Vector Stochastic Differential Equation Based Stochastic Analysisof Transformer

Rawid Banchuin[1] and Roungsan Chaisricharoen[2]

ABSTRACT: Previously, the stochastic behaviors of transformer were analyzed by using the stochastic differential equation approach. Unfortunately, only noise in the voltage source applied to the transformer, were taken into account. Therefore improved analysis has been performed in this work by also consider the formerly ignored random variations in elements/parameters of transformer. The resulting vector stochastic differential equations have been solved in Ito's sense. The Euler-Maruyama scheme has been adopted for determining the numerical solutions and the confidence intervals for means of stochastic currents have been used for verification. The stochastic properties of the transformer's electrical quantities have been studied and the influences of noise in the voltage source and random variations in elements/parameters of transformers have been analyzed. We have found that noise in the voltage source and random variations in the elements/parameters of the transformer respectively contribute high and low frequency fluctuations where the effects of variations in the elements/parameters are greater than that of noise in the voltage source.

Keywords: Color Noise, Stochastic Differential Equation, Ito Calculus, Transformer, White Noise

DOI: 10.37936/ecti-cit.2021151.188931
Received May 14, 2019; revised March 20, 2020; accepted June 20, 2020; available online December 25, 2020

1. INTRODUCTION

The stochastic differential equation (SDE) is applicable in various scientific areas such as chemistry, physics, mathematical biology and engineering fields when stochastic effects are to be considered [1]-[3]. In electrical and electronic engineering, the SDE covers various stochastic variations which occur in the systems as can be seen from many previous research [4]-[7]. Many previous studies have focused on the effects of stochastic variations in conventional passive electrical circuits which are uncoupled [8]-[13]. A transformer which is a coupled device, can be found in many electronic and electrical systems including those based on the up to date low voltage/low power VLSI technology [14]-[18]. In those system, for the transformers which are of the on-chip monolithic type, stochastic analysis has been performed in [19]. Unfortunately, merely noise in the voltage source applied to the transformer has been taken into account by [19]. In that work, the stochastic effects of random variations in the circuit elements/parameters of transformer, which also exist and play significant roles in the statistical/variability aware analysis and design, have been totally ignored.

Hence, the stochastic behaviors of the transformer have been analyzed by using an SDE based approach in our paper by considering both noise in the voltage source and such random variations in the circuit elements/parameters unlike [19]. As a result, the multidimensional SDEs of the transformer have been derived. Traditionally, the multidimensional SDE is often written in vector form and referred to as the vector SDE. Noise in the voltage source applied to the transformer has been considered as white noise, which is a Wiener process. On the other hand, random variations in the elements/parameters of the transformer have been considered as color noise which is an Ornstein-Uhlenbeck process. The resulting vector SDEs have been solved analytically and numerically by using numerical simulation software in the Ito sense. The Euler-Maruyama scheme has been adopted for determining the numerical solutions. The confidence intervals for means of stochastic currents have been compared to the deterministic results for verifying our SDEs. These deterministic

[1]Graduated school of IT and Faculty of Engineering, Siam University, Bangkok, Thailand., E-mail: rawid_b@yahoo.com
[2]School of IT, Mae Fah Luang University, Chiangrai, Thailand., E-mail: roungsan.cha@mfu.ac.th

results can be obtained from either the SPICE simulation of transformer circuit with neither noise nor random variations in circuit elements/parameters, or the numerical inverse Laplace transformation of the deterministic s-domain based responses. By using the obtained analytical and numerical solutions, the stochastic properties of the transformer's electrical quantities have been studied. The influences of noise in the voltage source and random variations in elements/parameters of transformers to those electrical quantities have been analyzed. The causes of high and low frequency stochastic variations of such electrical quantities in both transient and steady state have been pointed out.

In the subsequent section, the vector SDEs of transformers will be formulated, their analytical solutions will be determined, and the stochastic properties of the transformer's electrical quantities will be studied. The SDEs will be numerically solved in section 3. The influences of noise in the voltage source along with the random variations in elements/parameters of transformers to those electrical quantities, will be analyzed and the causes of high and low frequency variations in both transient and steady state will be pointed out. Further discussion on the obtained simulation results will be shown in section 4. The conclusion will be drawn in section 5. Since many stochastic calculus related variables have been introduced in this work, a table of their notations will be shown in the appendix for the convenience of those readers unfamiliar with them.

2. THE VECTOR SDES OF A TRANSFORMER, THEIR SOLUTIONS AND THE STOCHASTIC PROPERTIES OF THE TRANSFORMER'S ELECTRICAL QUANTITIES

For simplicity, the deterministic equations of the transformer and their solutions will be firstly formulated. Later, the vector SDEs will be derived by extending the deterministic equations. Finally, the solutions of these SDEs, which are interpreted in the Ito sense, will be determined. Consider the lossy/imperfectly coupled transformer circuit model [20] depicted in Fig.1, which is applicable to both up to date monolithic on-chip transformers and classical off-chip transformers. The deterministic voltage-current relationships of the transformer can be given by (1) and (2). Noted that $L_{11}(L_{22})$, $R_{11}(R_{22})$, $v_1(t)(v_2(t))$, and $i_1(t)(i_2(t))$ denote the primary(secondary) self inductance, primary(secondary) self resistance, primary(secondary) voltage, and primary(secondary) current respectively. Moreover, $R_{12}(M_{12})$ and $R_{21}(M_{21})$ stand for mutual resistance (inductance) from secondary to primary and vice versa. Note that R_{12} and R_{21} cannot be ignored in the on-chip transformer for accurate power consumption modeling [21] but they can be neglected for the off-chip transformer because its mutual impedances are purely inductive [19].

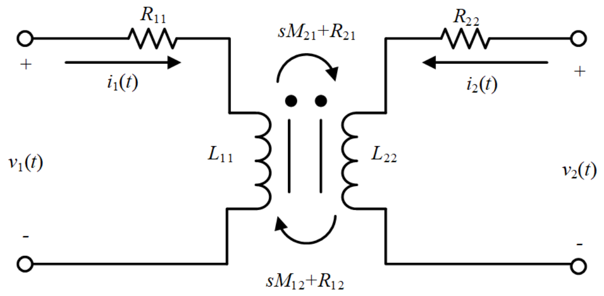

Fig.1: *The Transformer's Circuit Model [20].*

$$v_1(t) = L_{11}\frac{di_1}{dt} + R_{11}i_1(t) + M_{12}\frac{di_2}{dt} + R_{12}i_2(t), \quad (1)$$

$$v_2(t) = L_{22}\frac{di_2}{dt} + R_{22}i_2(t) + M_{21}\frac{di_1}{dt} + R_{21}i_1(t). \quad (2)$$

For covering the imperfect coupling completely, we use $i_2(t) = kn^{-1}i_1(t)$ where k and n respectively denote the coupling factor and turn ratio of the transformer. As a result, (1) and (2) become [19]

$$v_1(t)=(L_{11}+M_{12}kn^{-1})\frac{di_1}{dt}+(R_{11}+R_{12}kn^{-1})i_1(t), \quad (3)$$

$$v_2(t)=(L_{22}+M_{21}k^{-1}n)\frac{di_2}{dt}+(R_{22}+R_{21}k^{-1}n)i_2(t). \quad (4)$$

If $v_1(t)$ and $v_2(t)$ are known, $i_1(t)$ and $i_2(t)$ which are the deterministic solutions can be respectively obtained as given by (5) and (6) where t_0 stands for the initial time. For the off-chip transformer which employs lossless coupling and purely inductive mutual impedances, it's deterministic equations and solutions can be determined from those of the on-chip transformer by simply letting $k = 1$ and $R_{12} = R_{21} = 0$.

$$i_1(t) = e^{-\left(\frac{R_{11}+R_{12}kn^{-1}}{L_{11}+M_{12}kn^{-1}}\right)}[i_1(t_0) + \int_{t_0}^{t} \frac{v_1(\tau)e^{\left(\frac{R_{11}+R_{12}kn^{-1}}{L_{11}-M_{12}kn^{-1}}\right)}}{L_{11}+M_{12}kn^{-1}}d\tau], \quad (5)$$

$$i_2(t) = e^{-\left(\frac{R_{22}+R_{21}k^{-1}n}{L_{22}+M_{21}k^{-1}n}\right)}[i_2(t_0) + \int_{t_0}^{t} \frac{v_2(\tau)e^{\left(\frac{R_{22}+R_{21}k^{-1}n}{L_{22}-M_{21}k^{-1}n}\right)}}{L_{22}+M_{21}k^{-1}n}d\tau]. \quad (6)$$

Stochastically, this is not the case because noise in the voltage source along with random variations in the transformer circuit elements/parameters must be taken into account. By the effects of such noise, $v_1(t)$ and $v_2(t)$ become

$$V_1(t) = v_1(t) + \alpha\zeta_1(t), \tag{7}$$

$$V_2(t) = v_2(t) + \beta\zeta_2(t). \tag{8}$$

Note that $V_1(t)$ and $V_2(t)$ denote the stochastically varied primary and secondary voltage. Moreover, $\zeta_1(t)$ and $\zeta_2(t)$ are the zero mean white noise with α and β as their noise intensities. Finally, $\zeta_1(t) = dW_1(t)/dt$ and $\zeta_2(t) = dW_2(t)/dt$, where $W_1(t)$ and $W_2(t)$ are Wiener processes.

As a result of those random variations on the other hand, the following stochastically varied circuit elements/parameters whise effects have been ignored in [19], have been arisen.

$$R^*_{11} = R_{11} + \gamma_{11}\theta_{11}(t), \tag{9}$$

$$R^*_{12} = R_{12} + \gamma_{12}\theta_{12}(t), \tag{10}$$

$$R^*_{21} = R_{21} + \gamma_{21}\theta_{21}(t), \tag{11}$$

$$R^*_{22} = R_{22} + \gamma_{22}\theta_{22}(t), \tag{12}$$

$$L^*_{11} = L_{11} + \lambda_{11}\chi_{11}(t), \tag{13}$$

$$M^*_{12} = M_{12} + \lambda_{12}\chi_{12}(t), \tag{14}$$

$$M^*_{21} = M_{21} + \lambda_{21}\chi_{21}(t), \tag{15}$$

$$L^*_{22} = L_{22} + \lambda_{22}\chi_{22}(t), \tag{16}$$

$$n^* = n + v\eta(t), \tag{17}$$

$$k^* = k + \delta\kappa(t). \tag{18}$$

Of course, k^* , R^*_{12} and R^*_{21} are not existed in the off-chip transformer since the coupling and mutual impedances are lossless. Moreover, $\theta_{ij}(t)$ and $\chi_{ij}(t)$ (where $\{i\} = \{1, 2\}$ and $\{j\} = \{1, 2\}$), $\eta(t)$ and $\kappa(t)$ are the color noises that define the random variations in elements/parameters of the transformer. We use the color noise because the mentioned variations are very slowly changed with respect to time. Note also that γ_{ij}, λ_{ij}, v, and δ are noise intensities of $\theta_{ij}(t)$, $\chi_{ij}(t)$, $\eta(t))$ respectively. Since the color noise can be mathematically modelled as the Ornstein-Uhlenbeck process [10], [22], the SDEs of $\theta_{ij}(t)$, $\chi_{ij}(t)$, $\eta(t)$, and $\kappa(t)$ can be given by

$$d\theta_{ij}(t) = -\rho_{\theta,ij}\theta_{ij}(t)dt + \sigma_{i_{\theta,ij}}\rho_{\theta,ij}dW_{ij}(t), \tag{19}$$

$$d\chi_{ij}(t) = -\rho_{\chi,ij}\chi_{ij}(t)dt + \sigma_{i_{\chi,ij}}\rho_{\chi,ij}dX_{ij}(t), \tag{20}$$

$$d\eta(t) = -\rho_\eta\eta(t)dt + \sigma_\eta\rho_\eta dN(t), \tag{21}$$

$$d\kappa(t) = -\rho_\kappa\kappa(t)dt + \sigma_\kappa\rho_\kappa dN(t). \tag{22}$$

$W_{ij}(t)$, $X_{ij}(t)$, $N(t)$ and $K(t)$ are Wiener process with unit variance. Moreover, $\rho_{\theta,ij}$, $\rho_{\chi,ij}$, ρ_η, and ρ_κ are the speeds of mean reversions of $\theta_{ij}(t)$, $\chi_{ij}(t)$, $\eta(t)$, and $\kappa(t)$. In addition, $\sigma_{\theta,ij}$, $\sigma_{\chi,ij}$, σ_η, and σ_κ stand for the volatilities of $\theta_{ij}(t)$, $\chi_{ij}(t)$, $\eta(t)$, and $\kappa(t)$ normalized with respect to $\rho_{\theta,ij}$, $\rho_{\chi,ij}$, ρ_η, and ρ_κ respectively. From (19)-(22), it can be seen that $\theta_{ij}(t)$, $\chi_{ij}(t)$, $\eta(t)$, and $\kappa(t)$ are normally distributed with zero mean and variances of $0.5\sigma^2_{\theta,ij}\rho^2_{\theta,ij}$, $0.5\sigma^2_{\chi,ij}\rho^2_{\chi,ij}$, $0.5\sigma^2_\eta\rho^2_\eta$, and $0.5\sigma^2_\kappa\rho^2_\kappa$ respectively.

Now, we rearrange (3) and (4) as follows [19]

$$di_1(t) = \left(\frac{R_{11} + R_{12}kn^{-1}}{L_{11} + M_{12}k^1n}\right)i_1dt + (L_{11} + M_{12}kn^{-1})^{-1}v_1(t)dt, \tag{23}$$

$$di_2(t) = \left(\frac{R_{22} + R_{21}kn^{-1}}{L_{22} + M_{21}k^1n}\right)i_2dt + (L_{22} + M_{21}kn^{-1})^{-1}v_2(t)dt. \tag{24}$$

By taking both noise in voltage source along with the random variations in circuit elements/parameters into account, (23) and (24) become

$$\begin{aligned} dI_1(t) = &-\left[\frac{R_{11} + \gamma_{11}\theta_{11}(t) + \frac{(R_{12} + \gamma_{12}\theta_{12}(t))(k + \delta\kappa(t))}{n + \nu\eta(t)}}{L_{11} + \lambda_{11}\chi_{11}(t)\frac{(M_{12} + \lambda_{12}\chi_{12}(t))(k + \delta\kappa(t))}{n + \nu\eta(t)}}\right]I_1(t)dt \\ &+ \left[L_{11} + \lambda_{11}\chi_{11}(t) + \frac{(M_{12} + \lambda_{12}\chi_{12}(t))(k + \delta\kappa(t))}{n + \nu\eta(t)}\right]^{-1}V_1(t)dt \\ &+ \alpha\left[L_{11} + \lambda_{11}\chi_{11}(t) + \frac{(M_{12} + \lambda_{12}\chi_{12}(t))(k + \delta\kappa(t))}{n + \nu\eta(t)}\right]^{-1}dW_1(t) \end{aligned}, \tag{25}$$

$$dI_2(t) = -\left[\frac{R_{22}+\gamma_{22}\theta_{22}(t)+\frac{(R_{21}+\gamma_{21}\theta_{21}(t))(k+\delta\kappa(t))}{n+\nu\eta(t)}}{L_{22}+\lambda_{22}\chi_{22}(t)\frac{(M_{21}+\lambda_{21}\chi_{21}(t))(k+\delta\kappa(t))}{n+\nu\eta(t)}}\right]I_1(t)dt + \left[L_{22}+\lambda_{22}\chi_{22}(t)+\frac{(M_{21}+\lambda_{21}\chi_{21}(t))(k+\delta\kappa(t))}{n+\nu\eta(t)}\right]^{-1}V_2(t)dt + \beta\left[L_{22}+\lambda_{22}\chi_{22}(t)+\frac{(M_{21}+\lambda_{21}\chi_{21}(t))(k+\delta\kappa(t))}{n+\nu\eta(t)}\right]^{-1}dW_2(t) \quad (26)$$

which show that $I_1(t)$ and $I_2(t)$ are subjected to both additive noise caused by noise in the voltage source and multiplicative noise due to random variations in the transformer's elements/parameters.

By combining (19)-(22), (25) and (26), the vector SDEs of the transformer can be finally obtained as given by (27) and (28) where all vectors, matrices and their elements can be given by (29)-(44).

$$d\boldsymbol{X}_1(t)[\boldsymbol{A}_1(t)+\boldsymbol{B}_1(t)]dt+\boldsymbol{C}_1(t)d\boldsymbol{\Omega}_1(t), \quad (27)$$

$$d\boldsymbol{X}_2(t)[\boldsymbol{A}_2(t)+\boldsymbol{B}_2(t)]dt+\boldsymbol{C}_2(t)d\boldsymbol{\Omega}_2(t). \quad (28)$$

$$\boldsymbol{X}_1(t) = [\theta_{11}(t)\ \theta_{12}(t)\ \chi_{11}(t)\ \chi_{12}(t)\ \eta(t)\ \kappa(t)\ I_1(t)]^T, \quad (29)$$

$$\boldsymbol{A}_1(t) = \begin{bmatrix} -\rho_{\theta,11} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\rho_{\theta,12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\rho_{\chi,11} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\rho_{\chi,12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\rho_{\eta} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\rho_{\kappa} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & A_1(t) \end{bmatrix}, \quad (30)$$

$$A_1(t) = -\left[\frac{R_{11}+\gamma_{11}\theta_{11}(t)+\frac{(R_{12}+\gamma_{12}\theta_{12}(t))(k+\delta\kappa(t))}{n+\nu\eta(t)}}{L_{11}+\lambda_{11}\chi_{11}(t)\frac{(M_{12}+\lambda_{12}\chi_{12}(t))(k+\delta\kappa(t))}{n+\nu\eta(t)}}\right], \quad (31)$$

$$\boldsymbol{B}_1(t)[0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ B_1(t)]^T, \quad (32)$$

$$B_1(t) = \left[L_{11}+\lambda_{11}\chi_{11}(t)+\frac{(M_{12}+\lambda_{12}\chi_{12}(t))(k+\delta\kappa(t))}{n+\nu\eta(t)}\right]^{-1}V_1(t), \quad (33)$$

$$\boldsymbol{C}_1(t) = \begin{bmatrix} -\sigma_{\theta,11}\rho_{\theta,11} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\sigma_{\theta,12}\rho_{\theta,12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\sigma_{\chi,11}\rho_{\chi,11} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\sigma_{\chi,12}\rho_{\chi,12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\sigma_{\eta}\rho_{\eta} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\sigma_{\kappa}\rho_{\kappa} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & C_1(t) \end{bmatrix}, \quad (34)$$

$$C_1(t) = \alpha\left[L_{11}+\lambda_{11}\chi_{11}(t)+\frac{(M_{12}+\lambda_{12}\chi_{12}(t))(k+\delta\kappa(t))}{n+\nu\eta(t)}\right]^{-1}, \quad (35)$$

$$\boldsymbol{\Omega}_1(t) = [W_{11}(t)\ W_{12}(t)\ X_{11}(t)\ X_{12}(t)\ N(t)\ K(t)\ W_1(t)]^T, \quad (36)$$

$$\boldsymbol{X}_2(t) = [\theta_{22}(t)\ \theta_{21}(t)\ \chi_{22}(t)\ \chi_{21}(t)\ \eta(t)\ \kappa(t)\ I_2(t)]^T, \quad (37)$$

$$\boldsymbol{A}_2(t) = \begin{bmatrix} -\rho_{\theta,22} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\rho_{\theta,21} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\rho_{\chi,22} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\rho_{\chi,21} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\rho_{\eta} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\rho_{\kappa} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & A_2(t) \end{bmatrix}, \quad (38)$$

$$A_2(t) = -\left[\frac{R_{22}+\gamma_{22}\theta_{22}(t)+\frac{(R_{21}+\gamma_{21}\theta_{21}(t))(k+\delta\kappa(t))}{n+\nu\eta(t)}}{L_{22}+\lambda_{22}\chi_{22}(t)\frac{(M_{21}+\lambda_{21}\chi_{21}(t))(k+\delta\kappa(t))}{n+\nu\eta(t)}}\right], \quad (39)$$

$$\boldsymbol{B}_2(t) = [0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ B_2(t)]^T, \quad (40)$$

$$B_2(t) = \left[L_{22}+\lambda_{22}\chi_{22}(t)+\frac{(M_{21}+\lambda_{21}\chi_{21}(t))(k+\delta\kappa(t))}{n+\nu\eta(t)}\right]^{-1}V_2(t) \quad (41)$$

$$\boldsymbol{C}_1(t) = \begin{bmatrix} -\sigma_{\theta,22}\rho_{\theta,22} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\sigma_{\theta,21}\rho_{\theta,21} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\sigma_{\chi,22}\rho_{\chi,22} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\sigma_{\chi,21}\rho_{\chi,21} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\sigma_{\eta}\rho_{\eta} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\sigma_{\kappa}\rho_{\kappa} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & C_2(t) \end{bmatrix}, \quad (42)$$

$$C_2(t) = \beta\left[L_{22}+\lambda_{22}\chi_{22}(t)+\frac{(M_{21}+\lambda_{21}\chi_{21}(t))(k+\delta\kappa(t))}{n+\nu\eta(t)}\right]^{-1}, \quad (43)$$

$$\boldsymbol{\Omega}_2(t) = [W_{22}(t)\ W_{21}(t)\ X_{22}(t)\ X_{21}(t)\ N(t)\ K(t)\ W_2(t)]^T. \quad (44)$$

By applying the methodology adopted in [9], we define

$$\boldsymbol{\Phi}_1(t,t_0) = \boldsymbol{I} + \sum_{n=1}^{\infty}[\int_{t_0}^{t}\dots\int_{t_0}^{t}\int_{t_0}^{t}\boldsymbol{A}_1(t_1)\boldsymbol{A}_1(t_2)\dots\boldsymbol{A}_1(t_n dt_n dt_{n-1}\dots dt_1], \quad (45)$$

$$\boldsymbol{\Phi}_2(t,t_0) = \boldsymbol{I} + \sum_{n=1}^{\infty}[\int_{t_0}^{t}\dots\int_{t_0}^{t}\int_{t_0}^{t}\boldsymbol{A}_2(t_1)\boldsymbol{A}_2(t_2)\dots\boldsymbol{A}_2(t_n dt_n dt_{n-1}\dots dt_1], \quad (46)$$

Note that $t_0 < t_n < t_{n-1} < \ldots < t_2 < t_1 < t$ [9]. As a result, the solutions of our vector SDEs can be respectively obtained as given by (47) and (48) where the stochastic integrals have been interpreted in the Ito sense.

$$\boldsymbol{X}_1 = \boldsymbol{\Phi}_1(t,t_0)\boldsymbol{X}_1(t_0) + \int_{t_0}^{t}\boldsymbol{\Phi}_1(t,\tau)\boldsymbol{B}_1(\tau)d\tau + \int_{t_0}^{t}\boldsymbol{\Phi}_1(t,\tau)\boldsymbol{C}_1(\tau)d\boldsymbol{\Omega}_1(\tau), \quad (47)$$

$$\boldsymbol{X}_2 = \boldsymbol{\Phi}_2(t,t_0)\boldsymbol{X}_2(t_0) + \int_{t_0}^{t}\boldsymbol{\Phi}_2(t,\tau)\boldsymbol{B}_2(\tau)d\tau + \int_{t_0}^{t}\boldsymbol{\Phi}_1(t,\tau)\boldsymbol{C}_2(\tau)d\boldsymbol{\Omega}_2(\tau), \quad (48)$$

From $\boldsymbol{X}_1(t)$ and $\boldsymbol{X}_2(t)$, we have found that $I_1(t)$ and $I_2(t)$ which are the interested stochastic electrical quantities of the transformer, can be simply determined as given by (49) and (50) where time dependencies of $\boldsymbol{X}_1(t)$ and $\boldsymbol{X}_2(t)$ have not been shown for convenience.

$$I_1(t) = \boldsymbol{X}_1[7,1], \tag{49}$$

$$I_2(t) = \boldsymbol{X}_2[7,1]. \tag{50}$$

For the classical off-chip transformer, we have $k = 1$, $R_{12} = R_{21} = 0$ and

$$\boldsymbol{X}_1(t) = [\theta_{11}(t) \quad \chi_{11}(t) \quad \chi_{12}(t) \quad \eta(t) \quad I_1(t)]^T, \tag{51}$$

$$\boldsymbol{A}_1(t) = \begin{bmatrix} -\rho_{\theta,11} & 0 & 0 & 0 & 0 \\ 0 & -\rho_{\chi,11} & 0 & 0 & 0 \\ 0 & 0 & -\rho_{\chi,12} & 0 & 0 \\ 0 & 0 & 0 & -\rho_\eta & 0 \\ 0 & 0 & 0 & 0 & A_1(t) \end{bmatrix}, \tag{52}$$

$$A_1(t) = -\frac{R_{11} + \gamma_{11}\theta_{11}(t)}{L_{11} + \lambda_{11}\chi_{11}(t) + (M_{12} + \lambda_{12}\chi_{12}(t))(n + \kappa\nu(t))^1}, \tag{53}$$

$$\boldsymbol{B}_1(t) = [0 \quad 0 \quad 0 \quad 0 \quad B_1(t)]^T, \tag{54}$$

$$B_1(t) = \left[L_{11} + \lambda_{11}\chi_{11}(t) + \frac{M_{12} + \lambda_{12}\chi_{12}(t)}{n + \nu\eta(t)}\right]^{-1} V_1(t), \tag{55}$$

$$\boldsymbol{C}_1(t) = \begin{bmatrix} -\sigma_{\theta,11}\rho_{\theta,11} & 0 & 0 & 0 & 0 \\ 0 & -\sigma_{\chi,11}\rho_{\chi,11} & 0 & 0 & 0 \\ 0 & 0 & -\sigma_{\chi,12}\rho_{\chi,12} & 0 & 0 \\ 0 & 0 & 0 & -\sigma_\eta\rho_\eta & 0 \\ 0 & 0 & 0 & 0 & C_1(t) \end{bmatrix}, \tag{56}$$

$$C_1(t) = \alpha\left[L_{11} + \lambda_{11}\chi_{11}(t) + \frac{M_{12} + \lambda_{12}\chi_{12}(t)}{n + \nu\eta(t)}\right]^{-1}, \tag{57}$$

$$\boldsymbol{\Omega}_1(t) = [W_{11}(t) \quad X_{11}(t) \quad X_{12}(t) \quad N(t) \quad W_1(t)]^T, \tag{58}$$

$$\boldsymbol{X}_2(t) = [\theta_{22}(t) \quad \chi_{22}(t) \quad \chi_{21}(t) \quad N(t) \quad I_2(t)]^T, \tag{59}$$

$$\boldsymbol{A}_2(t) = \begin{bmatrix} -\rho_{\theta,22} & 0 & 0 & 0 & 0 \\ 0 & -\rho_{\chi,22} & 0 & 0 & 0 \\ 0 & 0 & -\rho_{\chi,21} & 0 & 0 \\ 0 & 0 & 0 & -\rho_\eta & 0 \\ 0 & 0 & 0 & 0 & A_2(t) \end{bmatrix}, \tag{60}$$

$$A_2(t) = -\frac{R_{22} + \gamma_{22}\theta_{22}(t)}{L_{22} + \lambda_{22}\chi_{22}(t) + (M_{21} + \lambda_{21}\chi_{21}(t))(n + \kappa\nu(t))^1}, \tag{61}$$

$$\boldsymbol{B}_2(t) = [0 \quad 0 \quad 0 \quad 0 \quad B_2(t)]^T, \tag{62}$$

$$B_2(t) = [L_{22} + \lambda_{22}\chi_{22}(t) + (M_{21} + \lambda_{21}\chi_{21}(t))(n + \nu\eta(t))]^{-1} V_2(t), \tag{63}$$

$$C_2(t) = \begin{bmatrix} -\sigma_{\theta,22}\rho_{\theta,22} & 0 & 0 & 0 & 0 \\ 0 & -\sigma_{\chi,22}\rho_{\chi,22} & 0 & 0 & 0 \\ 0 & 0 & -\sigma_{\theta,21}\rho_{\theta,21} & 0 & 0 \\ 0 & 0 & 0 & -\sigma_\eta\rho_\eta & 0 \\ 0 & 0 & 0 & 0 & C_2(t) \end{bmatrix}, \tag{64}$$

$$C_2(t) = \beta[L_{22} + \lambda_{22}\chi_{22}(t) + (M_{21} + \lambda_{21}\chi_{21}(t))(n + \nu\eta(t))]^{-1}, \tag{65}$$

$$\boldsymbol{\Omega}_2(t) = [\ W_{22}(t) \quad X_{22}(t) \quad X_{21}(t) \quad N(t) \quad W_2(t)\ ]^T. \tag{66}$$

As a result, $I_1(t)$ and $I_2(t)$ can be respectively obtained from $X_1(t)$ and $X_2(t)$ which can be solved by using (45)-(48) and (51)-(66), as follows

$$I_1(t) = \boldsymbol{X}_1[5,1], \tag{67}$$

$$I_2(t) = \boldsymbol{X}_2[5,1]. \tag{68}$$

At this point, we will study the stochastic properties of I1(t) and I2(t). The expectations of $I_1(t)$ and $I_2(t)$, called $E[I_1(t)]$ and $E[I_2(t)]$, are respectively equal to $i_1(t)$ and $i_2(t)$ given by (5) and (6) where k, R_{12} and $R_{21} = 0$ become 1, 0 and 0 respectively for the classical off-chip transformer. This is because noise in the voltage source along and the random variations in elements/parameters have zero means. According to these time variant expectations, $I_1(t)$ and $I_2(t)$ are non-stationary stochastic processes. It can also be seen that both $I_1(t)$ and $I_2(t)$ are non-stochastic processes despite the fact that the causes of their stochastic variations are. This is because $I_1(t)$ and $I_2(t)$ are also subject to the multiplicative color noise besides the additive white noise as mentioned above. Now, the correlation between $I_1(t)$ and $I_2(t)$ will be analyzed by comparing the cross correlation function of $I_1(t)$ and $I_2(t)$ at $t_1 = t_2 = t$ where t_1

and t_2 are arbitrary points on t-axis i.e. R_{I_1,I_2}, to $E[I_1(t)]E[I_2(t)]$. Such $R_{I_1,I_2}(t)$ can be defined as

$$R_{I_1,I_2}(t) = E[I_1(t)I_2(t)]. \quad (69)$$

In order to determine , the cross correlation matrix of $X_1(t)$ and $X_2(t)$ at $t_1 = t_2 = t$ i.e. $\boldsymbol{R}_{x_1,x_2}(t)$, must be formulated first. For arbitrary t1 and t2, the cross correlation matrix of $X_1(t)$ and $X_2(t)$ can be defined as

$$\boldsymbol{R}_{x_1,x_2}(t_1,t_2) = E[\boldsymbol{X}_1(t_1)\boldsymbol{X}_2^T(t_2)]. \quad (70)$$

As a result, $\boldsymbol{R}_{x_1,x_2}$ of the on-chip transformer can be given by

$$\boldsymbol{R}_{x_1,x_2}(t) = \begin{bmatrix} E[\theta_{11}(t)\theta_{22}(t)] & E[\theta_{11}(t)\theta_{21}(t)] & E[\theta_{11}(t)\chi_{22}(t)] & E[\theta_{11}(t)\chi_{21}(t)] & E[\theta_{11}(t)\eta(t)] & E[\theta_{11}(t)\kappa(t)] & E[\theta_{11}(t)I_2(t)] \\ E[\theta_{12}(t)\theta_{22}(t)] & E[\theta_{12}(t)\theta_{21}(t)] & E[\theta_{12}(t)\chi_{22}(t)] & E[\theta_{12}(t)\chi_{21}(t)] & E[\theta_{12}(t)\eta(t)] & E[\theta_{12}(t)\kappa(t)] & E[\theta_{12}(t)I_2(t)] \\ E[\chi_{11}(t)\theta_{22}(t)] & E[\chi_{11}\theta_{21}(t)] & E[\chi_{11}\theta_{22}(t)] & E[\chi_{11}(t)\chi_{21}(t)] & E[\chi_{11}(t)\eta(t)] & E[\chi_{11}(t)\kappa(t)] & E[\chi_{11}(t)I_2(t)] \\ E[\chi_{12}(t)\theta_{22}(t)] & E[\chi_{12}\theta_{21}(t)] & E[\chi_{12}\chi_{22}(t)] & E[\chi_{12}\chi_{21}(t)] & E[\chi_{12}(t)\eta(t)] & E[\chi_{12}(t)\kappa(t)] & E[\chi_{12}(t)I_2(t)] \\ E[\eta(t)\theta_{22}(t)] & E[\eta(t)\theta_{21}(t)] & E[\eta(t)\chi_{22}(t)] & E[\eta(t)\chi_{21}(t)] & 0.5\sigma_\eta^2\rho_\eta^2 & E[\eta(t)\kappa(t)] & E[\eta(t)I_2(t)] \\ E[\kappa(t)\theta_{22}(t)] & E[\kappa(t)\theta_{21}(t)] & E[\kappa(t)\chi_{22}(t)] & E[\kappa(t)\chi_{21}(t)] & E[\kappa(t)\eta(t)] & 0.5\sigma_\kappa^2\rho_\kappa^2 & E[\kappa(t)I_2(t)] \\ E[I_1(t)\theta_{22}(t)] & E[I_1(t)\theta_{21}(t)] & E[I_1(t)\chi_{22}(t)] & E[I_1(t)\chi_{21}(t)] & E[I_1(t)\eta(t)] & E[I_1(t)\kappa(t)] & E[I_1(t)I_2(t)] \end{bmatrix}, \quad (71)$$

On the other hand, R_{x_1,x_2} of the off-chip transformer can be obtained as follows

$$\boldsymbol{R}_{x_1,x_2}(t) = \begin{bmatrix} E[\theta_{11}(t)\theta_{22}(t)] & E[\theta_{11}(t)\chi_{22}(t)] & E[\theta_{11}(t)\chi_{21}(t)] & E[\theta_{11}(t)\eta(t)] & E[\theta_{11}(t)I_2(t)] \\ E[\chi_{11}(t)\theta_{22}(t)] & E[\chi_{11}(t)\chi_{22}(t)] & E[\chi_{11}(t)\chi_{21}(t)] & E[\chi_{11}(t)\eta(t)] & E[\chi_{11}(t)I_2(t)] \\ E[\chi_{11}(t)\theta_{22}(t)] & E[\chi_{12}(t)\chi_{22}(t)] & E[\chi_{12}(t)\chi_{21}(t)] & E[\chi_{12}(t)\eta(t)] & E[\chi_{12}(t)I_2(t)] \\ E[\eta_{11}(t)\theta_{22}(t)] & E[\eta(t)\chi_{22}(t)] & E[\eta(t)\chi_{21}(t)] & 0.5\sigma_\eta^2\rho_\eta^2 & E[\eta(t)I_2(t)] \\ E[I_1(t)\theta_{22}(t)] & E[I_1(t)\chi_{22}(t)] & E[I_1(t)\chi_{21}(t)] & E[I_1(t)\eta(t)] & E[I_1^1(t)] - I_2(t) \end{bmatrix}, \quad (72)$$

It should be mentioned here that (71) and (72) have been derived by keeping in mind that both $\eta(t)$ and $\kappa(t)$ employ zero mean. By using (70)-(72), we have found that $R_{I_1,I_2}(t) = \boldsymbol{R}_{x_1,x_2}[7,7]$ and $R_{I_1,I_2}(t) = \boldsymbol{R}_{x_1,x_2}$ for the on-chip and off-chip transformer respectively. In addition, all expectation operator termed elements of $\boldsymbol{R}_{x_1,x_2}$ can be analytically given in terms of the correlation coefficients. As an example, $E[\theta_{11}(t)\theta_{22}(t)]$ can be found as

$$E[\theta_{11}(t)\theta_{22}(t)] = 0.5P_{\theta_{11}\theta_{22}}\sigma_{\theta_{11}}\sigma_{\theta_{22}}\rho_{\theta_{11}}\rho_{\theta_{22}}. \quad (73)$$

Noted that $P_{\theta_{11}\theta_{22}}$ denotes the correlation coefficient of $\theta_{11}(t)$ and $\theta_{22}(t)$. In addition, (73) has been derived by keeping in mind that $\theta_{11}(t)$ and $\theta_{22}(t)$ employ zero means and $\theta_{11}(t)\theta_{22}(t)$ employs a normal product distribution [23], as $\theta_{11}(t)$ and $\theta_{22}(t)$ are normally distributed.

For both on-chip and off-chip transformers, it has been found that

$$R_{I_1,I_2} = P_{I_2I_2}\sqrt{(E[I_1^2(t)] - i_1^2(t))(E[I_2^2(t)] - i_2^2(t))} + i_1(t)i_2(t). \quad (74)$$

Note that $P_{I_1I_2}$ stands for the correlation coefficient of $I_1(t)$ and $I_2(t)$. In addition, (74) has been derived by keeping in mind that $E[I_1(t)] = i_1(t)$ and $E[I_2(t)] = i_2(t)$ which also yields,

$$E[I_1(t)]E[I_2(t)] = i_1(t)i_2(t). \quad (75)$$

As a result, it can be seen that $R_{I_2,I_2}(t) \neq E[I_1(t)]E[I_2(t)]$, and now $I_1(t)$ and $I_2(t)$ are stochastically correlated.

Next, let the covariance matrices of $\boldsymbol{X}_1(t)$ and $\boldsymbol{X}_2(t)$ be denoted by $\boldsymbol{V}_1(t)$ and $\boldsymbol{V}_2(t)$ respectively. These covariance matrices, which can be applied for determining the variances of its $I_1(t)$ and $I_2(t)$, can be defined as

$$\boldsymbol{V}_1(t) = E\left[\boldsymbol{X}_1(t) - E[\boldsymbol{X}_1(t)](\boldsymbol{X}_1(t) - E[\boldsymbol{X}_1(t)])^T\right], \quad (76)$$

$$\boldsymbol{V}_2(t) = E\left[\boldsymbol{X}_2(t) - E[\boldsymbol{X}_2(t)](\boldsymbol{X}_2(t) - E[\boldsymbol{X}_2(t)])^T\right]. \quad (77)$$

As a result, $\boldsymbol{V}_1(t)$ and $\boldsymbol{V}_2(t)$ of the on-chip transformer can be given by

$$\boldsymbol{V}_1(t) = \begin{bmatrix} 0.5\sigma_{\theta_{11}}^2\rho_{\theta_{11}}^2 & E[\theta_{11}(t)\theta_{12}(t)] & E[\theta_{11}(t)\chi_{11}(t)] & E[\theta_{11}(t)\chi_{12}(t)] & E[\theta_{11}(t)\eta(t)] & E[\theta_{11}(t)\kappa(t)] & E[\theta_{11}(t)I_1(t)] \\ E[\theta_{11}(t)\theta_{12}(t)] & 0.5\sigma_{\theta_{12}}^2\rho_{\theta_{12}}^2 & E[\theta_{12}(t)\chi_{11}(t)] & E[\theta_{12}(t)\chi_{12}(t)] & E[\theta_{12}(t)\eta(t) & E[\theta_{12}(t)\kappa(t)] & E[\theta_{12}(t)I_1(t)] \\ E[\theta_{11}(t)\chi_{11}(t)] & E[\theta_{12}\chi_{11}(t)] & 0.5\sigma_{\chi_{11}}^2\rho_{\chi_{11}}^2 & E[\chi_{11}(t)\chi_{12}(t)] & E[\chi_{11}(t)\eta(t)] & E[\chi_{11}(t)\kappa(t) & E[\chi_{11}(t)I_1(t) \\ E[\theta_{11}(t)\chi_{12}(t)] & E[\theta_{12}\chi_{12}(t)] & E[\chi_{11}\chi_{12}(t)] & 0.5\sigma_{\chi_{12}}^2\rho_{\chi_{12}}^2 & E[\chi_{12}(t)\eta(t)] & E[\chi_{12}(t)\kappa(t)] & E[\chi_{12}(t)I_1(t)] \\ E[\theta_{11}(t)\eta(t)] & E[\theta_{12}(t)\eta(t)] & E[\chi_{11}(t)\eta(t)] & E[\chi_{12}(t)\eta(t)] & 0.5\sigma_\eta^2\rho_\eta^2 & E[\eta(t)\kappa(t)] & E[\eta(t)I_1(t)] \\ E[\theta_{11}(t)\kappa(t)] & E[\theta_{12}(t)\kappa(t)] & E[\chi_{11}(t)\kappa(t)] & E[\chi_{12}(t)\kappa(t)] & E[\kappa(t)\eta(t)] & 0.5\sigma_\kappa^2\rho_\kappa^2 & E[\kappa(t)I_1(t)] \\ E[\theta_{11}(t)I_1(t)] & E[\theta_{12}(t)I_1(t)] & E[\chi_{11}(t)I_1(t)] & E[\chi_{12}(t)I_2(t)] & E[\eta(t)I_1(t)] & E[\kappa(t)I_1(t)] & E[I_1^2(t)] - i_1^2(t) \end{bmatrix}, \quad (78)$$

$$\boldsymbol{V}_2(t) = \begin{bmatrix} 0.5\sigma_{\theta_{22}}^2\rho_{\theta_{22}}^2 & E[\theta_{22}(t)\theta_{21}(t)] & E[\theta_{22}(t)\chi_{22}(t)] & E[\theta_{22}(t)\chi_{21}(t)] & E[\theta_{22}(t)\eta(t)] & E[\theta_{22}(t)\kappa(t)] & E[\theta_{22}(t)I_2(t)] \\ E[\theta_{22}(t)\theta_{21}(t)] & 0.5\sigma_{\theta_{21}}^2\rho_{\theta_{21}}^2 & E[\theta_{21}(t)\chi_{22}(t)] & E[\theta_{21}(t)\chi_{21}(t)] & E[\theta_{21}(t)\eta(t) & E[\theta_{21}(t)\kappa(t)] & E[\theta_{21}(t)I_2(t)] \\ E[\theta_{22}(t)\chi_{11}(t)] & E[\theta_{21}\chi_{22}(t)] & 0.5\sigma_{\chi_{22}}^2\rho_{\chi_{22}}^2 & E[\chi_{22}(t)\chi_{21}(t)] & E[\chi_{22}(t)\eta(t)] & E[\chi_{22}(t)\kappa(t) & E[\chi_{22}(t)I_2(t) \\ E[\theta_{22}(t)\chi_{21}(t)] & E[\theta_{21}\chi_{12}(t)] & E[\chi_{22}\chi_{12}(t)] & 0.5\sigma_{\chi_{21}}^2\rho_{\chi_{21}}^{21} & E[\chi_{21}(t)\eta(t)] & E[\chi_{21}(t)\kappa(t)] & E[\chi_{21}(t)I_2(t)] \\ E[\theta_{22}(t)\eta(t)] & E[\theta_{21}(t)\eta(t)] & E[\chi_{11}(t)\eta(t)] & E[\chi_{21}(t)\eta(t)] & 0.5\sigma_\eta^2\rho_\eta^2 & E[\eta(t)\kappa(t)] & E[\eta(t)I_2(t)] \\ E[\theta_{22}(t)\kappa(t)] & E[\theta_{21}(t)\kappa(t)] & E[\chi_{11}(t)\kappa(t)] & E[\chi_{21}(t)\kappa(t)] & E[\kappa(t)\eta(t)] & 0.5\sigma_\kappa^2\rho_\kappa^2 & E[\kappa(t)I_2(t)] \\ E[\theta_{22}(t)I_2(t)] & E[\theta_{21}(t)I_1(t)] & E[\chi_{22}(t)I_2(t)] & E[\chi_{21}(t)I_2(t)] & E[\eta(t)I_2(t)] & E[\kappa(t)I_2(t)] & E[I_1^2(t)] - i_2^2(t) \end{bmatrix}. \quad (79)$$

Since the variance of any random variable can be given by its mean square subtracted by the square of its mean, the variances of $I_1(t)$ and $I_2(t)$ of the on-chip transformer are respectively equal to $\boldsymbol{V}_1[7,7]$ and $\boldsymbol{V}_2[7,7]$.

For the off-chip transformer on the other hand, we have

$$\boldsymbol{V}_1(t) = \begin{bmatrix} 0.5\sigma_{\theta_{11}}^2\rho_{\theta_{11}}^2 & E[\theta_{11}(t)\chi_{11}(t)] & E[\theta_{11}(t)\chi_{12}(t)] & E[\theta_{11}(t)\eta(t)] & E[\theta_{11}(t)I_1(t)] \\ E[\theta_{11}(t)\chi_{11}(t)] & 0.5\sigma_{\chi_{11}}^2\rho_{\chi_{11}}^2 & E[\chi_{11}(t)\chi_{12}(t)] & E[\chi_{11}(t)\eta(t)] & E[\chi_{11}(t)I_1(t)] \\ E[\theta_{11}(t)\chi_{12}(t)] & E[\chi_{11}(t)\chi_{12}(t)] & 0.5\sigma_{\chi_{12}}^2\rho_{\chi_{12}}^2 & E[\chi_{12}(t)\eta(t)] & E[\chi_{12}(t)I_1(t)] \\ E[\theta_{11}(t)\eta(t)] & E[\chi_{11}(t)\eta(t)] & E[\chi_{12}(t)\eta(t)] & 0.5\sigma^2\rho_\eta^2 & E[\eta(t)I_1(t)] \\ E[\theta_{11}(t)I_1(t)] & E[\chi_{11}(t)I_1(t)] & E[\chi_{12}(t)I_1(t)] & E[\eta(t)I_1(t)] & E[I_1^2(t)] - i_1^2(t) \end{bmatrix}, \quad (80)$$

$$\boldsymbol{V}_2(t) = \begin{bmatrix} 0.5\sigma_{\theta_{22}}^2\rho_{\theta_{22}}^2 & E[\theta_{22}(t)\chi_{22}(t)] & E[\theta_{22}(t)\chi_{21}(t)] & E[\theta_{22}(t)\eta(t)] & E[\theta_{22}(t)I_2(t)] \\ E[\theta_{22}(t)\chi_{22}(t)] & 0.5\sigma_{\chi_{22}}^2\rho_{\chi_{22}}^2 & E[\chi_{22}(t)\chi_{21}(t)] & E[\chi_{22}(t)\eta(t)] & E[\chi_{22}(t)I_1(t)] \\ E[\theta_{22}(t)\chi_{21}(t)] & E[\chi_{22}(t)\chi_{21}(t)] & 0.5\sigma_{\chi_{12}}^2\rho_{\chi_{12}}^2 & E[\chi_{21}(t)\eta(t)] & E[\chi_{21}(t)I_1(t)] \\ E[\theta_{22}(t)\eta(t)] & E[\chi_{22}(t)\eta(t)] & E[\chi_{22}(t)\eta(t)] & 0.5\sigma^2\rho_\eta^2 & E[\eta(t)I_2(t)] \\ E[\theta_{22}(t)I_2(t)] & E[\chi_{22}(t)I_2(t)] & E[\chi_{22}(t)I_2(t)] & E[\eta(t)I_2(t)] & E[I_2^2(t)] - i_2^2(t) \end{bmatrix}, \quad (81)$$

Thus the variances of its $I_1(t)$ and $I_2(t)$ can be respectively given by $\boldsymbol{V}_1[5,5]$ and $\boldsymbol{V}_2[5,5]$.

Before we proceed further, the second moment matrices of $\boldsymbol{X}_1(t)$ and $\boldsymbol{X}_2(t)$ i.e. $\boldsymbol{P}_1(t)$ and $\boldsymbol{P}_2(t)$, which can be applied for determining the second moments of $I_1(t)$ and $I_2(t)$, will be derived. These second moment matrices can be defined as

$$\boldsymbol{P}_1(t) = \boldsymbol{V}_1(t) + E[\boldsymbol{X}_1(t)]E[\boldsymbol{X}_1^T(t)]. \quad (82)$$

$$\boldsymbol{P}_2(t) = \boldsymbol{V}_2(t) + E[\boldsymbol{X}_2(t)]E[\boldsymbol{X}_2^T(t)]. \quad (83)$$

For the on-chip transformer, we have

$$E[\boldsymbol{X}_1(t)] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & i_1(t) \end{bmatrix}^T, \quad (84)$$

$$E[\boldsymbol{X}_2(t)] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & i_2(t) \end{bmatrix}^T. \quad (85)$$

For the off-chip device, it can be seen that

$$E[\mathbf{X_1(t)}] = \begin{bmatrix} 0 & 0 & 0 & 0 & i_1(t) \end{bmatrix}^{\mathbf{T}}, \quad (86)$$

$$E[\mathbf{X_2(t)}] = \begin{bmatrix} 0 & 0 & 0 & 0 & i_2(t) \end{bmatrix}^{\mathbf{T}}. \quad (87)$$

As a result, $\boldsymbol{P}_1(t)$ and $\boldsymbol{P}_2(t)$ of the for the on-chip transformer can be given by (88) and (89), thus the second moment of $I_1(t)$ and $I_2(t)$ of the on-chip transformer are respectively equal to $\boldsymbol{P}_1[7,7]$ and $\boldsymbol{P}_2[7,7]$. In addition, $\boldsymbol{P}_1(t)$ and $\boldsymbol{P}_2(t)$ of the off-chip transformer can be given by (90) and (91) therefore the second moment of $I_1(t)$ and $I_2(t)$ of the off-chip transformer are respectively equal to $\boldsymbol{P}_1[5,5]$ and $\boldsymbol{P}_2[5,5]$.

$$\boldsymbol{P}_1(t) = \begin{bmatrix} 0.5\sigma_{\theta_{11}}^2\rho_{\theta_{11}}^2 & E[\theta_{11}(t)\theta_{12}(t)] & E[\theta_{11}(t)\chi_{11}(t)] & E[\theta_{11}(t)\chi_{12}(t)] & E[\theta_{11}(t)\eta(t)] & E[\theta_{11}(t)\kappa(t)] & E[\theta_{11}(t)I_1(t)] \\ E[\theta_{11}(t)\theta_{12}(t)] & 0.5\sigma_{\theta_{12}}^2\rho_{\theta_{12}}^2 & E[\theta_{12}(t)\chi_{11}(t)] & E[\theta_{12}(t)\chi_{12}(t)] & E[\theta_{12}(t)\eta(t) & E[\theta_{12}(t)\kappa(t)] & E[\theta_{12}(t)I_1(t)] \\ E[\theta_{11}(t)\chi_{11}(t)] & E[\theta_{12}\chi_{11}(t)] & 0.5\sigma_{\chi_{11}}^2\rho_{\chi_{11}}^2 & E[\chi_{11}(t)\chi_{12}(t)] & E[\chi_{11}(t)\eta(t)] & E[\chi_{11}(t)\kappa(t) & E[\chi_{11}(t)I_1(t) \\ E[\theta_{11}(t)\chi_{12}(t)] & E[\theta_{12}\chi_{12}(t)] & E[\chi_{11}\chi_{12}(t)] & 0.5\sigma_{\chi_{12}}^2\rho_{\chi_{12}}^2 & E[\chi_{12}(t)\eta(t)] & E[\chi_{12}(t)\kappa(t)] & E[\chi_{12}(t)I_1(t)] \\ E[\theta_{11}(t)\eta(t)] & E[\theta_{12}(t)\eta(t)] & E[\chi_{11}(t)\eta(t)] & E[\chi_{12}(t)\eta(t)] & 0.5\sigma_\eta^2\rho_\eta^2 & E[\eta(t)\kappa(t)] & E[\eta(t)I_1(t)] \\ E[\theta_{11}(t)\kappa(t)] & E[\theta_{12}(t)\kappa(t)] & E[\chi_{11}(t)\kappa(t)] & E[\chi_{12}(t)\kappa(t)] & E[\kappa(t)\eta(t)] & 0.5\sigma_\kappa^2\rho_\kappa^2 & E[\kappa(t)I_1(t)] \\ E[\theta_{11}(t)I_1(t)] & E[\theta_{12}(t)I_1(t)] & E[\chi_{11}(t)I_1(t)] & E[\chi_{12}(t)I_2(t)] & E[\eta(t)I_1(t)] & E[\kappa(t)I_1(t)] & E[I_1^2(t)] \end{bmatrix}, \quad (88)$$

$$\boldsymbol{P}_2(t) = \begin{bmatrix} 0.5\sigma_{\theta_{22}}^2\rho_{\theta_{22}}^2 & E[\theta_{22}(t)\theta_{21}(t)] & E[\theta_{22}(t)\chi_{22}(t)] & E[\theta_{22}(t)\chi_{21}(t)] & E[\theta_{22}(t)\eta(t)] & E[\theta_{22}(t)\kappa(t)] & E[\theta_{22}(t)I_2(t)] \\ E[\theta_{22}(t)\theta_{21}(t)] & 0.5\sigma_{\theta_{21}}^2\rho_{\theta_{21}}^2 & E[\theta_{21}(t)\chi_{22}(t)] & E[\theta_{21}(t)\chi_{21}(t)] & E[\theta_{21}(t)\eta(t) & E[\theta_{21}(t)\kappa(t)] & E[\theta_{21}(t)I_2(t)] \\ E[\theta_{22}(t)\chi_{11}(t)] & E[\theta_{21}\chi_{22}(t)] & 0.5\sigma_{\chi_{22}}^2\rho_{\chi_{22}}^2 & E[\chi_{22}(t)\chi_{21}(t)] & E[\chi_{22}(t)\eta(t)] & E[\chi_{22}(t)\kappa(t) & E[\chi_{22}(t)I_2(t) \\ E[\theta_{22}(t)\chi_{21}(t)] & E[\theta_{21}\chi_{12}(t)] & E[\chi_{22}\chi_{12}(t)] & 0.5\sigma_{\chi_{21}}^2\rho_{\chi_{21}}^{21} & E[\chi_{21}(t)\eta(t)] & E[\chi_{21}(t)\kappa(t)] & E[\chi_{21}(t)I_2(t)] \\ E[\theta_{22}(t)\eta(t)] & E[\theta_{21}(t)\eta(t)] & E[\chi_{11}(t)\eta(t)] & E[\chi_{21}(t)\eta(t)] & 0.5\sigma_\eta^2\rho_\eta^2 & E[\eta(t)\kappa(t)] & E[\eta(t)I_2(t)] \\ E[\theta_{22}(t)\kappa(t)] & E[\theta_{21}(t)\kappa(t)] & E[\chi_{11}(t)\kappa(t)] & E[\chi_{21}(t)\kappa(t)] & E[\kappa(t)\eta(t)] & 0.5\sigma_\kappa^2\rho_\kappa^2 & E[\kappa(t)I_2(t)] \\ E[\theta_{22}(t)I_2(t)] & E[\theta_{21}(t)I_1(t)] & E[\chi_{22}(t)I_2(t)] & E[\chi_{21}(t)I_2(t)] & E[\eta(t)I_2(t)] & E[\kappa(t)I_2(t)] & E[I_1^2(t)] \end{bmatrix}, \quad (89)$$

$$\boldsymbol{P}_1(t) = \begin{bmatrix} 0.5\sigma_{\theta_{11}}^2\rho_{\theta_{11}}^2 & E[\theta_{11}(t)\chi_{11}(t)] & E[\theta_{11}(t)\chi_{12}(t)] & E[\theta_{11}(t)\eta(t)] & E[\theta_{11}(t)I_1(t)] \\ E[\theta_{11}(t)\chi_{11}(t)] & 0.5\sigma_{\chi_{11}}^2\rho_{\chi_{11}}^2 & E[\chi_{11}(t)\chi_{12}(t)] & E[\chi_{11}(t)\eta(t)] & E[\chi_{11}(t)I_1(t)] \\ E[\theta_{11}(t)\chi_{12}(t)] & E[\chi_{11}(t)\chi_{12}(t)] & 0.5\sigma_{\chi_{12}}^2\rho_{\chi_{12}}^2 & E[\chi_{12}(t)\eta(t)] & E[\chi_{12}(t)I_1(t)] \\ E[\theta_{11}(t)\eta(t)] & E[\chi_{11}(t)\eta(t)] & E[\chi_{12}(t)\eta(t)] & 0.5\sigma^2\rho_\eta^2 & E[\eta(t)I_1(t)] \\ E[\theta_{11}(t)I_1(t)] & E[\chi_{11}(t)I_1(t)] & E[\chi_{12}(t)I_1(t)] & E[\eta(t)I_1(t)] & E[I_1^2(t)] \end{bmatrix}, \quad (90)$$

$$\boldsymbol{P}_2(t) = \begin{bmatrix} 0.5\sigma_{\theta_{22}}^2\rho_{\theta_{22}}^2 & E[\theta_{22}(t)\chi_{22}(t)] & E[\theta_{22}(t)\chi_{21}(t)] & E[\theta_{22}(t)\eta(t)] & E[\theta_{22}(t)I_2(t)] \\ E[\theta_{22}(t)\chi_{22}(t)] & 0.5\sigma_{\chi_{22}}^2\rho_{\chi_{22}}^2 & E[\chi_{22}(t)\chi_{21}(t)] & E[\chi_{22}(t)\eta(t)] & E[\chi_{22}(t)I_1(t)] \\ E[\theta_{22}(t)\chi_{21}(t)] & E[\chi_{22}(t)\chi_{21}(t)] & 0.5\sigma_{\chi_{12}}^2\rho_{\chi_{12}}^2 & E[\chi_{21}(t)\eta(t)] & E[\chi_{21}(t)I_1(t)] \\ E[\theta_{22}(t)\eta(t)] & E[\chi_{22}(t)\eta(t)] & E[\chi_{22}(t)\eta(t)] & 0.5\sigma^2\rho_\eta^2 & E[\eta(t)I_2(t)] \\ E[\theta_{22}(t)I_2(t)] & E[\chi_{22}(t)I_2(t)] & E[\chi_{22}(t)I_2(t)] & E[\eta(t)I_2(t)] & E[I_2^2(t)] \end{bmatrix}, \quad (91)$$

3. THE NUMERICAL SOLUTIONS AND THE INFLUENCES OF NOISE AND VARIATIONS IN THE TRANSFORMER'S CIRCUIT ELEMENTS/PARAMETERS

In order to solve the above vector SDEs numerically, the often cited Euler-Maruyama scheme with strong order of convergence given by 0.5 has been used. The Euler-Maruyama approximation of a general vector SDE given by (92) where $\boldsymbol{X}(t), f(t,\boldsymbol{X}(t)), g(t,\boldsymbol{X}(t))$ and $\boldsymbol{W}(t)$ denote the arbitrary stochastic vector, drift term, diffusion term and a vector of Wiener processes respectively, can be given by (93), where tn stands for an arbitrary nth instant within a time interval in which $\boldsymbol{X}(t)$ is a valid solution of (92) with $\boldsymbol{X}(t_0)$ as the initial random vector.

$$d\boldsymbol{X}(t) = f(t,\boldsymbol{X}(t))dt + g(t,\boldsymbol{X}(t)d\boldsymbol{W}(t), \quad (92)$$

$$\boldsymbol{X}(t_{n+1}) = \boldsymbol{X}(t_n) + f(t_n,\boldsymbol{X}(t_n))(t_{n+1}-t_n) + g(t_n,\boldsymbol{X}(t_n))[\boldsymbol{W}(t_{n+1}) - \boldsymbol{W}(t_n)]. \quad (93)$$

Similarly to [19], both step up and step down transformers, which have $n > 1$ and $n < 1$ respectively, have been considered. The step up transformer with $n = 2$ will be considered first followed by the step down one with $n = 1/2$, where k = 0.9 has been assumed for both transformers. Moreover, we let $\alpha = \beta = \gamma_{11} = \gamma_{12} = \gamma_{21} = \gamma_{22} = \lambda_{11} = \lambda_{12} = \lambda_{21} = \lambda_{22} = \nu = \delta = 1/7$, $\sigma_{\theta,11}, \rho_{\theta,11} = \sigma_{\theta,12}, \rho_{\theta,12} = \sigma_{\theta,21}, \rho_{\theta,21} = \sigma_{\theta,22}, \rho_{\theta,22} = \sigma_{\chi,11}, \rho_{\chi,11} = \sigma_{\chi,12}, \rho_{\chi,12} = \sigma_{\chi,21}, \rho_{\chi,21} = \sigma_{\chi,22}, \rho_{\chi,22} = \sigma_\eta\rho_\eta = \sigma_\kappa\rho_\kappa = 2$. These equalities among noise and random variation sources have been assumed for ceteris paribus as their influences to $I_1(t)$ and $I_2(t)$ will be comparatively analyzed. We also let $I_1(0) = I_2(0) = 0$ A.

Now, consider the step up transformer. By applying a step function which is mathematically equivalent to the DC voltage, as $v_1(t)$ thus yields $v_2(t)$ as a step function, 200 realizations of $I_1(t)$ and I2(t) obtained from the numerical solutions of our vector SDEs can be simulated as shown in Figs.2 and 3. Note that units of the current and t in these and following figures are respectively A and s. If $v_1(t)$ is a sinusoidal function, which is mathematically equivalent to the AC voltage, $v_2(t)$ is also a sinusoidal function, and 200 realizations of $I_1(t)$ and $I_2(t)$ can be simulated as depicted in Figs.4 and 5. Similar results for the step down transformer can be simulated as depicted in Figs. 6-9. In Figs.2-9, the confidence intervals for means of $I_1(t)$ and $I_2(t)$ and $i_1(t)$

and $i_2(t)$ which are the deterministic counterparts of both stochastic currents, have also been included. The means of $I_1(t)$ and $I_2(t)$ have been computed as if $I_1(t)$ and $I_2(t)$ are subjected only to additive noise despite the fact that they are also subjected to the multiplicative noise as stated above by applying the central limit theorem [24]. In addition, $i_1(t)$ and $i_2(t)$ can be obtained by applying the numerical inverse Laplace transform to the deterministic results obtained from the s-domain transformer model [25]. By following [10], the strong agreements between the confidence intervals and deterministic responses verify our vector SDEs.

At this point, the stochastic effect of noise in the voltage source and the random variations in n, k and primary/secondary elements will be studied. The step up transformer will be analyzed first. After letting all noise intensities but α and β be temporarily zero (thus only noise in the voltage source has been taken into account) and applying a step function as $v_1(t)$, 200 realizations of $I_1(t)$ and $I_2(t)$ of the step up transformer in this scenario can be simulated as depicted in Figs.10 and 11, which reveal that noise in the voltage source contributes the high frequency fluctuations in $I_1(t)$ and $I_2(t)$. Such high frequency fluctuations can be seen even in the early parts of the transient states. Now, consider only the effect of random variation in n by temporarily letting only ν be nonzero and applying a step function as $v_1(t)$. Here, 200 realizations of $I_1(t)$ and $I_2(t)$ can be simulated as depicted in Figs.12 and 13. For studying the effect of random variation in k, we temporarily let only δ be nonzero and apply a step function as $v_1(t)$. As a result, 200 realizations of $I_1(t)$ and $I_2(t)$ of the step up transformer in this case can be simulated as depicted in Figs.14 and 15. It can be seen from these figures that both random variations in n and k contribute the low frequency fluctuations in $I_1(t)$ and $I_2(t)$, which can also be seen in their transient states. These low frequency fluctuations employ greater magnitudes than the high frequency fluctuations caused by noise in the voltage source. For the classical off-chip transformer which has lossless coupling, there is neither k nor its variation, so the related current fluctuations do not exist.

Now, the effects of random variations in the primary resistive elements will be studied. Of course, only $I_1(t)$ will be effected by these variations. With only the variation in R_{11} being taken into account by temporarily letting only γ_{11} be nonzero and applying a step function as $v_1(t)$, 200 realizations of $I_1(t)$ can be simulated as depicted in Fig.16. In order to study the effect of variation in R_{12}, we temporarily let all intensities but γ_{12} be zero. As a result, 200 realizations of $I_1(t)$ can be simulated as depicted in Fig.17. These figures show that the random variations in both primary resistive elements contribute the low frequency fluctuation in $I_1(t)$. This low frequency fluctuation can be obviously seen in the steady state and has a larger magnitude than the high frequency fluctuation caused by noise in the voltage source. However, for the off-chip transformer, the random variation in R_{11} has been found to be the only resistive variation that causes the low frequency fluctuation in $I_1(t)$. This is because there is neither R_{12} nor its variation, as the mutual impedances of the off-chip transformer are purely inductive.

Next, we analyze the stochastic effects of random variations in the primary inductive elements. Similarly to the variations in the primary resistive elements, random variations in the primary inductive elements affect $I_1(t)$ only. With only random variation in L_{11} being taken into account by temporarily letting only λ_{11} be nonzero and applying a step function as $v_1(t)$, 200 realizations of $I1(t)$ can be simulated as depicted in Fig.18. In order to study the effect of variation in M_{12}, we temporarily let all intensities but λ_{12} be zero. As a result, 200 realizations of $I_1(t)$ can be simulated as depicted in Fig.19. Both figures show that the variations in both primary inductive elements also contribute the low frequency fluctuation in $I_1(t)$. However, this fluctuation can be obviously seen in the transient. In addition, the fluctuation due to the random variation in L_{11} has been found to be of greater magnitude than that caused by the variation in M_{12}.

The effects of random variations in the secondary resistive elements will be now considered. Of course, these variations affect $I_2(t)$ only. With only the random variation in R_{22} being taken into account by temporarily letting only γ_{22} be nonzero and applying a step function as $v_1(t)$, 200 realizations of $I_2(t)$ can be simulated as depicted in Fig.20. For studying the effect of variation in R_{21} on the other hand, we temporarily let all noise intensities but γ_{21} be zero then 200 realizations of $I_2(t)$ can be simulated as depicted Fig.21. These figures show that the random variations in the secondary resistive elements contribute the low frequency fluctuation in $I_2(t)$, which can be obviously seen in the steady state. This low frequency fluctuation has a larger magnitude than the high frequency fluctuation caused by noise in the voltage source. However, for the classical off-chip transformer, the random variation in R_{22} has been found to be the only resistive variation that causes the low frequency fluctuation in $I_2(t)$. This is because there is neither R_{21} nor its random variation, as the mutual impedances of the off-chip transformer are purely inductive.

As the last study on the step up transformer, we study the effects of random variations in the secondary inductive elements where only $I_2(t)$ is affected by these variations. With only the random variation in L_{22} being considered by temporarily letting only λ_{22} be nonzero, 200 realizations of $I_2(t)$ can be simulated as depicted in Fig.22. For considering the ef-

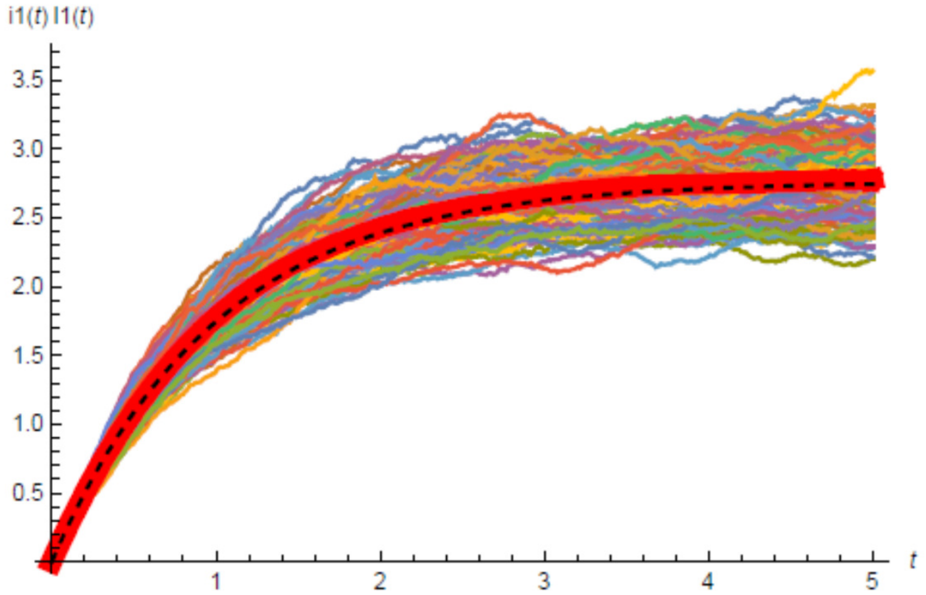

Fig.2: *200 Realizations of $I_1(t)(A)$ of Step Up Transformer, The Confidence Interval for Its Mean (Shade Area) and $i_1(t)(A)$ vs. t (s): $v_1(t)$ As A Step Function*

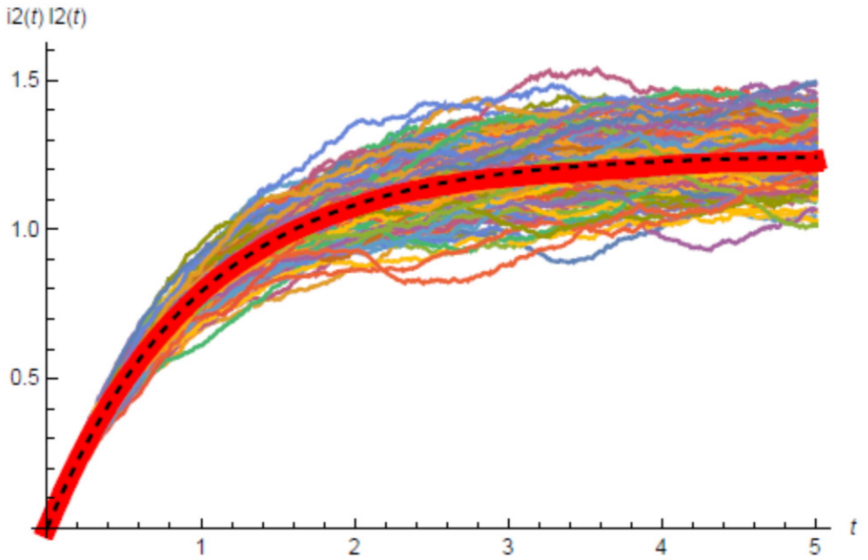

Fig.3: *200 Realizations of $I_2(t)(A)$of Step Up Transformer, The Confidence Interval for Its Mean (Shade Area) and $i_2(t)$ (A) vs. t (s): $v_1(t)$ As A Step Function*

fect of variation in $M21$, we temporarily let all noise intensities except λ_{21} be zero. As a result, 200 realizations of $I_2(t)$ can be simulated as depicted Fig.23. Both figures reveal that the random variations in both secondary inductive elements contribute the low frequency fluctuation in $I2(t)$, which can be obviously seen in the transient. In addition, the fluctuation caused by the random variation in M_{21} has been found to be of greater magnitude than the fluctuation due to the variation in L_{22}.

For the stochastic analysis of the step down transformer, its stochastically varied $I_1(t)$ and $I_2(t)$ induced by noise in the voltage source, random variation in n and that in k can be simulated as depicted in Figs.24-29. The stochastically varied $I_1(t)$ induced by random variation in the primary elements have been simulated as depicted in Figs.30-33. Those of the stochastically varied $I_2(t)$ due to random variation in the secondary elements have been simulated as displayed in Figs.34-37. From these figures, it can be seen that almost all stochastic behaviors of the step down transformer are similar to those of the step up transformer, except that the low frequency fluctuation in $I_2(t)$ due to the random variation in L_{22}

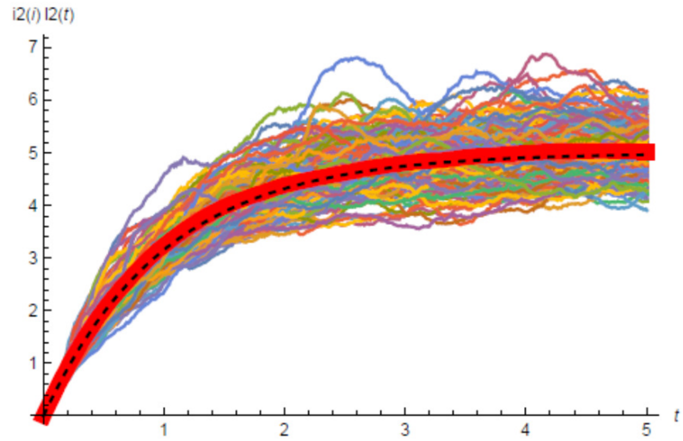

Fig.4: *200 Realizations of $I_1(t)(A)$ of Step Up Transformer, The Confidence Interval for Its Mean (Shade Area) and $i_1(t)$ (A) vs. t (s): $v_1(t)$ As A Sinusoidal Function*

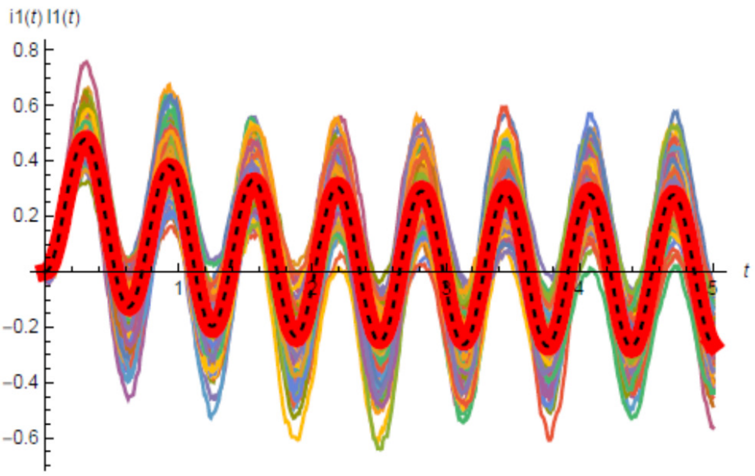

Fig.5: *200 Realizations of $I_2(t)$ (A) of Step Up Transformer, The Confidence Interval for Its Mean (Shade Area) and $i_2(t)$ (A) vs. t (s): $v_1(t)$ As A Sinusoidal Function*

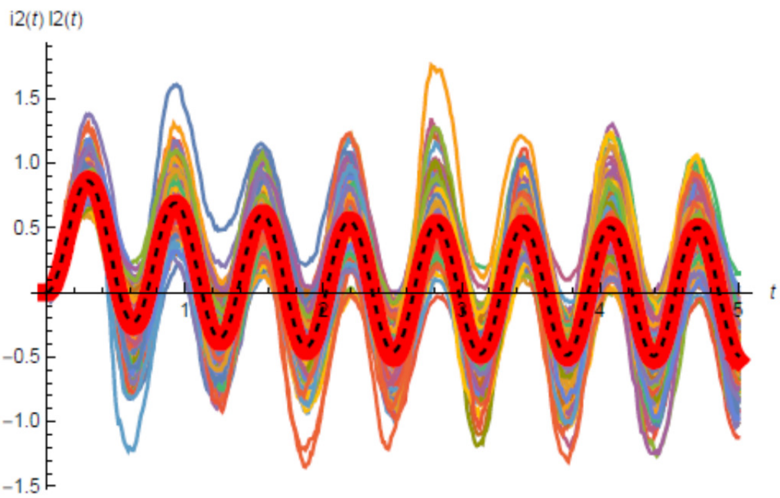

Fig.6: *200 Realizations of $I_1(t)(A)$of Step Down Transformer, The Confidence Interval for Its Mean (Shade Area) and $i_1(t)(A)$ vs. t (s): $v_1(t)$ As A Step Function*

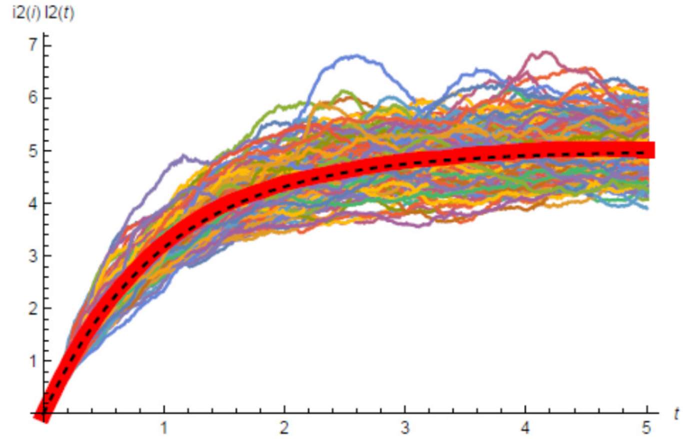

Fig.7: *200 Realizations of $I_2(t)$ (A) of Step Down Transformer, The Confidence Interval for Its Mean (Shade Area) and $i_2(t)$ (A) vs. t (s): $v_1(t)$ As A Step Function*

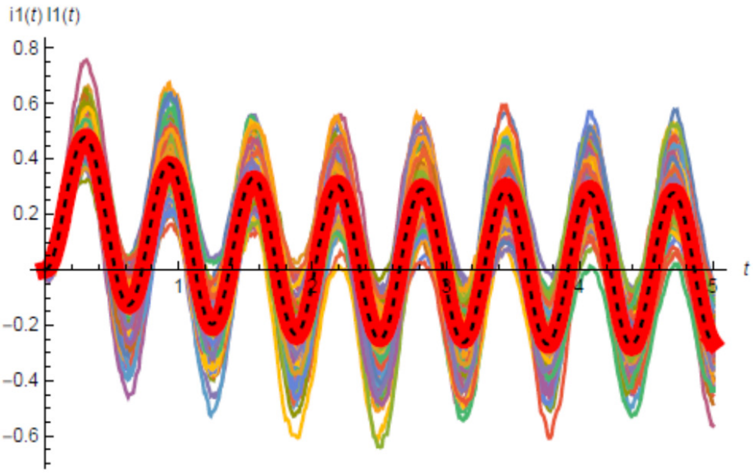

Fig.8: *200 Realizations of $I_1(t)$ (A) of Step Down Transformer, The Confidence Interval for Its Mean (Shade Area) and $i_1(t)$ (A) vs. t (s): $v_1(t)$ As A Sinusoidal Function*

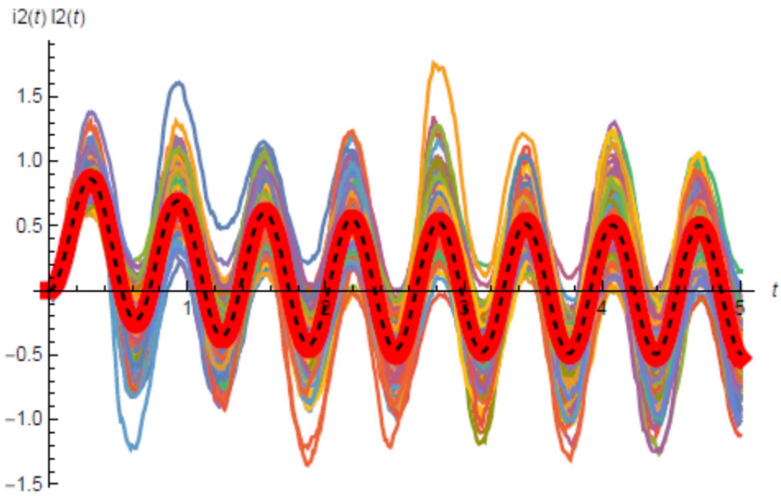

Fig.9: *200 Realizations of $I_2(t)$ (A) of Step Down Transformer, The Confidence Interval for Its Mean (Shade Area) and $i_2(t)$ (A) vs. t (s): $v_1(t)$ As A Sinusoidal Function*

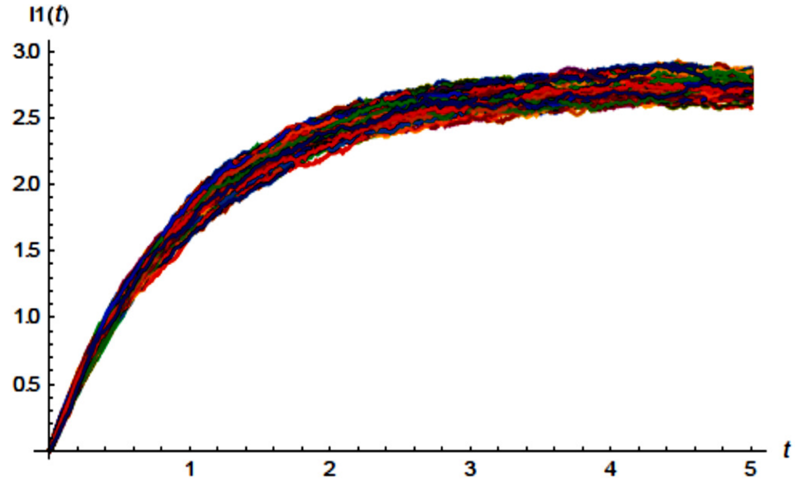

Fig.10: *200 Realizations of $I_1(t)$ (A) of Step Up Transformer Due to Noise in The Voltage Source vs. t (s): $v_1(t)$ As A Step Function*

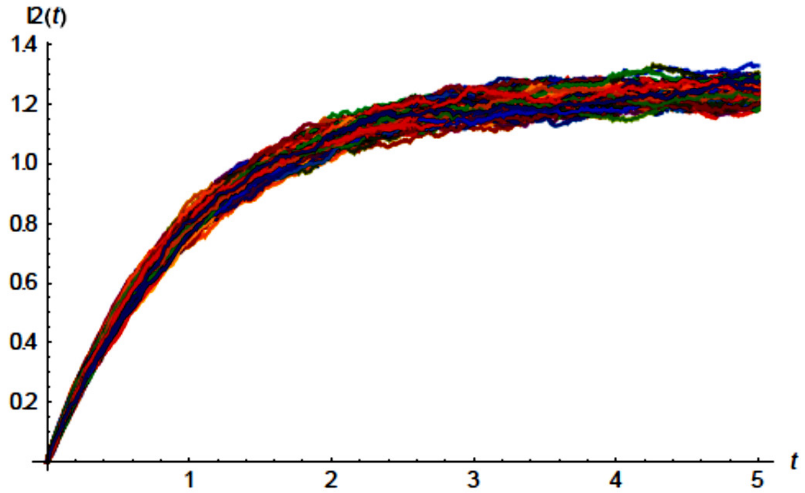

Fig.11: *200 Realizations of $I_2(t)$ (A) of Step Up Transformer Due to Noise in The Voltage Source vs. t (s): $v_1(t)$ As A Step Function*

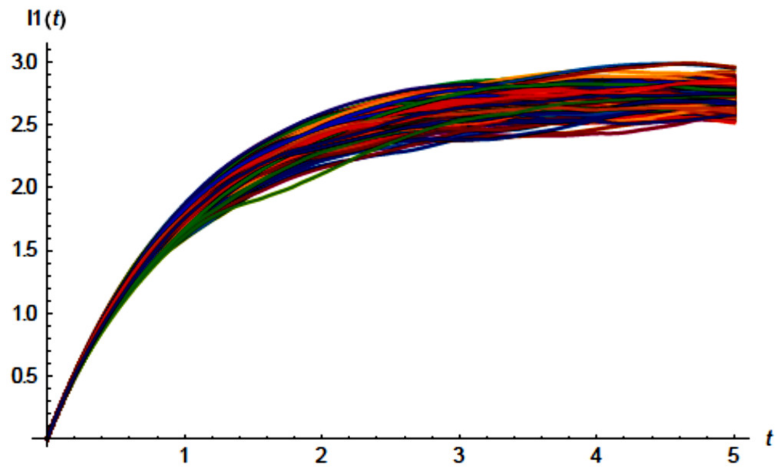

Fig.12: *200 Realizations of $I_1(t)$ (A) of Step Up Transformer Due to Random Variation in n vs. t (s): $v_1(t)$ As A Step Function*

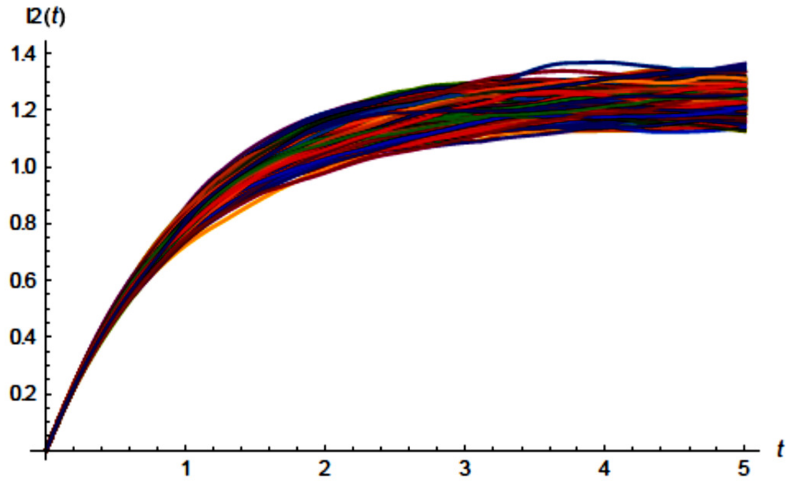

Fig.13: *200 Realizations of $I_2(t)$ (A) of Step Up Transformer Due to Random Variation in n vs. t (s): $v_1(t)$ As A Step Function*

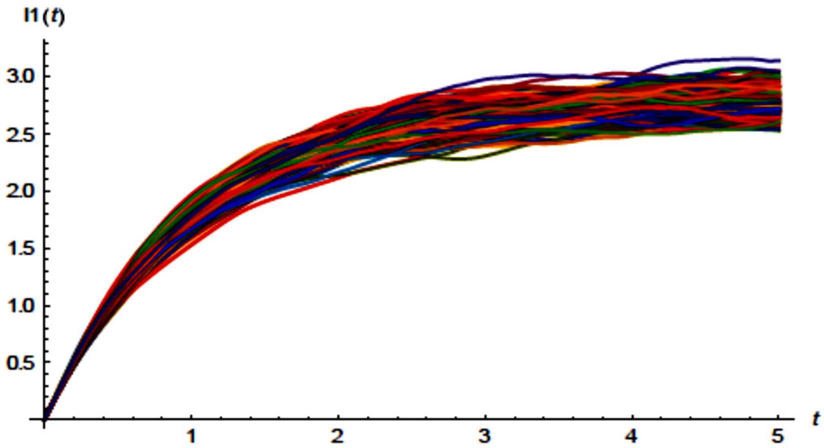

Fig.14: *200 Realizations of $I_1(t)$ (A) of Step Up Transformer Due to Random Variation in k vs. t (s): $v_1(t)$ As A Step Function*

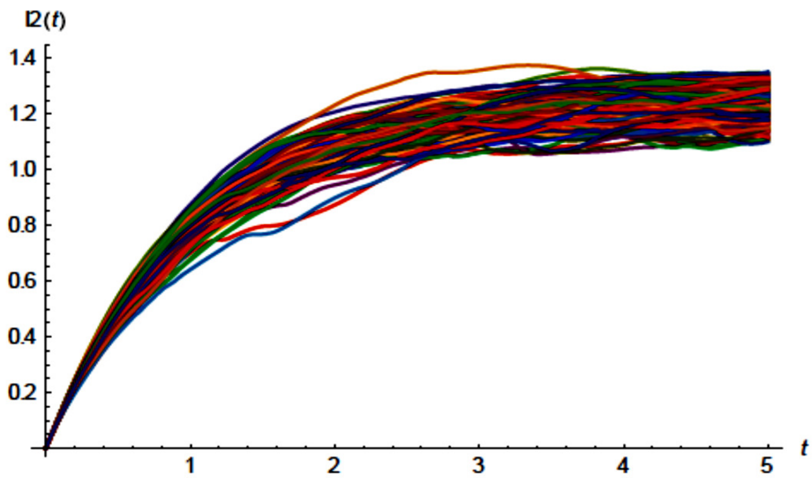

Fig.15: *200 Realizations of $I_2(t)$ (A) of Step Up Transformer Due to Random Variation in k vs. t (s): $v_1(t)$ As A Step Function*

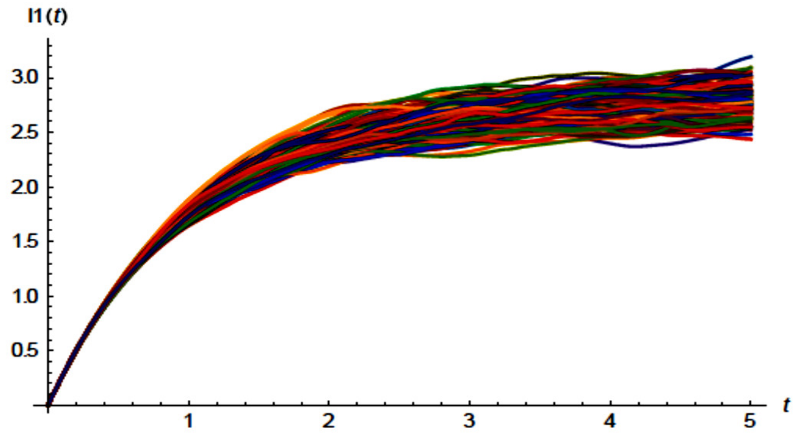

Fig.16: *200 Realizations of $I_1(t)(A)$ of Step Up Transformer Due to Random Variation in R_{11} vs. t (s): $v_1(t)$ As A Step Function*

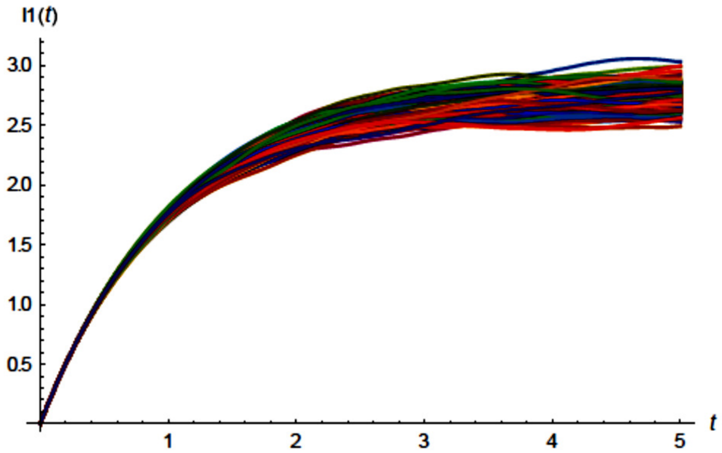

Fig.17: *200 Realizations of $I_1(t)(A)$ of Step Up Transformer Due to Random Variation in R_{12} vs. t (s): $v_1(t)$ As A Step Function*

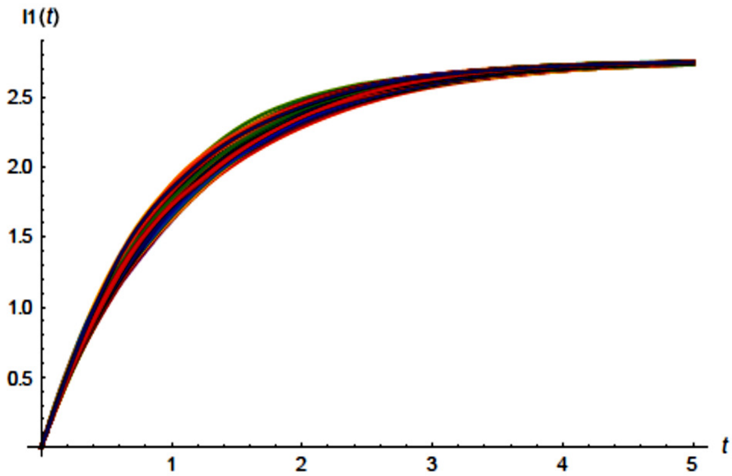

Fig.18: *200 Realizations of $I_1(t)$ (A) of Step Up Transformer Due to Random Variation in L_{11} vs. t (s): $v_1(t)$ As A Step Function*

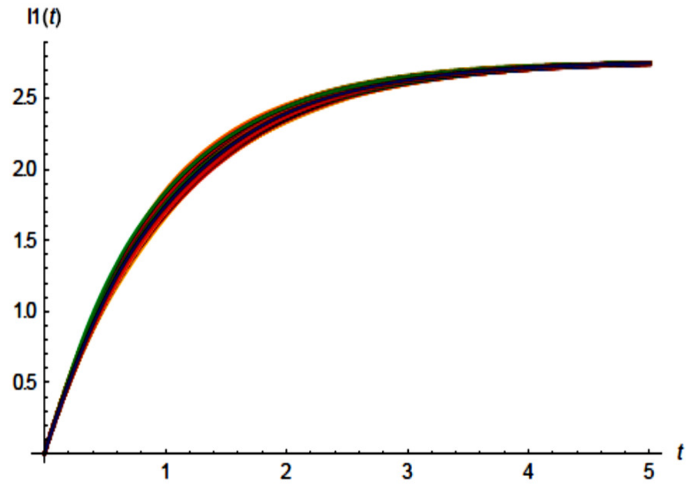

Fig.19: *200 Realizations of $I_1(t)$ (A) of Step Up Transformer Due to Random Variation in M_{12} vs. t (s): $v_1(t)$ As A Step Function*

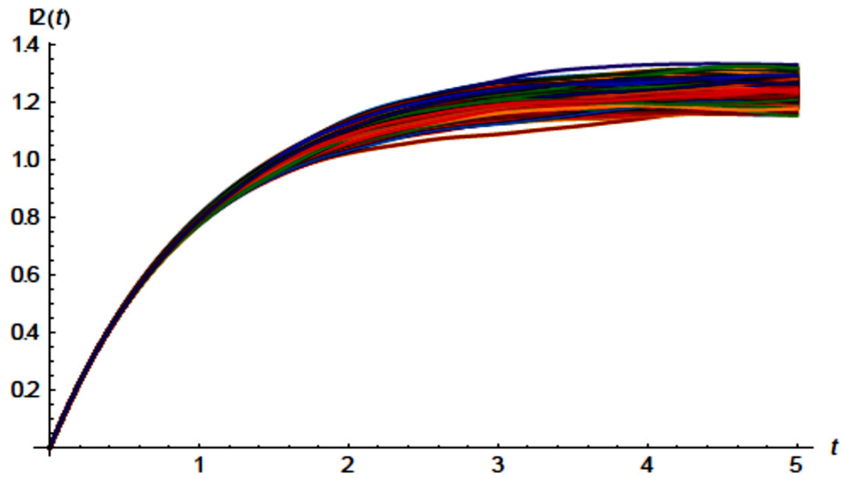

Fig.20: *200 Realizations of $I_2(t)$ (A) of Step Up Transformer Due to Random Variation in R_{22} vs. t (s): $v_1(t)$ As A Step Function*

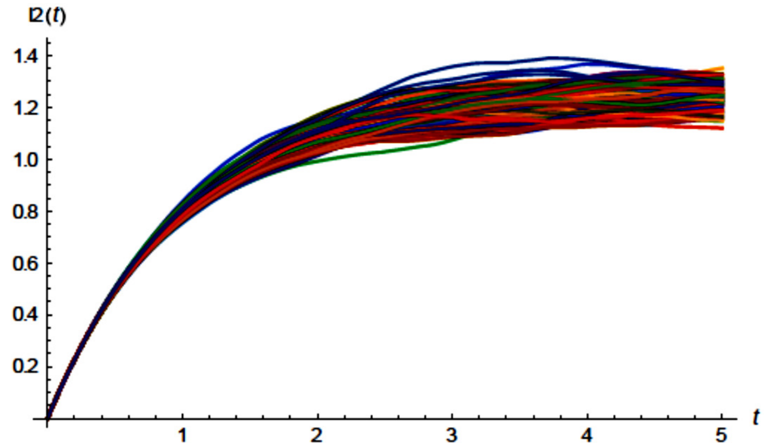

Fig.21: *200 Realizations of $I_2(t)$ (A) of Step Up Transformer Due to Random Variation in R_{21} vs. t (s): $v_1(t)$ As A Step Function*

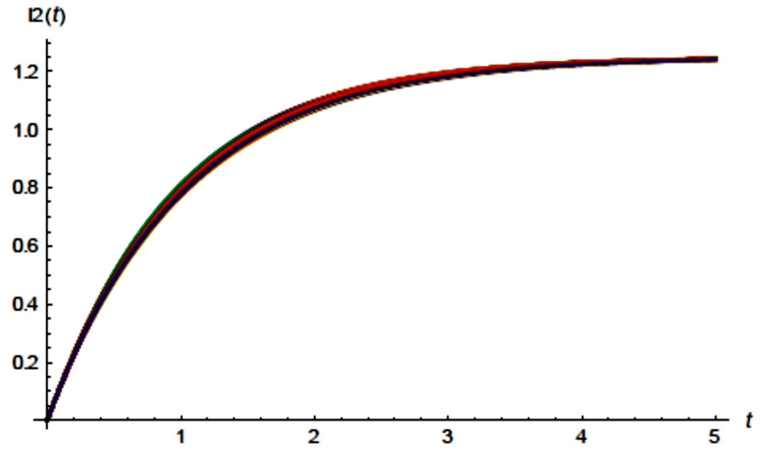

Fig.22: *200 Realizations of $I_2(t)$ (A) of Step Up Transformer Due to Random Variation in L_{22} vs. t (s): $v_1(t)$ As A Step Function*

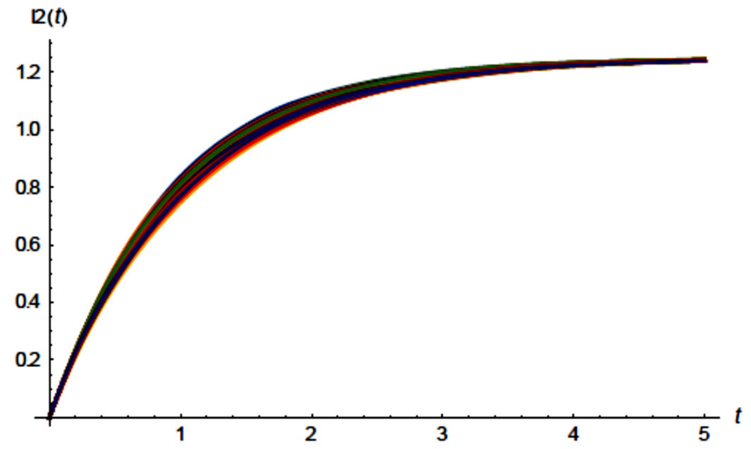

Fig.23: *200 Realizations of $I_2(t)$ (A) of Step Up Transformer Due to Random Variation in M_{21} vs. t (s): $v_1(t)$ As A Step Function*

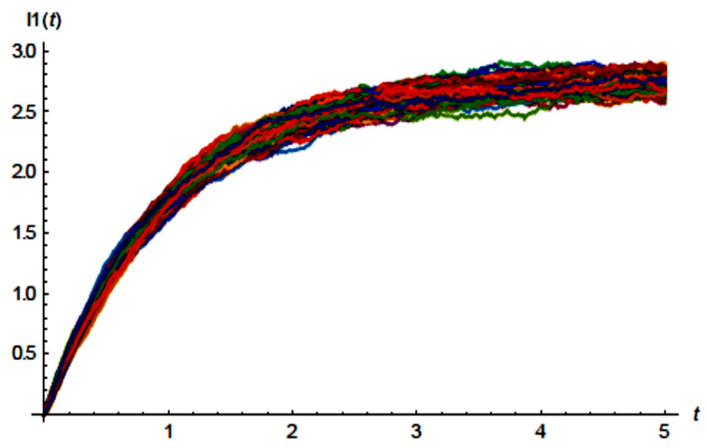

Fig.24: *200 Realizations of $I_1(t)$ (A) of Step Down Transformer Due to Noise in The Voltage Source vs. t (s): $v_1(t)$ As A Step Function*

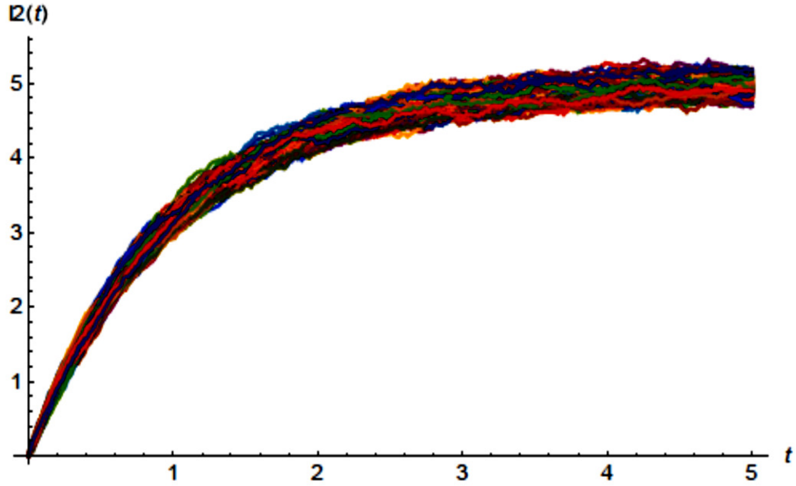

Fig.25: *200 Realizations of $I_2(t)$ (A) of Step Down Transformer Due to Noise in The Voltage Source vs. t (s): $v_1(t)$ As A Step Function*

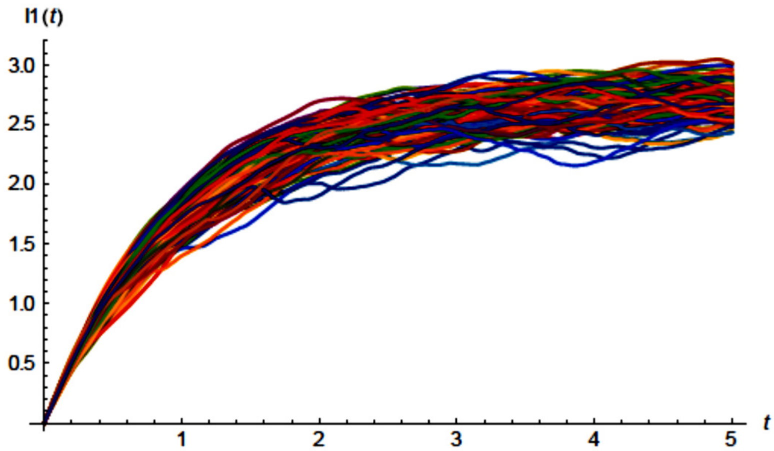

Fig.26: *200 Realizations of $I_1(t)$ (A) of Step Down Transformer Due to Random Variation in n vs. t (s): $v_1(t)$ As A Step Function*

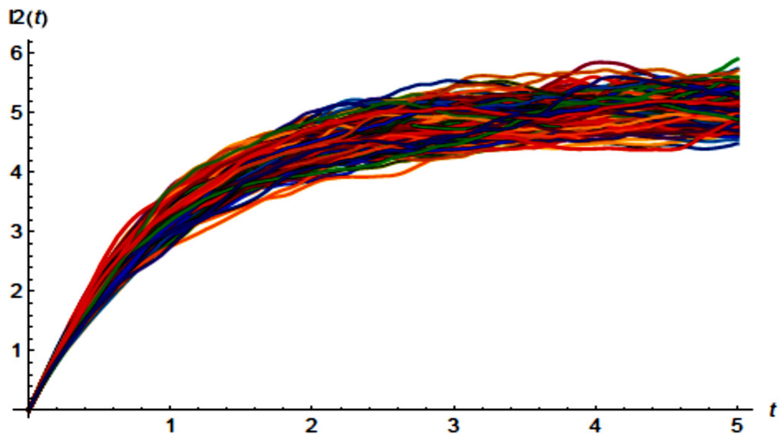

Fig.27: *200 Realizations of $I_2(t)$ (A) of Step Down Transformer Due to Random Variation in n vs. t (s): $v_1(t)$ As A Step Function*

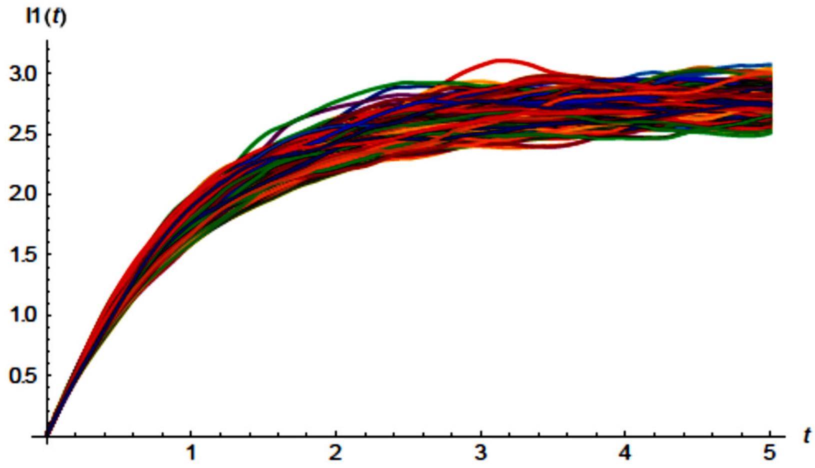

Fig.28: *200 Realizations of* $I_1(t)$ *(A) of Step Down Transformer Due to Random Variation in* k *vs.* t *(s):* $v_1(t)$ *As A Step Function*

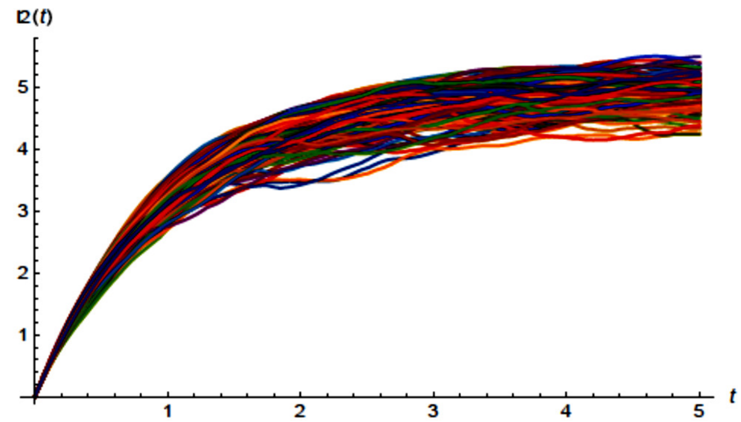

Fig.29: *200 Realizations of* $I_2(t)$ *(A) of Step Down Transformer Due to Random Variation in* k *vs.* t *(s):* $v_1(t)$ *As A Step Function*

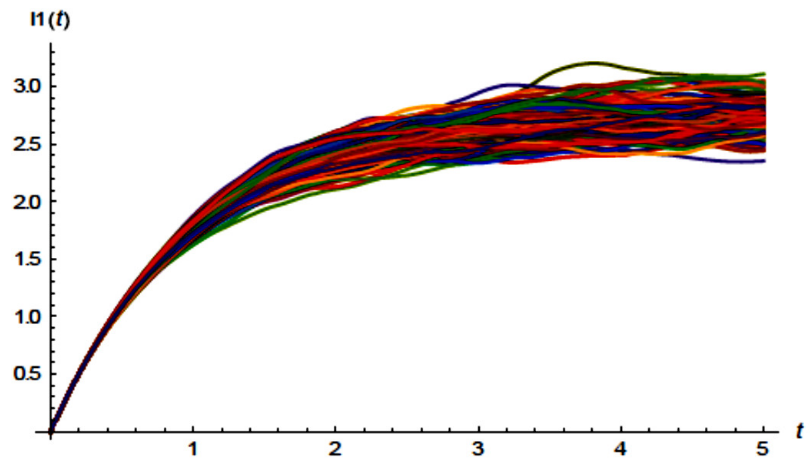

Fig.30: *200 Realizations of* $I_1(t)$ *(A) of Step Down Transformer Due to Random Variation in* R_{11} *vs.* t *(s):* $v_1(t)$ *As A Step Function*

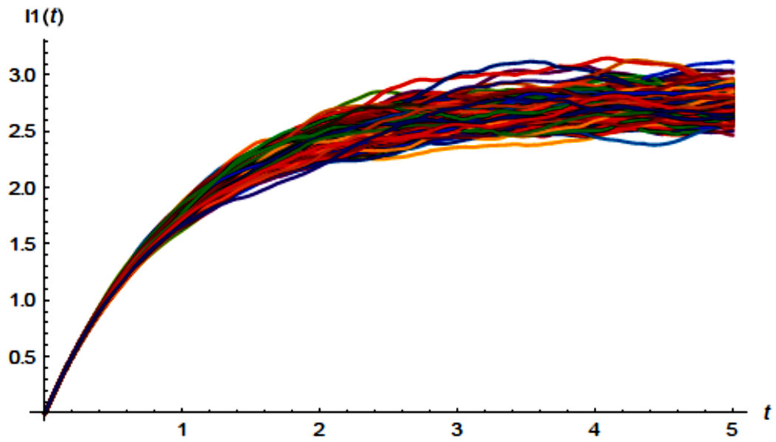

Fig.31: *200 Realizations of $I_1(t)$ (A) of Step Down Transformer Due to Random Variation in R_{12} vs. t (s): $v_1(t)$ As A Step Function*

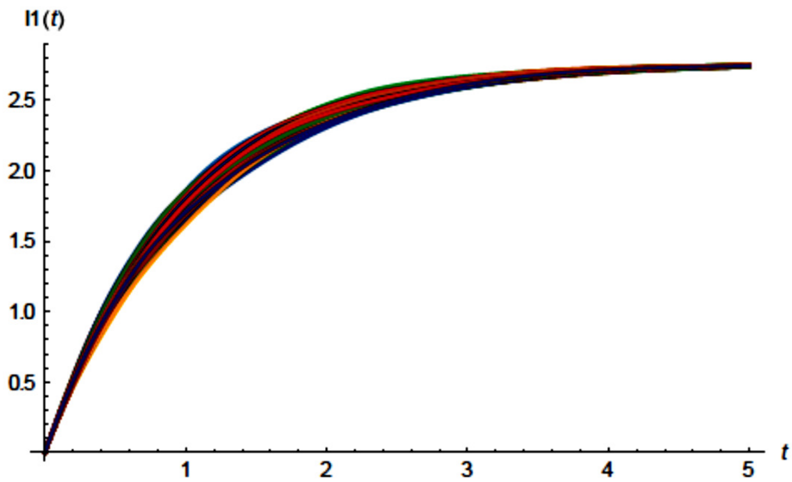

Fig.32: *200 Realizations of $I_1(t)$ (A) of Step Down Transformer Due to Random Variation in L_{11} vs. t (s): $v_1(t)$ As A Step Function*

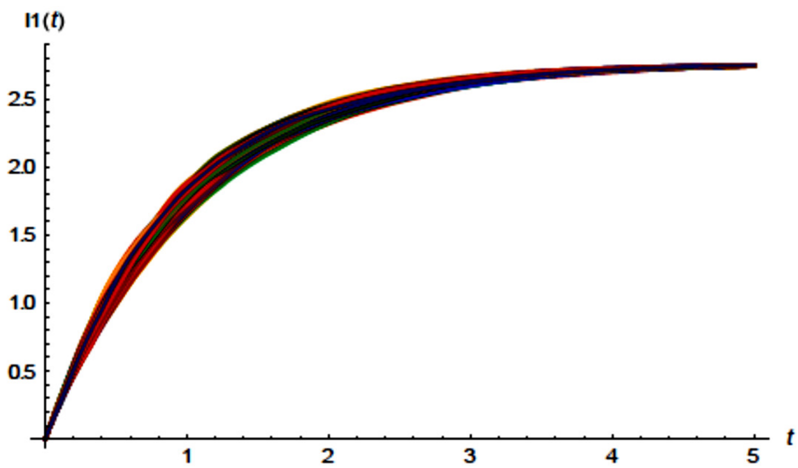

Fig.33: *200 Realizations of $I_1(t)$ (A) of Step Down Transformer Due to Random Variation in M_{12} vs. t (s): $v_1(t)$ As A Step Function*

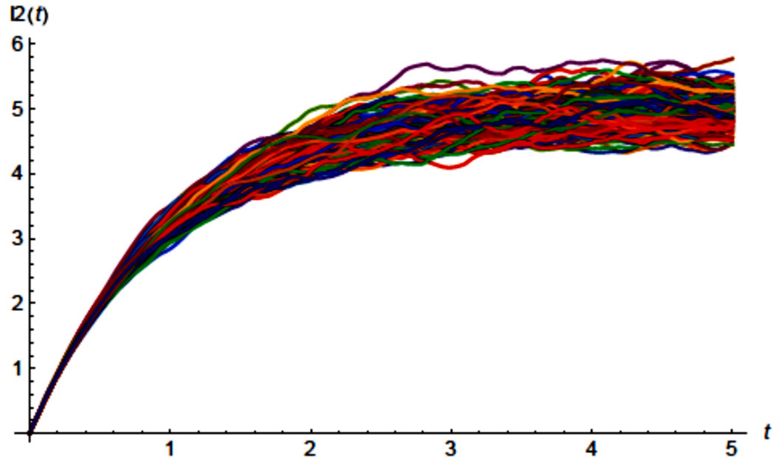

Fig.34: *200 Realizations of $I_2(t)$ (A) of Step Down Transformer Due to Random Variation in R_{22} vs. t (s): $V_1(t)$ As A Step Function*

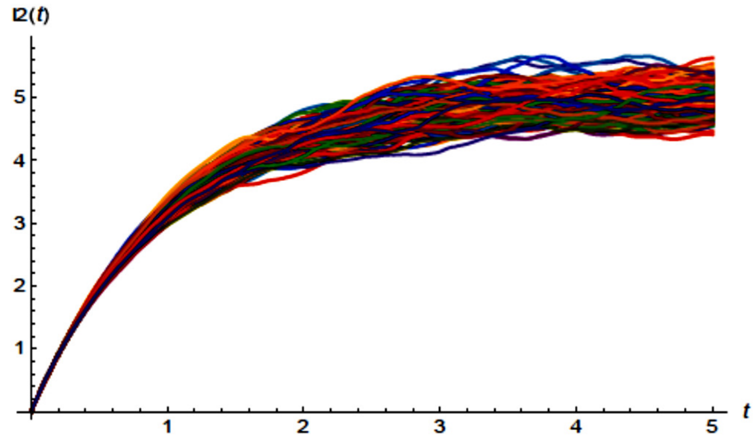

Fig.35: *200 Realizations of $I_2(t)$ (A) of Step Down Transformer Due to Random Variation in R_{21} vs. t (s): $v_1(t)$ As A Step Function*

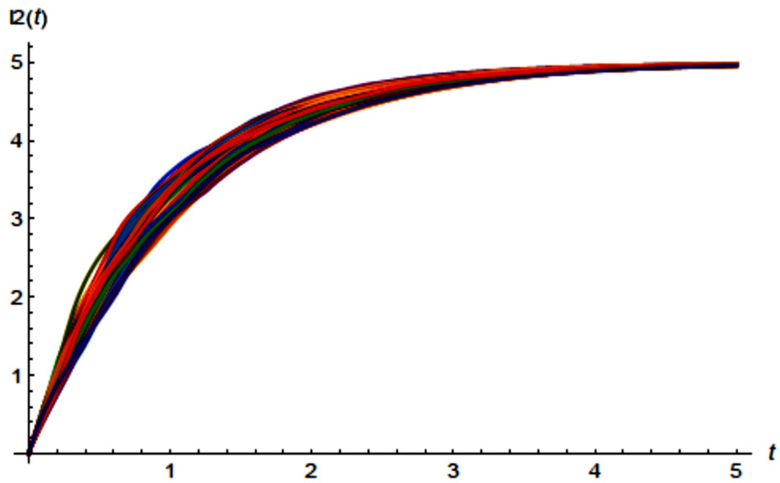

Fig.36: *200 Realizations of $I_2(t)$ (A) of Step Down Transformer Due to Random Variation in L_{22} vs. t (s): $v_1(t)$ As A Step Function*

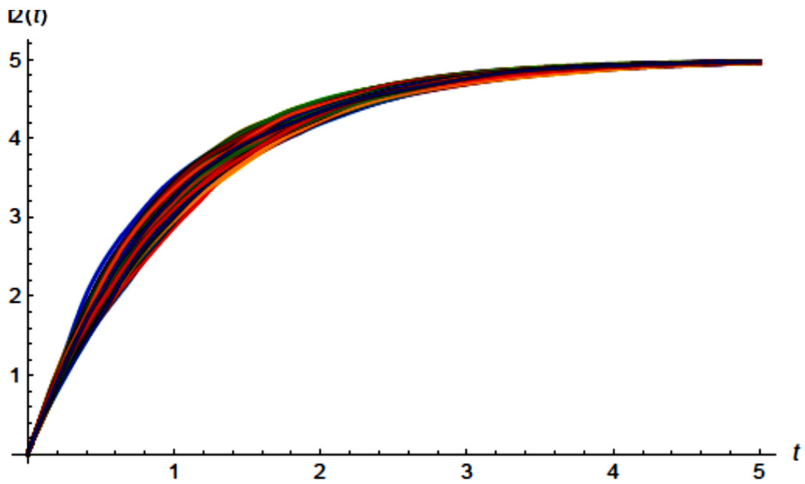

Fig.37: *200 Realizations of $I_2(t)$ (A) of Step Down Transformer Due to Random Variation in M_{21} vs. t (s): $v_1(t)$ As A Step Function*

has been found to be of greater magnitude than that caused by the random variation in M_{21}. Similarly to the case of step up transformer, the step down off-chip transformer which employs lossless coupling and mutual impedance, has no current fluctuations induced by variation in k as there is no such variation. Moreover, the random variations in self resistive elements have been found to be the only resistive sources of low frequency fluctuations in the currents.

4. FURTHER DISCUSSION

In summary, it has been shown in section 3 that noise in the voltage source and random variations in the parameters/elements of the transformer respectively contribute the high frequency and low frequency fluctuations in $I_1(t)$ and $I_2(t)$. This is because the variation of such noise changes very fast, but the variations of those elements/parameters change very slowly with respected to time. Moreover, it can be seen that the effect of random variations in the elements/parameters to the transformer's performance is greater than that of noise in the voltage source. This is because the low frequency stochastic fluctuations in $I_1(t)$ and $I_2(t)$ caused by those random variations have larger magnitudes than those high frequency fluctuations caused by such noise. In addition, it has been found that the random variation in n is the most critical cause of stochastic variations in transformer's performances, as it produces such large magnitude low frequency fluctuations in $I_1(t)$ and $I_2(t)$ in both on-chip and off-chip transformer, unlike the random variation in k that affects the on-chip transformer only. Since it has been found that the effects of random variations in inductive elements are small compared with the others, particularly at steady state, the simplified vector SDEs of transformer at such state can also be given by (27) and (28), but with the following vector, matrices, and their elements for the on-chip transformer.

$$\boldsymbol{X}_1(t) = \begin{bmatrix} \theta_{11}(t) & \theta_{12}(t) & \eta(t) & \kappa(t) & I_1(t) \end{bmatrix}^T, \quad (94)$$

$$\boldsymbol{A}_1(t) = \begin{bmatrix} -\rho_{\theta,11} & 0 & 0 & 0 & 0 \\ 0 & -\rho_{\theta,12} & 0 & 0 & 0 \\ 0 & 0 & -\rho_\eta & 0 & 0 \\ 0 & 0 & 0 & -\rho_\kappa & 0 \\ 0 & 0 & 0 & 0 & A_1(t) \end{bmatrix} \quad (95)$$

$$A_1(t) = -\left[\frac{R_{11} + \gamma_{11}\theta_{11}(t) + \frac{(R_{12}+\gamma_{12}\theta_{12}(t))(n+v\eta(t))}{k+\delta\kappa(t)}}{L_{11} + \frac{M_{12}(n+v\eta(t))}{k+\delta\kappa(t)}}\right], \quad (96)$$

$$\boldsymbol{B}_1(t) = \begin{bmatrix} 0 & 0 & 0 & 0 & B_1(t) \end{bmatrix}^T, \quad (97)$$

$$\boldsymbol{B}_1(t) = \left[L_{11} + \frac{M_{12}(n+v\eta(t))}{k+\delta\kappa(t)}\right]^{-1} V_1(t), \quad (98)$$

$$\boldsymbol{C}_1(t) = \begin{bmatrix} -\sigma_{\theta,11}\rho_{\theta,11} & 0 & 0 & 0 & 0 \\ 0 & -\sigma_{\theta,12}\rho_{\theta,12} & 0 & 0 & 0 \\ 0 & 0 & -\sigma_\eta\rho_\eta & 0 & 0 \\ 0 & 0 & 0 & -\sigma_\kappa\rho_\kappa & 0 \\ 0 & 0 & 0 & 0 & C_1(t) \end{bmatrix} \quad (99)$$

$$C_1(t) = \beta\left[L_{11} + \frac{M_{12}(n+v\eta(t))}{k+\delta\kappa(t)}\right]^{-1}, \quad (100)$$

$$\boldsymbol{\Omega}_1(t) = \begin{bmatrix} W_{11}(t) & W_{12}(t) & N(t) & K(t) & W_1(t) \end{bmatrix}^T, \quad (101)$$

$$\boldsymbol{X}_2(t) = \begin{bmatrix} \theta_{22}(t) & \theta_{21}(t) & \eta(t) & \kappa(t) & I_2(t) \end{bmatrix}^T, \quad (102)$$

$$\boldsymbol{A}_2(t) = \begin{bmatrix} -\rho_{\theta,22} & 0 & 0 & 0 & 0 \\ 0 & -\rho_{\theta,21} & 0 & 0 & 0 \\ 0 & 0 & -\rho_\eta & 0 & 0 \\ 0 & 0 & 0 & -\rho_\kappa & 0 \\ 0 & 0 & 0 & 0 & A_2(t) \end{bmatrix}, \quad (103)$$

$$A_2(t) = -\left[\frac{R_{22} + \gamma_{22}\theta_{22}(t) + \frac{(R_{21}+\gamma_{21}\theta_{21}(t))(n+v\eta(t))}{k+\delta\kappa(t)}}{L_{22} + \frac{M_{21}(n+v\eta(t))}{k+\delta\kappa(t)}}\right], \quad (104)$$

$$\boldsymbol{B}_2(t) = \begin{bmatrix} 0 & 0 & 0 & 0 & B_2(t) \end{bmatrix}^T, \quad (105)$$

$$\boldsymbol{B}_2(t) = \left[L_{22} + \frac{M_{21}(n + v\eta(t))}{k + \delta\kappa(t)}\right]^{-1} V_2(t), \quad (106)$$

$$\boldsymbol{C}_2(t) = \begin{bmatrix} -\sigma_{\theta,22}\rho_{\theta,22} & 0 & 0 & 0 & 0 \\ 0 & -\sigma_{\theta,21}\rho_{\theta,21} & 0 & 0 & 0 \\ 0 & 0 & -\sigma_\eta\rho_\eta & 0 & 0 \\ 0 & 0 & 0 & -\sigma_\kappa\rho_\kappa & 0 \\ 0 & 0 & 0 & 0 & C_2(t) \end{bmatrix} \quad (107)$$

$$C_2(t) = \beta\left[L_{22} + \frac{M_{21}(n + v\eta(t))}{k + \delta\kappa(t)}\right]^{-1}, \quad (108)$$

$$\boldsymbol{\Omega}_2(t) = \begin{bmatrix} W_{22}(t) & W_{21}(t) & N(t) & K(t) & W_2(t) \end{bmatrix}^T. \quad (109)$$

For the off-chip transformer, we have

$$\boldsymbol{X}_1(t) = \begin{bmatrix} \theta_{11}(t) & \eta(t) & I_1(t) \end{bmatrix}^T, \quad (110)$$

$$\boldsymbol{A}_1(t) = \begin{bmatrix} -\rho_{\theta,11} & 0 & 0 \\ 0 & -\rho_\eta & 0 \\ 0 & 0 & A_1(t) \end{bmatrix}, \quad (111)$$

$$A_1(t) = -\frac{R_{11} + \gamma_{11}\theta_{11}(t)}{L_{11} + M_{12}(n + \kappa\nu(t))^{-1}}, \quad (112)$$

$$\boldsymbol{B}_1(t) = \begin{bmatrix} 0 & 0 & B_1(t) \end{bmatrix}^T, \quad (113)$$

$$B_1(t) = \left[L_{11} + \frac{M_{12}}{n + v\eta(t)}\right]^{-1} V_1(t), \quad (114)$$

$$\boldsymbol{C}_1(t) = \begin{bmatrix} -\sigma_{\theta,11}\rho_{\theta,11} & 0 & 0 \\ 0 & -\sigma_\eta\rho_\eta & 0 \\ 0 & 0 & C_1(t) \end{bmatrix}, \quad (115)$$

$$C_1(t) = \alpha\left[L_{11} + \frac{M_{12}}{n + v\eta(t)}\right]^{-1}, \quad (116)$$

$$\boldsymbol{\Omega}_1(t) = \begin{bmatrix} W_{11}(t) & N(t) & W_1(t) \end{bmatrix}^T, \quad (117)$$

$$\boldsymbol{X}_2(t) = \begin{bmatrix} \theta_{22}(t) & \eta(t) & I_2(t) \end{bmatrix}^T, \quad (118)$$

$$\boldsymbol{A}_2(t) = \begin{bmatrix} -\rho_{\theta,22} & 0 & 0 \\ 0 & -\rho_\eta & 0 \\ 0 & 0 & A_2(t) \end{bmatrix}, \quad (119)$$

$$A_2(t) = \frac{R_{22} + \gamma_{22}\theta_{22}(t)}{L_{22} + M_{21}(n + v\eta(t))}, \quad (120)$$

$$\boldsymbol{B}_2(t) = \begin{bmatrix} 0 & 0 & B_2(t) \end{bmatrix}^T, \quad (121)$$

$$B_2(t) = [L_{22} + M_{21}(n + v\eta(t))]^{-1} V_2(t), \quad (122)$$

$$\boldsymbol{C}_2(t) = \begin{bmatrix} -\sigma_{\theta,22}\rho_{\theta,22} & 0 & 0 \\ 0 & -\sigma_\eta\rho_\eta & 0 \\ 0 & 0 & C_2(t) \end{bmatrix}, \quad (123)$$

$$C_2(t) = \beta\left[L_{22} + \frac{M_{21}}{n + v\eta(t)}\right]^{-1}, \quad (124)$$

$$\boldsymbol{\Omega}_2(t) = \begin{bmatrix} W_{22}(t) & N(t) & W_2(t) \end{bmatrix}^T. \quad (125)$$

5. CONCLUSIONS

The stochastic behaviors of transformers have been analyzed by using an SDE based approach in this research. Both noise in the voltage source and the random variations in circuit elements/parameters of the transformer have been taken into account, unlike our previous work [19]. Noise in the voltage source applied to the transformer has been considered as white noise, which is a Wiener process. The random variations in the elements/parameters of the transformer have been considered as color noise, which is an Ornstein-Uhlenbeck process. The resulting vector SDEs of the transformer have been solved both analytically and numerically in the Ito's sense. The Euler-Maruyama scheme with strong order of convergence has been applied for determining the numerical solutions. The confidence intervals for the means of stochastic currents have been used for verifying our vector SDEs. In addition, the stochastic properties of these currents, their stochastic correlation and the influences of noise in the voltage source and random variations in the elements/parameters of the transformer have been analyzed and discussed. Compared to [19], a much more thorough analysis has been presented in this work. The results proposed in this work have been found to be beneficial to statistical/variability aware analysis and designing of transformer based electronic and electrical systems. For further studies, the multiple winding transformer and fractional order mutual inductance [26], which is a transformer with fractional order impedances, should be considered.

References

[1] B. Øksendal, *Stochastic Differential Equations, An Introduction with Applications*, Springer-Verlag, New York, 2000.

[2] S. Cyganowski, P. Kloeden, J. Ombach, *From Elementary Probability to Stochastic Differential Equations with Maple*, Springer-Verlag, New York, 2000.

[3] M. Carletti, "Numerical Solution of Stochastic Differential Problems in The Biosciences," *Journal of Computational and Applied Mathematics*, Vol. 185, No. 2, pp. 422-40, 2006.

[4] D. Ham, A. Hajimiri, "Complete Noise Analysis for CMOS Switching Mixers via Stochastic Differential Equations," *Proceeding of IEEE 2000 Custom Integrated Circuits Conference (CICC'00)*, pp. 439-442., 2000.

[5] C.D. Charalambous and N. Menemenlis, "A State-Space Approach in Modeling Multipath Fading Channels Via Stochastic Differential Equations," *Proceeding of 2001 IEEE International Conference on Communications (ICC'01)*, pp. 2251-5, 2001

[6] K. Wang, M. L. Crow, "Numerical Simulation of Stochastic Diffe-rential Algebraic Equations for Power System Transient Stability with Random Loads," *Proceeding of 2011 IEEE Power and Energy Society General Meeting (PES'11)*, pp. 1-8, 2011.

[7] A. Zjajo, Q. Tang, M. Berkelaar, J. P. de Gyvez, A. Di Bucchianico, N. van der Meijs, "Stochastic Analysis of Deep-Submicrometer CMOS Process for Reliable Circuits Designs," *IEEE Transactions on Circuits and Systems I: Regular Papers,* Vol. 58, No. 1, pp. 164-175, 2011.

[8] E. Kolarova, "Modelling RL Electrical Circuits by Stochastic Diferential Equations," *EUROCON 2005 - The International Conference on "Computer as a Tool"*, Belgrade, pp.1236-1238, 2005.

[9] T.K. Rawat and H. Parthasarathy, "On Stochastic Modeling of Linear Circuits," *International Journal of Circuit Theory and Applications*, Vol. 38, No. 3, pp. 259-274, 2010.

[10] F. Rahman, N. Parisa, "A Stochastic Perspective of RL Electrical Circuit Using Different Noise Terms," *COMPEL: The International Journal for Computation and Mathematics in Electrical and Electronic Engineering*, Vol. 30, No. 2, pp. 812-822, 2011

[11] T.K. Rawat and H. Parthasarathy, "Modeling of An RC Circuit Using A SDEs," *Science & Technology Asia*, Vol. 13, No. 2, pp. 40-7, 2008.

[12] L. Brancik and E. Kolarova, "Analysis of Higher-Order Electrical Circuits With Noisy Sources via Stochastic Differential Equations Approach," *Proceedings of 22nd International Conference Radioelektronika*, pp. 1-4, 2012.

[13] L. Brancik and E. Kolarova, "Simulation of Higher-Order Electrical Circuits with Stochastic Parameters via SDEs," *Advances in Electrical and Computer Engineering*, Vol., 13, No. 1, pp. 17-22, 2013

[14] J.J. Zhou and D.J. Allstot, "Monolithic Transformers and Their Application in A differential CMOS RF Low-Noise Amplifier," *IEEE Journal of Solid-State Circuits*, Vol. 33, No. 12, pp. 2020–2027, 1998.

[15] J.R. Long, "Monolithic Transformers for Silicon RF IC Design," *IEEE Journal of Solid-State Circuits*, Vol. 35, No. 9, pp. 1368-1382, 2000.

[16] K.T. Ng, B. Rejaei, and J.N. Burghartz, "Substrate Effects in Monolithic RF Transformers on Silicon," *IEEE Transaction on Microwave Theory and Techniques*, Vol. 50, No. 1, pp. 377-383, 2002

[17] A. Tang, F. Yuan and E. Law, "CMOS Active Transformers and Their Applications in Voltage-Controlled Quadrature Oscillators," *Analog Integrated Circuits and Signal Processing*, Vol. 62, No. 1, pp. 83-90, 2010.

[18] A. Tang, G. Zhu and F. Yuan, "Current-Mode Phase-Locked Loop with Constant-Q Active Inductor CCO and Active Transformer Loop Filter," *Analog Integrated Circuits and Signal Processing*, Vol. 74, No. 2, pp. 365-375, 2013.

[19] R. Banchuin and R. Chaisricharoen, "An SDE based Stochastic Analysis of Transformer," *Proceedings of Joint International Conference on Digital Arts, Media and Technology with ECTI Northern Section Conference on Electrical, Electronics, Computer and Telecommunications Engineering (ECTI DAMT-NCON)*, pp. 310-313, 2019.

[20] F. Yuan, "CMOS Gyrator-C Active Transformers," *IET Circuits Devices and Systems*, Vol. 1, No. 6, pp. 494-508, 2007.

[21] Y. Mayevskiy et.al., "A New Compact Model for Monolithic Transformers in Silicon-Based RFICs," *IEEE Microwave and Wireless Components Letter*, Vol. 15, No.6, 2005, pp. 2203-2210.

[22] A.H. Jazwinski, *Stochastic Process and Nonlinear Filtering Theory*, Academic Press, New York, 1970.

[23] S. Nadarajah and T.K. Pogány, "On The Distribution of The Product of Correlated Normal Random Variables," *Comptes Rendus Mathematique*, Vol. 354, No. 2, pp. 201-204, 2016

[24] P. Kloeden, E. Platen, H. Schurz, *Numerical Solution of SDE Through Computer Experiments*, Springer-Verlag, New York, 1997.

[25] R. Banchuin and R. Chaisricharoen, "A Tensor Algebraic Model of On-chip Monolithic Transformer," *Proceedings of 2018 15th International Conference on Electrical Engineering/Electronics, Computer, Telecommunications and Information Technology (ECTI-CON'18)*, pp. 544-547, 2018.

[26] A. Soltan, A.G. Radwan, and A.M. Soliman, "Fractional-Order Mutual Inductance: Analysis and Design," *International Journal of Circuit Theory and Applications*, Vol. 44, No. 1, pp. 85-97, 2016.

APPENDIX

Table 1: *Notations of crucial stochastic calculus related variables.*

Variable	Notation
$\zeta_1(t)$	Zero mean white noise that interferes $v_1(t)$
$\zeta_2(t)$	Zero mean white noise that interferes $v_2(t)$
α	Noise intensity of $\zeta_1(t)$
β	Noise intensity of $\zeta_2(t)$
$\theta_{11}(t)$	Color noise (Ornstein-Uhlenbeck process) defining the random variation in R_{11}
$\theta_{12}(t)$	Color noise (Ornstein-Uhlenbeck process) defining the random variation in R_{12}
$\theta_{21}(t)$	Color noise (Ornstein-Uhlenbeck process) defining the random variation in R_{21}
$\theta_{22}(t)$	Color noise (Ornstein-Uhlenbeck process) defining the random variation in R_{22}
$\chi_{11}(t)$	Color noise (Ornstein-Uhlenbeck process) defining the random variation in L_{11}
$\chi_{12}(t)$	Color noise(Ornstein-Uhlenbeck process) defining the random variation in M_{12}
$\chi_{21}(t)$	Color noise (Ornstein-Uhlenbeck process) defining the random variation in M_{21}
$\chi_{22}(t)$	Color noise (Ornstein-Uhlenbeck process) defining the random variation in L_{22}
$\eta(t)$	Color noise(Ornstein-Uhlenbeck process) defining the random variation in n
$\kappa(t)$	Color noise (Ornstein-Uhlenbeck process) defining the random variation in k
γ_{11}	Noise intensity of $\theta_{11}(t)$
γ_{12}	Noise intenity of $\theta_{12}(t)$
γ_{21}	Noise intensity of $\theta_{21}(t)$
γ_{22}	Noise intensity of $\theta_{22}(t)$
λ_{11}	Noise intensity of $\chi_{11}(t)$
λ_{12}	Noise intensity of $\chi_{12}(t)$
λ_{21}	Noise intensity of $\chi_{21}(t)$
λ_{22}	Noise intensity of $\chi_{22}(t)$
ν	Noise intensity of $\eta(t)$
δ	Noise intensity of $\kappa(t)$
$W_1(t)$	Wiener process defining $\zeta_1(t)$
$W_2(t)$	Wiener process defining $\zeta_2(t)$
$W_{11}(t)$	Basis Wiener process of $\theta_{11}(t)$
$W_{12}(t)$	Basis Wiener process of $\theta_{12}(t)$
$W_{21}(t)$	Basis Wiener process of $\theta_{21}(t)$
$W_{22}(t)$	Basis Wiener process of $\theta_{22}(t)$
$X_{11}(t)$	Basis Wiener process of $\chi_{11}(t)$
$X_{12}(t)$	Basis Wiener process of $\chi_{12}(t)$
$X_{21}(t)$	Basis Wiener process of $\chi_{21}(t)$
$X_{22}(t)$	Basis Wiener process of $\chi_{22}(t)$
$N(t)$	Basis Wiener process of $\eta(t)$
$K(t)$	Basis Wiener process of $\kappa(t)$
$\rho_{\theta,11}$	Speed of mean reversion of $\theta_{11}(t)$
$\rho_{\theta,12}$	Speed of mean reversion of $\theta_{12}(t)$
$\rho_{\theta,21}$	Speed of mean reversion of $\theta_{21}(t)$
$\rho_{\theta,22}$	Speed of mean reversion of $\theta_{22}(t)$
$\rho_{\chi,11}$	Speed of mean reversion of $\chi_{11}(t)$
$\rho_{\chi,12}$	Speed of mean reversion of $\chi_{12}(t)$
$\rho_{\chi,21}$	Speed of mean reversion of $\chi_{21}(t)$
$\rho_{\chi,22}$	Speed of mean reversion of $\chi_{22}(t)$
ρ_{η}	Speed of mean reversion of $\eta(t)$
ρ_{κ}	Speed of mean reversion of $\kappa(t)$

$\sigma_{\theta,11}$	Volatility of $\theta_{11}(t)$ normalized with respected to $\rho_{\theta,11}$
$\sigma_{\theta,12}$	Volatility of $\theta_{12}(t)$ normalized with respected to $\rho_{\theta,12}$
$\sigma_{\theta,21}$	Volatility of $\theta_{21}(t)$ normalized with respected to $\rho_{\theta,21}$
$\sigma_{\theta,22}$	Volatility of $\theta_{22}(t)$ normalized with respected to $\rho_{\theta,22}$
$\sigma_{\chi,11}$	Volatility of $\chi_{11}(t)$ normalized with respected to $\rho_{\chi,11}$
$\sigma_{\chi,12}$	Volatility of $\chi_{12}(t)$ normalized with respected to $\rho_{\chi,12}$
$\sigma_{\chi,21}$	Volatility of $\chi_{21}(t)$ normalized with respected to $\rho_{\chi,21}$
$\sigma_{\chi,22}$	Volatility of $\chi_{22}(t)$ normalized with respected to $\rho_{\chi,22}$
σ_{η}	Volatility of $\eta(t)$ normalized with respected to ρ_{η}
σ_{κ}	Volatility of $\kappa(t)$ normalized with respected to ρ_{κ}

Rawid Banchuin received the B.Eng. degree in electrical engineering from Mahidol University, Bangkok, Thailand in 2000, the degree of M.Eng. in computer engineering and Ph.D. in electrical and computer engineering from King Mongkut's University of Technology Thonburi, Bangkok, Thailand in 2003 and 2008 respectively. At the present, he is an assistant professor of the Graduated School of Information Technology and Faculty of Engineering, Siam University, Bangkok, Thailand. His research interests are computation and mathematics in electrical and electronic engineering especially the fractional order and memristive devices, circuits and systems.

Roungsan Chaisricharoen received B.Eng., M.Eng. and Ph.D. degrees from the department of computer engineering, King Mongkut's University of Technology Thonburi, Bangkok, Thailand. He is an assistant professor at the school of information technology, Mae Fah Luang University, Chiang Rai, Thailand. His research interests are computational intelligence, analog circuits and devices, wireless networks and optimization techniques. At the present, Asst. Prof. Dr. Roungsan Chaisricharoen is a member of ECTI board committee, public relations personal of ECTI.