System Modelling and Controller designed for Thermoelectric Generator using a First Order Plus Dead Time

Rung-aroon Photoon[1] and **Warit Wichakool**[2], Non-members

ABSTRACT

This paper discusses the design, implementations and verification of Thermoelectric generator (TEG) modules using a first order plus dead time (FOPDT) in a Matlab/Simulink™ program. The models proposed in this paper were designed to improve the thermoelectric generator performance under the condition of the inlet fuel control system with performed load connections. The tuning for the proportional-integral-derivative (PID) controller was done with Dahlin synthesis tuning and Quarter decay ration tuning formulas. The comparisons result of simulations model from real TEG machine were examined and analysed against controller from simulation model to study the TEG system.

Keywords: Thermoelectric Generator, FOPDT.

1. INTRODUCTION

A basic thermoelectric generator machine is shown in Fig. 1.

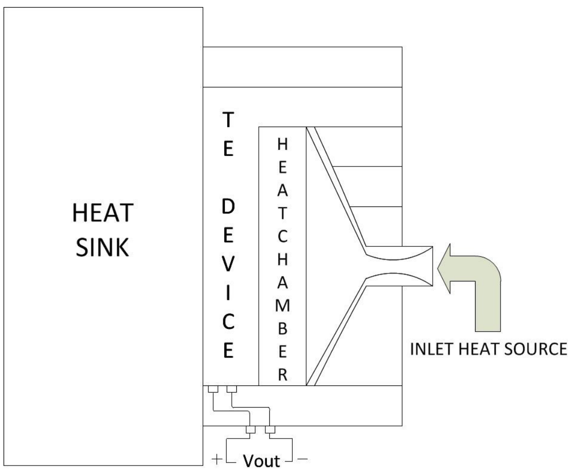

Fig.1: *Basic thermoelectric machine.*

A thermoelectric generators (TEG) comprises basically of three main sections: a heat source, a heat sink and a thermoelectric (TE) device. The heat source supply thermal energy while the heat sink is important in creating a temperature gradient across the TE device. The TE device basically works by converting the thermal energy in the thermal reservoir into an electrical energy. In terms of design, simplicity is one of the upside of TEG over other heat engines. In additional, an absence of moving parts makes TEG favorable for miniaturization.

The mathematical models of TEG are available which can be used to simulate their corresponding behaviors and performance analysis. Modeling and simulation enable us to analyze, design, and optimize the TEG by omitting the design cycle, which is easier. Recently, SPICE software is used to design an equivalent circuit model of TEG for easy model analysis and for further extraction of model parameter from specifications in commercial. Most authors studied transient behavior using SPICE software in order to analyze the voltage output under different thermal reservoirs and electrical load conditions. However, the results were not able to predict the exact time to reach a steady state condition [1]-[10]. The circuit models designed using SPICE is more suitable for simulation of power electronics applications.

For the simulation of control objective, it is a better option to build a TEG model using a Matlab/Simulink package. Aati Kane studied performance of thermoelectric module (TEM) using Matlab/Simulink [11]. The Seebeck, Peltier and Thomson coefficients were experimentally with respect to temperature dependent to extract voltage and current output when TEM working in dynamic mode. Huan-Liang Tsi studied and performed TEM model using Matlab/Simulink and extract voltage and current at standard temperature and compare to commercial TEC1-12710 module [12]. However, both cases used SPICE model to analyze the voltage output behavior under different thermal reservoirs.

The system model is simplified by utilizing a first order plus dead time (FOPDT) to model the TEG module [13]-[14]. A complicated higher-ordered system model with a simple dead time delay and a lower-ordered system can be represented using the FOPDT model. Even though we are able to determine the dead time delay and the system time constant through a graphical method [15]-[16], its accuracy rely on the drawing of the line tangent to the

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[1,2]The authors are with Department of Electrical Engineering Faculty of Engineering, Prince of Songkla University Hat Yai, Songkhla 90112, Thailand., E-mail:rung1515@gmail.com and warit.wi@psu.ac.th

process reaction curve at the point of maximum rate of change. Thus, to reduce the model error, the fitting curve was designed following the qualifying dynamic process behavior, which was proposed by Douglas J. Cooper [17]. By examining the FOPDT model and process step running of commercial TEG machine, this model is selected due to the fact that the method of step running of commercial TEG is identical to process step testing procedure of FOPDT model. A typical plot of voltage output over time of commercial TEG machine to the response curve of FOPDT model seems to be indistinguishable because both have dead time presented. A dead time is essential in the system step operating of commercial TEG machine in avoiding damage on TE device due to the high pressure at the first step input. Similarly, the process reaction from FOPDT model describes the dead time as the amount of delay for the first begin of model responses behavior. As a result, a simple PID controller can be designed using Zeigler-Nichols and Dahlin synthesis tuning process to analyze the performance of commercial TEG in simulation [18].

This paper illustrates the steps in modeling a simple TEG system using the FOPDT model in Matlab/Simulink. In addition, the paper discusses the method for using the determined model to design a simple controller to enhance the TEG system performance. This paper is organized as follows. First, we describe the theory of the FOPDT and the extraction of its parameters. Next, the experimental setup is depicted. The real TEG system response is shown and verified. Finally, the results of simulated close-loop control obtained from the TEG system are shown and discussed.

2. THEORY AND SYSTEM MODEL

A simple block diagram of a TEG system is shown in Fig. 2

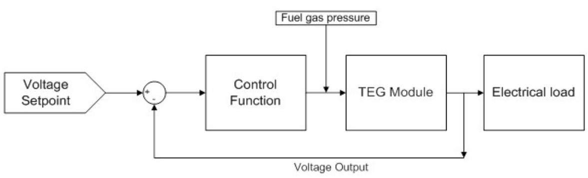

Fig.2: *Block diagram of a TEG system model.*

The system model comprises of three parts, a control function, a TEG module and an electrical load. First, our concentration starts on modelling the TEG process module. The model used to predict a transfer function of the TEG is the FOPDT model. The reason we use FOPDT model is that this model can work on both dynamic test on the actual system or computer simulation of the process. It is simple to perform a step test which matches the process step running of a real commercial TEG machine. The three difference parameters in low order transfer function are needed for identifications. Furthermore, the basic and main design methods and tuning techniques in the practical controllers such as PID controller are generalized and developed for the FOPDT transfer function. The proper experimental data consist of dynamic information that characterize our controller output to determine a process variable (PV) behaviour. A FOPDT model is a proper method to quantify by assigning numerical values to key aspects of this controller output (CO) to the process variable relationship which is used in controller. The FOPDT curve is shown in Fig. 3. The combination of a FOPDT process can be expressed in a differential equation as [18]

$$\tau_p \frac{dPV(t)}{dt} + PV(t) = K.CO(t - \theta_p). \quad (1)$$

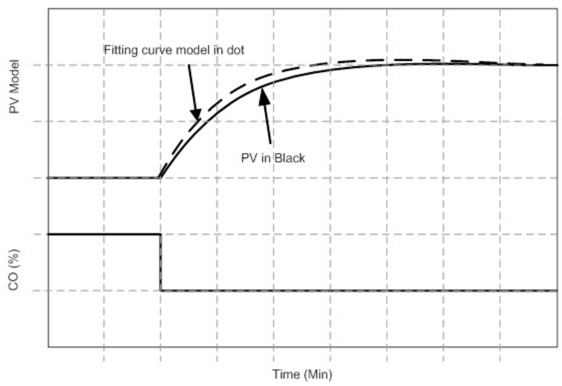

Fig.3: *FOPDT curve.*

When CO changes, the parameter K represents the direction and how far the process variable moves. A time constant τ_p describes the time taken for the process variable to respond. A variable t is an active time and variable θ_p describes the amount of delay that occurs before the process variable starts to change.

The equation of transfer function of FOPDT can be represented as

$$G(s) = \left(\frac{Ke^{-t_0 s}}{\tau s + 1} \right). \quad (2)$$

The model has three characteristic parameters to be determined. The variable K is a process gain, representing the direction and how far of process variable moves. It can be determined by the steady state level of the process output. The variable τ is a time constant, describing the time required for the process variable response. Finally the variable t_0 is a dead time, which describes how much delay occurs before the process variable first begins to change.

The TEG transfer function $G(s)$ can be approximated by a first-order model for the purpose of characterizing the dynamic response of the process. The characterized process includes the dynamic behavior of the output signal that can be modelled by a stable FOPDT model. In order to identify the system parameters, we used a step response of the open loop TEG module, as shown in Fig. 4.

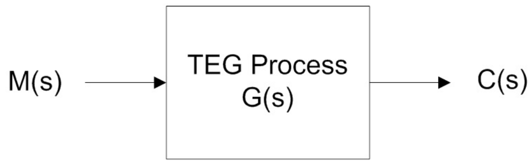

Fig.4: *Block diagram for open loop test.*

The step test procedure was carried out as follows. First, the input step change $m(t)$ was applied to the process and the process output response $c(t)$ was measured. The resulting plot of $c(t)$ versus time must cover entire test period of the step test until the system reaches a new steady state. A typical step response plot also known as a process reaction curve is shown in Fig. 5.

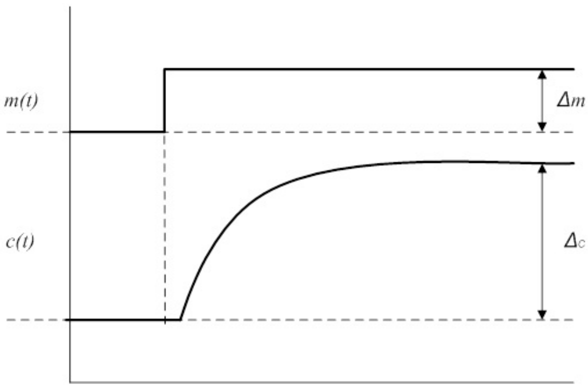

Fig.5: *Process reaction curve to a step response.*

From the block diagram for open loop test shown in Fig. 4, the process output of the FOPDT model in (1) is given by

$$C(s) = \left(\frac{Ke^{-t_0 s}}{\tau s + 1}\right)\left(\frac{1}{s}\right). \tag{3}$$

Expanding this expression by partial fractions, we obtain

$$C(s) = Ke^{-t_0 s}\left[\frac{1}{s} - \frac{\tau}{\tau s + 1}\right]. \tag{4}$$

We can obtain the time domain response of the system using the inverse Laplace transform. The process output is given by

$$c(t) = Ku(t - t_0)[1 - e^{-(t-t_0)/\tau}]. \tag{5}$$

Where the unit step function is included to indicate explicitly that

$$c(t) = 0 \quad \text{,for t} \le \text{t}_0. \tag{6}$$

The perturbation of the transmitter output from its initial value is given by

$$\Delta c = c(t) - c(t = 0). \tag{7}$$

The steady state value of the process reaction curve can be calculated by

$$\lim_{t\to\infty} c(t) = K. \tag{8}$$

From Fig. 4, because the model response must match the process reaction curve at steady state, we can calculate the steady gain of the process, where is the step change in input response as

$$K = \frac{\Delta c}{\Delta m}. \tag{9}$$

After the process reaction is obtained, the control system can be designed by using a closed loop Quarter decay ratio compare to a Dahlin synthesis method.

The Quarter decay ratio method is a trial and error tuning method based on sustained oscillations which was first proposed by Ziegler-Nichols (1942). On the other hand, the Dahlin synthesis was originally introduced by Dahlin (1968), who defined the tuning parameter as the reciprocal of the close loop time constant. The connection between process transfer function and the modes of a PID controller can be established from the controller synthesis. These methods can propose a set of formulas based on the parameters of first order model fit to the process reaction curve.

Regarding to FOPDT equation, the results in the following synthesized controller transfer function as

$$G_c(s) = \left(\frac{\tau s + 1}{Ke^{-t_0 s}}\right)\left(\frac{e^{-t_0 s}}{\tau_c s + 1 - e^{-t_0 s}}\right). \tag{10}$$

In order to convert the algorithm of Eq. 10 to the standard PID form we obtain the following synthesized controller transfer function as

$$G_c(s) = \left(\frac{\tau}{K(\tau_c + t_0)}\right)\left(1 + \frac{1}{\tau s}\right)\left(\frac{1 + \frac{\tau_0}{2}s}{1 + \tau s}\right). \tag{11}$$

The Quarter Decay ratio PID controllers is recommended for slow processes or the processes with dead time. The standard PID form we obtain the following Quarter Decay ratio controller transfer function as

$$G_c(s) = K\left(1 + \frac{1}{\tau s}\right)\left(\frac{\tau_d s}{\alpha\tau_d s + 1}\right). \tag{12}$$

Equation (12) shows that derivative portion is multiplied by the term $1/(\alpha\tau_d s + 1)$. This term, which can be recognized as the transfer function of a first order system with gain of unity and a time constant equal to $\alpha\tau_d$, is referred as a filter. The filter does not usually effect the performance of the controller because the time constant is small and the variable τ_d is a derivative time.

The equivalent to an actual PID controller with tuning parameter of Quarter decay ratio and Dahlin synthesis shown in Table 1. The variable K represents the process gain, K_c represents the proportional gain, T_i represents the integral time, T_d represents the derivative time, t_0 represents the dead time, and τ_c represents the closed loop time constant while τ represents the time constant.

Table 1: *Quarter decay and Dahlin synthesis formulas.*

PROCESS	PID Quarter decay ratio	PID Dahlin Synthesis
FOPDT $G(s) = \left(\frac{Ke^{-t_0 s}}{\tau s+1}\right)$	$K_c = \frac{1.2}{K}\left(\frac{t_0}{\tau}\right)^{-1}$	$K_c = \frac{\tau}{K(t_0+\tau_c)}$
	$T_i = 2t_0$	$T_i = \tau$
	$T_d = 0.5t_0$	$T_d = 0.5\tau$

Fig.6: *Experimental setup.*

3. EXPERIMENTAL SETUP

The test rig was setup by using a commercially available TEG1120 as shown in Fig. 6.

In this research, we measured and recorded the values of the output voltage over times which were used to plot the response of process reaction curve. The experiments were done over the period of six months to verify that data were accurate enough in determining the parameter of the FOPDT function. At the inlet, the fuel gas that we applied to the pressure regulator was about 40% of maximum rating of the TEG. The maximum pressure rating for the TEG machine used should be at 4 psi, which means that the pressure applied was 1.6 psi. We then increased the pressure using a step input change with the steady state voltage set at 50% of the maximum rating value, which is 2 psi. By controlling the fuel gas pressure at this range, we can determine the minimum gas pressure required to get a good stable TEG voltage under shorter amount of time.
The voltage outputs versus times were taken into account to plot for process reaction curve or open loop step response shown in Fig. 7. We used the incremental step response as an input for parameter extraction process.

Using a curve fitting method mentioned before, we obtained the parameters of FOPDT in (1) as follows, $K = 2.425$, $t_0 = 1.5$ minutes and $\tau = 7.5$ minutes.

Using a Simulink model as shown in Fig. 8, we verified the extracted parameters obtained. A determination of process transfer was done by comparing the incremental step response of the experimental result and the simulation result was shown in Fig. 9.

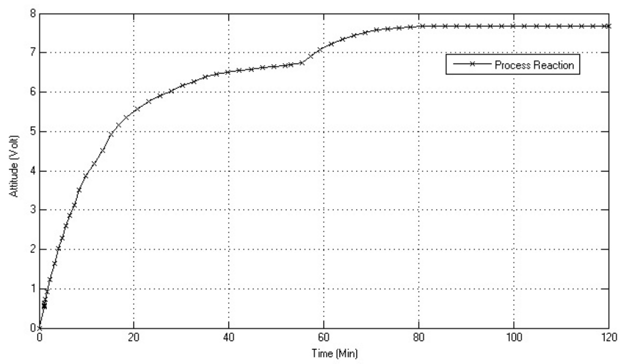

Fig.7: *Process reaction curve from commercial TEG machine.*

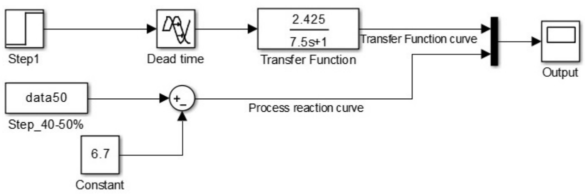

Fig.8: *Simulation block diagram for verifying the obtained FOPDT model for the TEG system.*

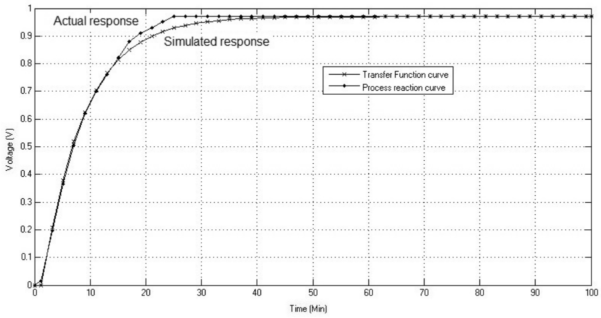

Fig.9: *Comparison between the actual response and the simulated response using a FOPDT model.*

The incremental step response from the experiment and the Simulink model were closely fitted together as shown in Fig. 9. The deviation between transfer function curve and process reaction curve is caused from calculation of proper time constant with maintain the same value between simulated and actual response. Alternatively, the actual response shows a small overshoot, which indicates a damped second-ordered system. However, the deviation is not too big. As a result, the FOPDT can be used to model a complicated system such as the TEG for further study purposes.

The controller tuning was done under no load connection to get the actual result. Practically, different machines will have different amount of load, which can result in different voltage drops. As the load varies, it is better to exclude the load from the tuning process to obtain more accurate result.

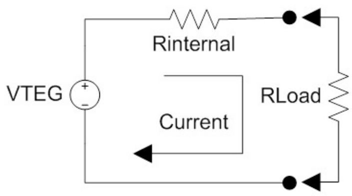

Fig.10: *An equivalent circuit of TEG.*

Fig. 10 shows the equivalent circuit for the TEG. Thus based from the voltage divider theory, the internal load can be estimated. By measuring the corresponding voltage and current across each of the load used, we can estimate the voltage drop across the internal resistance. Next, this voltage drop and the measured current values can be used to calculate the internal load resistance value. The calculated internal load values for each of the corresponding load used and the average internal load are shown in Table 2 below.

We used the derived transfer function model to design a simple controller to study its behaviour under a closed-loop control system. Tuning formulas of

Table 2: *Internal resistance estimation.*

Rload	Voltage (Measured)	Current (Measured)	Internal Load (Calculated)	Average Internal Load (Calculated)
1 Ω	5.30 V	5.30 A	0.28 Ohm	0.31 Ohm
2 Ω	6.00 V	3.00 A	0.27 Ohm	
5 Ω	6.50 V	1.30 A	0.23 Ohm	
10 Ω	6.50 V	0.65 A	0.46 Ohm	

Quarter decay ratio and Dahlin synthesis are given in Table 3.

Table 3: *Quarter decay ratio and Dahlin synthesis parameters.*

PROCESS	PID Quarter decay ratio	PID Dahlin synthesis
FOPDT	$K_c = 2.474$	$K_c = 1.134$
$G(s) = \left(\dfrac{2.425e^{-1.5s}}{7.5s+1}\right)$	$T_i = 3.000$	$T_i = 7.500$
	$T_d = 0.750$	$T_d = 3.750$

The comparison results of voltage output which operate on different electrical load values were used in the model and their respective results were compared with voltage output from physical machine.

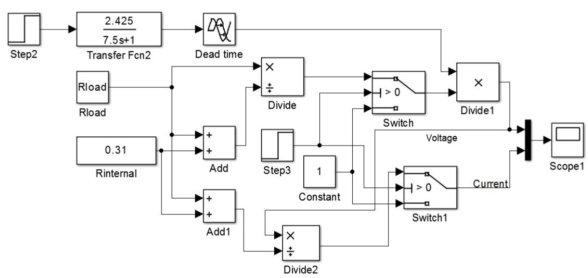

Fig.11: *Completed simulation model with electric load connection.*

The electric loads of 1 Ohm, 2 Ohm, 5 Ohm, and 10 Ohm together with internal load were selected to be used in the model. The switch for each of the electrical load will be activated when the completed model achieve its steady state time.

Fig. 12 above represents the output voltage of the system model with electrical load connected when the system is operating at the steady state. The voltage output current and internal resistance is based on the voltage divider theory. Fig. 13 shows the output voltage and current obtained from the physical machine. The electrical load used is connected when the voltage reach 6.8 volt.

Table 4 above shows the different output voltage and current with respect to the load of the system model used. By comparing the voltage and current values obtained from simulation and physical machine, we can conclude that both sets of result are almost the same. One of the hiding conditions is the internal

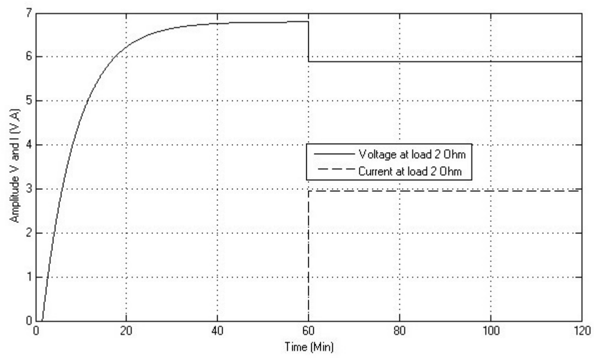

Fig.12: *The system step response using completed simulation model with electric load connection.*

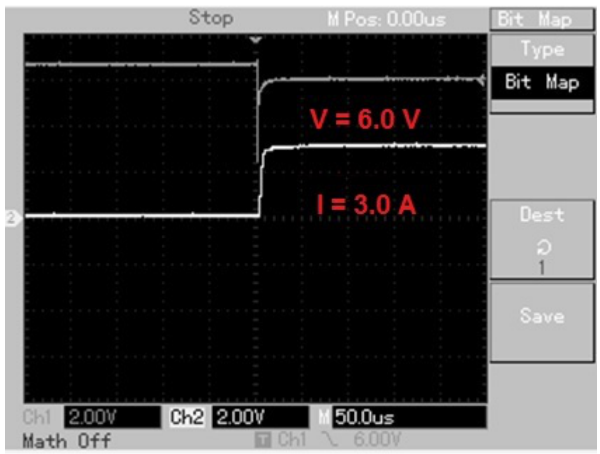

Fig.13: *The output response from physical machine.*

Table 4: *Comparison of step response system model and physical machine.*

Rload	Voltage (Measured)	Voltage (Simulated)	Current (Measured)	Current (Simulated)
1 Ω	5.30 V	5.19 V	5.30 A	5.19 A
2 Ω	6.00 V	5.89 V	3.00 A	2.94 A
5 Ω	6.50 V	6.40 V	1.30 A	1.28 A
10 Ω	6.50 V	6.60 V	0.65 A	0.66 A

load that might have effect on the output voltage and current when electric load is connected.

Next, we tried to equip the system with the Dahlin synthesis PID controller together with electric load connection. This step is to validate that the PID controller can control output response back to the steady state voltage even the output voltage drop due to the electric load. The complete simulation model with PID controller was designed and PID parameters given to the PID block to simulate step response of both voltage and current.

4. RESULTS AND DISCUSSION

We simulated the completed closed-loop control of the TEG system without an electrical load using the controller parameter described in Table II. The unit step response of the controlled system is shown in Fig. 14. And the completed of experimental data collection from real TEG machine was compared with data response from the Dahlin synthesis tuning.

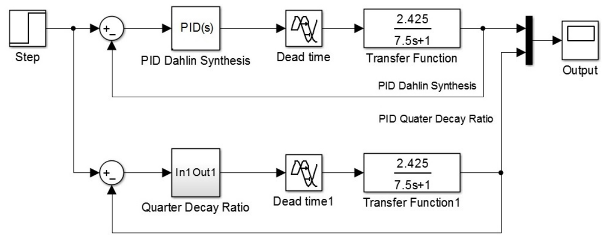

Fig.14: *Completed simulation model for PID controller.*

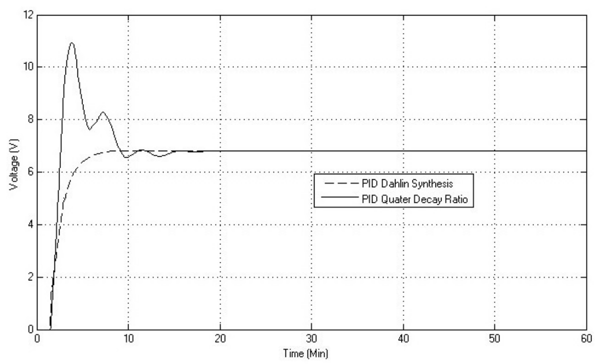

Fig.15: *The system step response using the proposed method for PID controller.*

The simulation results confirm that the proposed method can be applied to the simulation of thermoelectric generator control. Fig. 15 shows that the Quarter decay ratio tuning parameters results in a significant high overshoot, more oscillatory behaviour, and a longer settling time. More details on the step responses are shown in Table 5.

Table 5: *Comparison of step response system model and physical machine.*

STEP RESPONSE	Quarter decay	Dahlin synthesis
Rise time	2 minutes	8 minutes
Peak time	5 minutes	-
Settling time	15 minutes	10 minutes

Fig. 16 shows that the system response for both voltage and current at the electric load when using PID controller.

5. CONCLUSION

This paper has shown that the parameter extraction of the TEG model using the FOPDT as a system model. We can utilize this model for designing a controller in order to enhance the system performance of the TEG system. Finally, it is possible to take

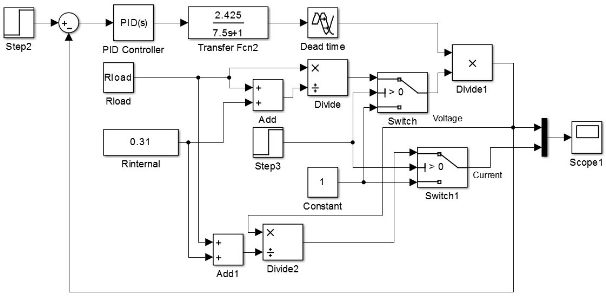

Fig.16: *Completed simulation model for PID controller with electric load connection.*

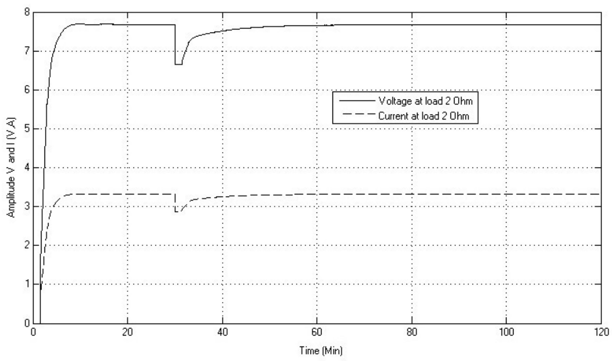

Fig.17: *The system step response using completed simulation model for PID controller with electric load connection.*

into account an electrical load in the simulation for dynamic response study.

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Rung-aroon Photoon received his Bachelor degree in Electrical Engineering from Mahanakorn University, Thailand in 2007. Currently, he is a Electrical Trainer at Chevron Thailand Exploration and Production. His research interests include system modeling, control theory and electronics application.

Warit Wichakool received the S.B. and M.Eng. degrees in Electrical Engineering and Computer Science from the Massachusetts Institute of Technology, Cambridge, USA in 2001, and his Ph.D. degree in Electrical Engineering at Massachusetts Institute of Technology, Cambridge, USA in 2011. Currently he is a lecturer at Electrical Engineering Department, Faculty of Engineering, Prince of Songkla University, Songkhla, Thailand. His research interests include system modeling, control theory and power electronics applications.