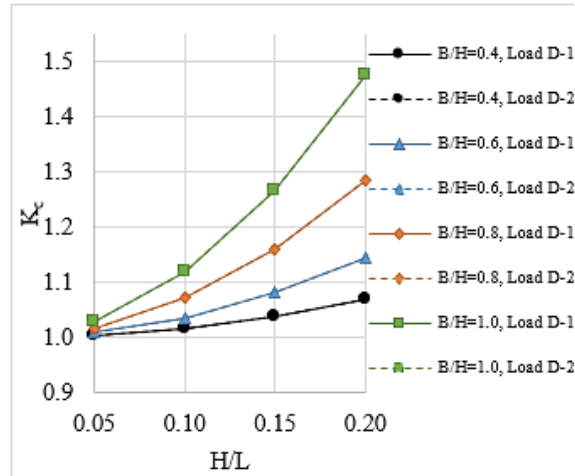
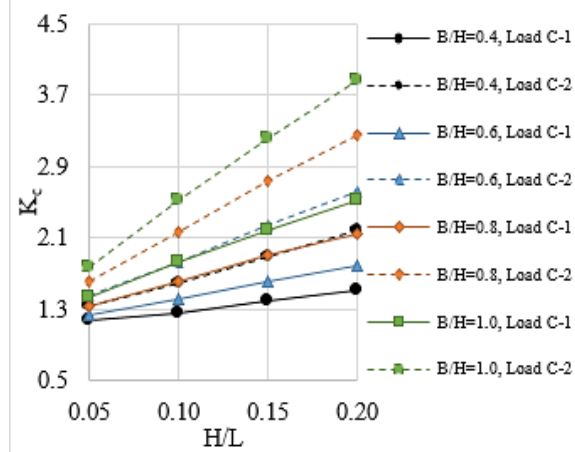


Figure 9 Variation of K_c with respect to H/L



(b) $A_s/A_f = 0.9$, $t_f/t_w = 1.9$



(c) $A_s/A_f = 0.9$, $t_f/t_w = 2.5$

Figure 9 (Cont.) Variation of K_c with respect to H/L

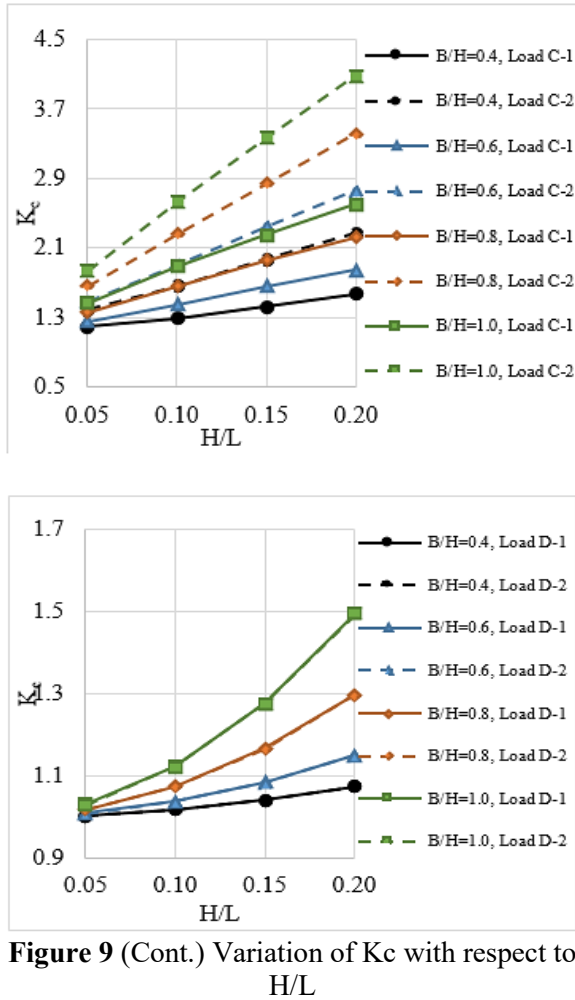


Figure 9 (Cont.) Variation of K_c with respect to H/L

Figure 9, 10, 11 and 12 show some more characteristic results of K_c and may be summarized as follow:

1. K_c grows with increase of H/L .
2. H/L on K_c is small for B/H equal to 0.4 under Load C-1 and Load D-1.
3. B/H has considerable influence on K_c : as B/H become larger, K_c increase in general.
4. B/H on K_c is small for H/L equal to 0.05 under Load C-1, Load C-2 and Load D-1.
5. K_c grows with the increase of t_f/t_w for large H/L .
6. K_c grows with the increase of A_s/A_f for large H/L .

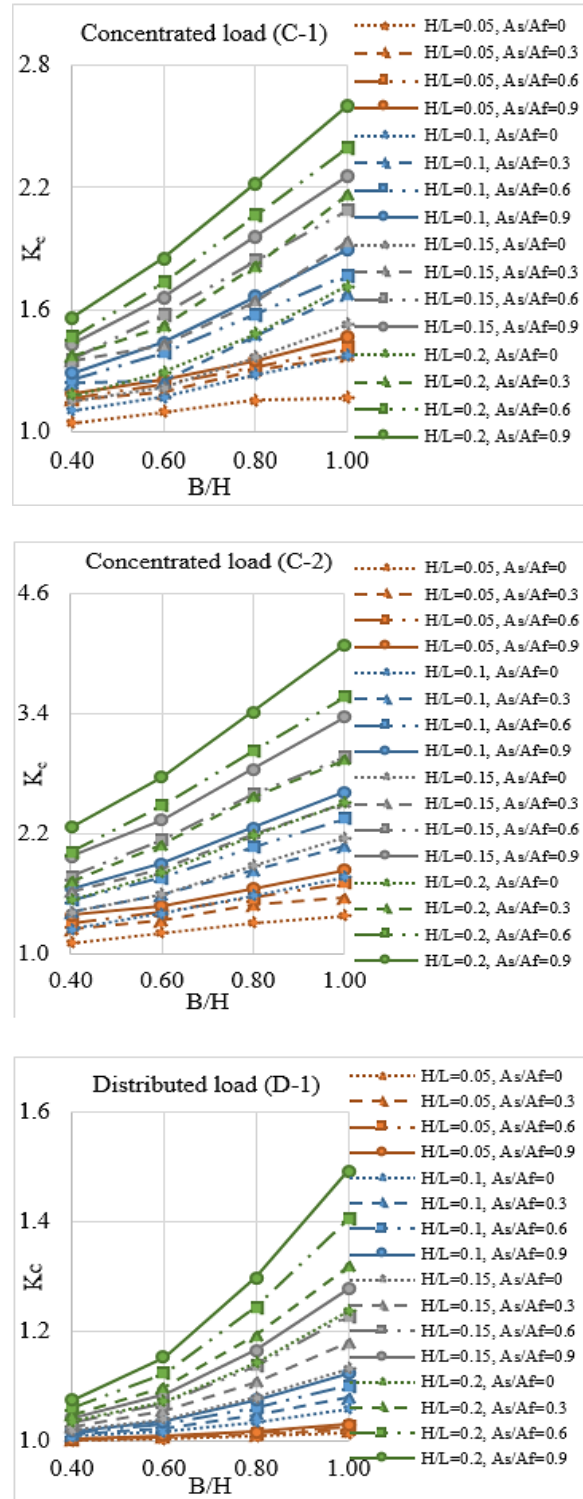


Figure 10 Variation of K_c with respect to B/H ($t_f/t_w = 2.5$)

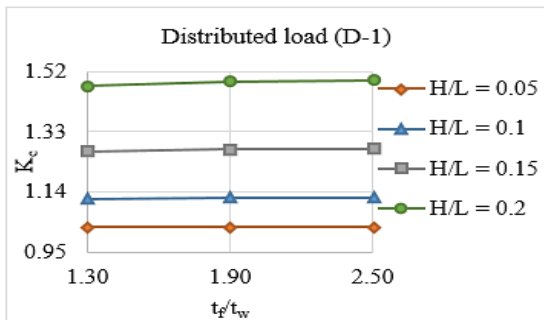
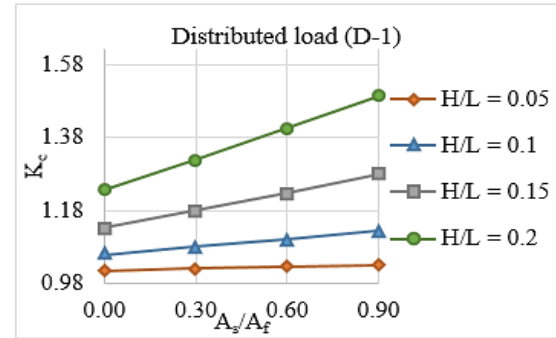
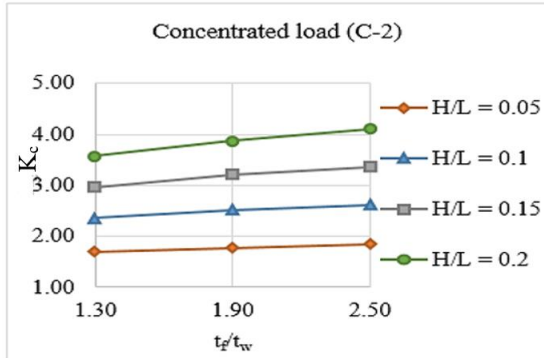
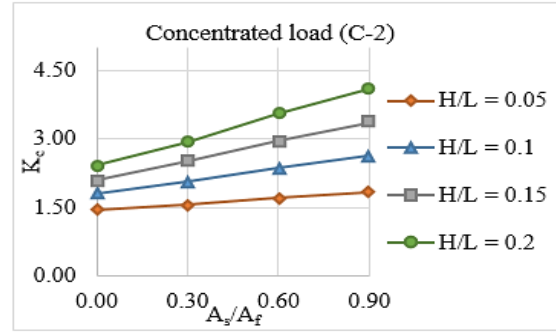
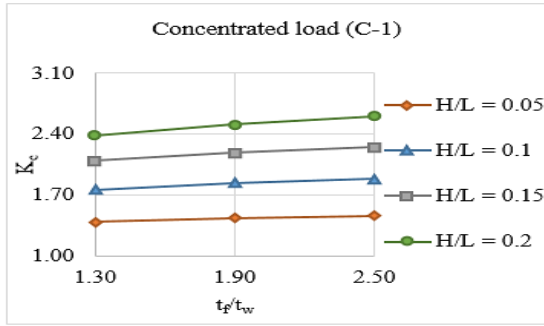


Figure 12 (Cont.) Variation of K_c with respect to A_s/A_f ($B/H=1.0, t_f/t_w=2.5$)

Figure 11 Variation of K_c with respect to t_f/t_w ($B/H=1.0, A_s/A_f=0.9$)

7. EMPIRICAL FORMULAS

For the discussion in the first section [6], the numerical results give the below formulas:

Concentrated load (Load C-1):

$$K_c = \phi \times a_1 \times \left(\frac{H}{L}\right) + 1 \quad (2)$$

Where

$$\phi = \left(1 + \frac{A_s}{A_f}\right)^{1.1}$$

$$a_1 = b_1 \times \left(\frac{B}{H}\right)^{c_1}$$

$$b_1 = 0.832 \times \ln\left(\frac{t_f}{t_w}\right) + 2.77$$

$$c_1 = -0.034 \times \ln\left(\frac{t_f}{t_w}\right) + 1.744$$

Concentrated load (Load C-2):

$$K_c = \phi \times a_2 \times \left(\frac{H}{L}\right) + 1 \quad (3)$$

Where

$$\phi = \left(1 + \frac{A_s}{A_f}\right)^{1.2}$$

$$a_2 = b_2 \times \left(\frac{B}{H}\right)^{c_2}$$

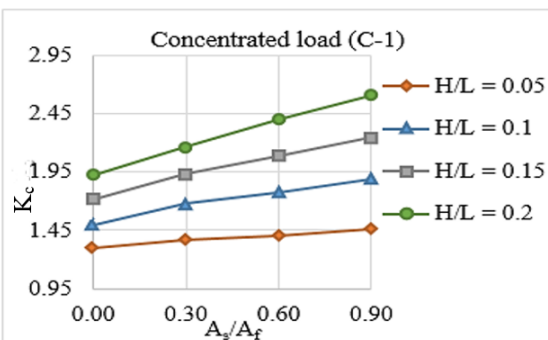


Figure 12 Variation of K_c with respect to A_s/A_f ($B/H=1.0, t_f/t_w=2.5$)

$$b_2 = 1.756 \times \ln\left(\frac{t_f}{t_w}\right) + 6.101$$

$$c_2 = 0.053 \times \ln\left(\frac{t_f}{t_w}\right) + 1.202$$

Distributed load (Load D-1):

$$K_c = \emptyset \times a_3 \times \left(\frac{H}{L}\right)^2 + 1 \quad (4)$$

Where

$$\emptyset = \left(1 + \frac{A_s}{A_f}\right)^{1.15}$$

$$a_3 = b_3 \times \left(\frac{B}{H}\right)^{c_3}$$

$$b_3 = 1.225 \times \ln\left(\frac{t_f}{t_w}\right) - 0.494 \left(\frac{t_f}{t_w}\right) + 6.001$$

$$c_3 = -0.041 \times \ln\left(\frac{t_f}{t_w}\right) - 0.006 \left(\frac{t_f}{t_w}\right) + 2.371$$

It is noted that the above formulas are applicable for $0.05 \leq \frac{H}{L} \leq 0.2$, $0.4 \leq \frac{B}{H} \leq 1.0$, $1.3 \leq \frac{t_f}{t_w} \leq 2.5$ and $0 \leq \frac{A_s}{A_f} \leq 1.0$.

7.1 Accuracy of the proposed formulas

Figure 13 shows K_c established by the proposed formulas of Eq. (2), Eq. (3) and Eq. (4) with the FEA results. To find the accuracy, the percentage error of the stress concentration factor calculated from this formula [1]:

$$\varepsilon_i = \frac{K_{cEmp} - K_{cFEA}}{K_{cFEA}} \times 100(\%) \quad (5)$$

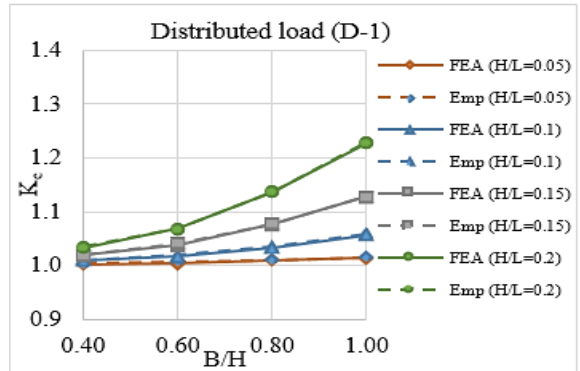
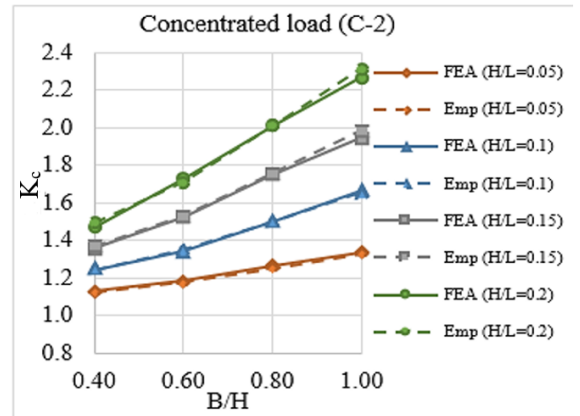
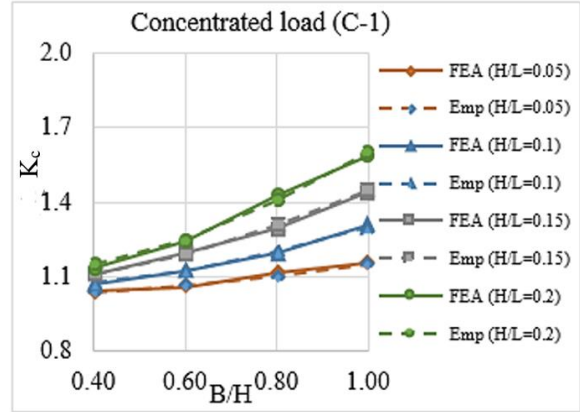
Where K_{cEmp} and K_{cFEA} are the K_c values obtained by the proposed formulas and the present finite element analysis, respectively [6]. The accuracy of the proposed formula for each loading condition is calculated as mean square error by the following equation [1]:

$$\bar{\varepsilon} = \sqrt{\frac{1}{N} \sum_{i=1}^N \varepsilon_i^2} \quad (6)$$

Where ε_i is the error computed by Eq. (5) for a present finite element result and N is the number of the present finite element results for a loading condition [1]. N in Eq. (6) is equal 192. Using Eq. (6), the mean square error is 1.92% for Load C-1, 2.84% for Load C-2 and 1.41% for Load D-1.

Figure 13 shows K_c due to proposed formulas and finite element analysis for the cross section of $t_f/t_w = 1.3$ and $A_s/A_f = 0, 0.9$ under Load C-1, Load C-2 and Load D-1.

(a) $t_f/t_w = 1.3$ and $A_s/A_f = 0$



(b) $t_f/t_w = 1.3$ and $A_s/A_f = 0.9$

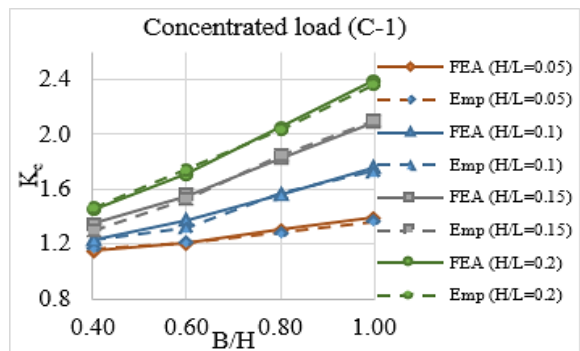


Figure 13 K_c due to proposed formulas and finite element analysis

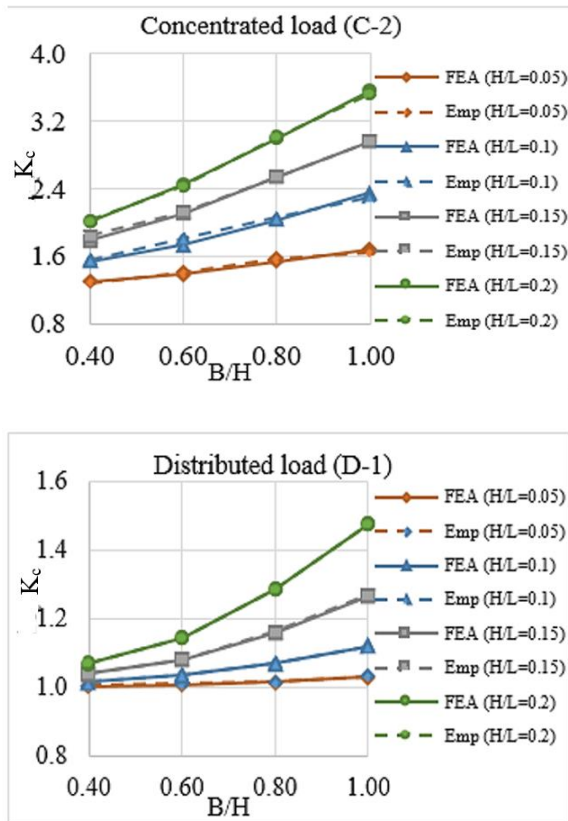


Figure 13 (Cont.) K_c due to proposed formulas and finite element analysis

8. CONCLUSIONS

Three-dimensional finite element analysis of steel box girder with longitudinal stiffeners is operated to show the shear lag effect. The stress concentration factor, K_c is used to determine the shear lag effect on stress concentration. Shell elements are used for all models. Concentrated load and distributed load are employed to various cross-sections of steel box girders. The results show that the stress concentration factor increases with the increase of proportions of geometric properties. From the numerical results, the empirical formulas are proposed to calculate the stress concentration factors. And it is verified that the results of proposed formulas are similar to the present finite element results.

For future study, it is recommended that the study of stress concentration due to shear lag of steel box girder with longitudinal stiffeners should be compared with data from actual steel box girder sample.

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