An Evaluation of Significant Lightness Difference Effective Term for CIEDE2000 Comparative Image Colour Pixels

Bongkarn Homnan[1], Member and **Watit Benjapolakul**[2], Non-member

ABSTRACT

As CIEDE2000 standard has developed the colour difference formula, the colour difference formula of compared colours in similar colour space has been evaluated and emphasized in this paper. The lightness difference effective term has been distinctively separated from the small informative colour-difference of CIEDE2000. In addition, the power of the lightness difference effective term can clearly magnify the lightness difference which illustrated categorized surfaces of the lightness difference effective term for total colour differences in the CIEDE2000 standard. All projected four categories are classified by compared image colour pixels based on the colour space's three forward transform functions of the lightness of the colour, the chroma between red/magenta and green, and the chroma between yellow and blue. Results show that the lightness difference effective term gives mainly three different surfaces in four categories. Furthermore, in first category, the most two maximum statistical frequencies can be expressed with the left side along with the right side of the probability mass function of the lightness difference effective term; in third category, the obvious maximum statistical frequency can be expressed with only the left side of the probability mass function; and in second and fourth categories, the maximum statistical frequency can be expressed most absolutely with the left side of the probability mass function.

Keywords: CIEDE2000, Colour, Different, Image, Lightness

1. INTRODUCTION

1.1 Abbreviations

CIE	the International Commission on Illumination
CIE31	the chromacity model of CIE 1931
CIE64	the chromacity model of CIE 1964
CIE94	the colour difference model of CIE 1994
CIEDE2000	the colour difference model of CIE 2000
CIELAB	the colour difference model of CIE 1976
CIELUV	the colour space adopted by CIE 1976
HSL	Hue, Saturation, Lightness primaries
RGB	Red, Green, Blue primaries
XYZ	X,Y, Z primaries

1.2 Symbols

α	the slope of the forward transform function
β	the intercept of the forward transform function
$\Delta\theta$	the angle difference
$\Delta C'_{ab}$	the chroma difference of CIEDE2000
ΔC^*_{ab}	the chroma difference of CIE94
ΔE^*_{00}	the total colour difference of CIEDE2000
ΔE^*_{ab}	the total colour difference of CIELAB
ΔE^*_{94}	the total colour difference of CIE94
$\Delta E^*_{94}(k_L : k_C : k_H)$	the ratio format of $k_L : k_C : k_H$ of CIE94
$\Delta H'{}_{ab}$	the hue difference of CIEDE2000
$\Delta H^*{}_{ab}$	the hue difference of CIE94
$\Delta h'{}_{ab}$	the a, b hue difference of CIEDE2000
$\Delta L'$	the lightness difference of CIEDE2000
$\Delta L'{}_F$	the lightness difference effective term
ΔL^*	the lightness difference of CIE94
Δx	the x difference, $x_2 - x_1$
λ	the wavelength
A	the denominator of $\dfrac{\Delta L^*}{A}$ or $\dfrac{\Delta L^*}{A}$
a'	the a chroma of CIEDE2000
a^*	the a chroma of CIELAB or CIE94
B	the denominator of $\dfrac{\Delta C^*_{ab}}{B}$ or

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[1] The author is with Research Service Center, Dhurakij Pundit University, Laksi, Bangkok, 10210, Thailand, Email: bongkarn@dpu.ac.th
[2] The author is with Department of Electrical Engineering, Faculty of Engineering, Chulalongkorn University, Pathumwan, Bangkok, 10330, Thailand, Email: watit.b@chula.ac.th

	$\frac{\Delta C^*_{ab}}{B}$
$b^{'}$	the b chroma of CIEDE2000
b^*	the b chroma of CIELAB or CIE94
B	the denominator of $\frac{\Delta C^*_{ab}}{B}$ or $\frac{\Delta C^*_{ab}}{B}$
$\Delta C^{'}{}_{ab}$	the a, b chroma of CIEDE2000
$\Delta C^*{}_{ab}$	the a, b chroma of CIE94
D	the compensation term for hue rotation
$f(t)$	the forward transform function of t
G	the gain of the scaling factor for a^* of CIE94
$\Delta h^{'}{}_{ab}$	the a, b hue angle of CIEDE2000
$\Delta h^*{}_{ab}$	the a, b hue angle of CIE94
K_1	the weighting factor for chroma
K_2	the weighting factor for hue
k_C	the weighting factor of the compensation for chroma
k_H	the weighting factor of the compensation for hue
k_L	the weighting factor of the compensation for lightness
$L^{'}$	the lightness of CIEDE2000
L^*	the lightness of CIELAB, CIELUV, or CIE94
N	the normalized factor
R_C	the 2 time prediction of the a b chroma of CIELAB C^*_{ab}
R_T	the interactive term between ΔC^*_{ab} and ΔH^*_{ab}
S_C	the compensation factor for chroma
S_H	the compensation factor for hue
S_L	the compensation factor for lightness
T	the gain of the scaling factor for S_H of CIE94
$\bar{x}$	the average value of the variable x
x°	the x degree
(L^*, a^*, b^*)	the tristimulus values of the L^*, a^*, b^* chromaticity coordinate
(R, G, B)	the tristimulus values of the RGB chromaticity coordinate
(r, g, b)	the tristimulus values of the norm RGB chromaticity coordinate
$(r(\lambda), g(\lambda), b(\lambda))$	the tristimulus values of the norm RGB chromaticity coordinate for 2° observer in the environment of the wavelength λ
$(\bar{r}_{10}, \bar{g}_{10}, \bar{b}_{10})$	the average tristimulus values of the norm RGB chromaticity coordinate for 10° observer
(X, Y, Z)	the tristimulus values of the XYZ chromaticity coordinate
(x, y, z)	the tristimulus values of the norm XYZ chromaticity coordinate
$(x(\lambda), y(\lambda), z(\lambda))$	the tristimulus values of the norm XYZ chromaticity coordinate for 2° observer in the environment of the wavelength λ
$(\bar{x}_{10}, \bar{y}_{10}, \bar{z}_{10})$	the average tristimulus values of the norm XYZ chromaticity coordinate for 10° observer
(X_n, Y_n, Z_n)	the tristimulus values of the reference neutral white point

1.3 Fundamental

The colour distances of colour non-uniformities are continuing studied because normally human eyes are still more sensitive to certain colours than other colours. Forwarding society, the International Commission on Illumination (CIE) met the colour difference model of CIE 1976 (CIELAB), the colour difference model of CIE 1994 (CIE94) and the colour difference model of CIE 2000 (CIEDE2000) standards have developed and investigated the colour difference formula [1, 2, 3, 4, 5]. That is, the colour difference formula depends on only the two compared colours in similar colour space. The colour difference is very informative and very important. For example, the exponential function of an angle difference has been introduced to use [6]. CIELAB, the first colour difference formula, has been the colour difference formula computing the lightness of CIELAB L^*, the a chroma of CIELAB a^*, and the b chroma of CIELAB b^*. This colour difference of CIELAB $\Delta E^*{}_{ab}$ is the Euclidean distance of two coordinates-compared colours of tristimulus values of the L^*, a^*, b^* chromaticity coordinate (L^*, a^*, b^*). However, this formula rates these colours too highly as opposed to other colours [3]. Since CIE94 standard did not not adequately determine the perceptual uniformity problem especially in the saturated regions even though the hue difference of CIE94 ΔH^*_{ab}, the compensation factor for lightness S_L,the compensation factor for chroma S_C, and the compensation factor for hue S_H have been introduced, CIEIDE2000 refined their definition, adding corrections [1, 3] including 1) a hue rotation term or the interactive term between ΔC^*_{ab} and $\Delta H^{'}{}_{ab}, R_T$, to deal with the problematic blue region of hue angles in the neighborhoods of $0^\circ - 275^\circ$, 2) the compensation for neutral colours-the a chroma of CIEDE2000 $a^{'}$ and the chroma of CIEDE2000 $b^{'}$, 3) the compensation factor for lightness S_L, 4) the compensation factor for chroma S_L, including $a^{'}$ and chroma $b^{'}$ and, 5) the compensation factor for hue S_H including $h^{'}_{ab}$.

This paper extends the small informative colour difference. It has been magnified and expressed in the lightness difference effective term $\Delta L_F^{'}$ of the total colour difference of CIEDE2000 $\Delta E_{00}^{'}$. All projected four categories are classified by compared image colour pixels based on the colour space's three forward transform functions of the lightness of the colour, the a chroma of CIELAB a^* between red/magenta and green, and the b chroma of CIELAB b^* between yellow and blue as depicted in Fig. 1. The paper consists of section 1. Introduction, section 2. The colour different improvement, section 3. Colour space transformation, section 4. Experimental results and analysis, and section 5. Conclusions and future work.

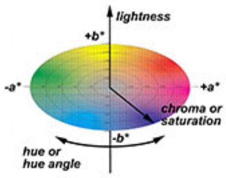

Fig.1: *CIELAB colour space with lightness axis and hue including saturation, the a chroma of CIELAB a^*, and the b chroma of CIELAB b^* [7] (Artwork: Karl Schmetzer).*

2. THE COLOUR DIFFERENCE IMPROVEMENT

As explained in the previous section, the comparative parameters [1, 2, 3, 8] among three colour spaces-CIELAB, CIE94, and CIEDE2000 can be listed in Table 5. CIELAB, the total colour difference model of CIE 1976, is a chromatic value colour space. A related colour space, the colour space adoped by CIE 1976 (CIELUV), preserves the lightness of CIELUV L^* as in (L^*, a^*, b^*). However, CIELUV has a different representation of the chromaticity components compared with CIELAB. The total colour difference of CIE94 listed in Table 5 is the objective function should be minimized to obtain the improved colour difference formula. Regarding Table 5, the chroma of CIE94 C_{ab}^* of two colour samples, image pixels in an computer vision 's work, of a and b is the geometric average chroma of two colour samples and was introduced to improve ΔE_{94}^*. CIE94 still has large errors in saturated blue colour and for near-neutral colours [6]. Thus, the a, b hue angle of CIEDE2000 $h_{ab}^{'}$ and its average $\bar{h}_{ab}^{'}$ and the a, b hue difference of CIEDE2000 $\Delta h_{ab}^{'}$ which are in similar degree (°) unit were introduced to the total colour difference of CIEDE2000 $\Delta E_{00}^{'}$. And, the final angle based on an periodic angle $0° - 360°$ can be made positive by an addition of the degree period of 360 if necessary.

Additionally, in CIE94 and CIEDE2000, the weighting factors of the compensation for lightness, chroma, and hue k_L, k_C, and kH are assigned to 1.0. The ratio format of $k_L : k_C : k_H$ is $\Delta E_{94}^*(k_L : k_C : k_H)$. Notably, in the textile industry k_L is assigned to 2.0 [1, 9] and it is denoted as $\Delta E_{94}^*(2 : 1 : 1)$.

CIEDE2000 introduced following features- 1) the interactive term between ΔC_{ab}^* and $\Delta H_{ab}^{'}$, R_T, to deal with the problematic blue region of hue angles in the neighborhoods of $0° - 275°$ and to improve the prediction of blue colour difference by increasing $|R_T|$ thus decreasing total colour difference of CIEDE2000 $\Delta E_{00}^{'}$, 2) the compensation for neutral colours-the a chroma of CIEDE2000 $a^{'}$ and the b chroma of CIEDE2000 $b^{'}$, by introducing the gain of the scaling factor for a^* of CIE94, G, to improve the prediction of the near-neutral colour difference by direct increasing the chroma difference of CIEDE2000 $\Delta C_{ab}^{'}$ and indirect increasing R_T and the compensation factor for hue S_H, 3) the compensation factor for lightness S_L, by increasing the unity S_L of CIE94 with the factor relevant to general industrial uses of the lightness around mean of the lightness range, 50%, of Hue, Saturation, and Lightness primaries (HSL) [10]. 4) the compensation factor for chroma S_C, including the a chorma of CIEDE2000 $a^{'}$ and the b chroma of CIE94 b^*, also by the gain of the scaling factor for a^* of CIE94, G, and 5) the compensation factor for hue S_H including $h_{ab}^{'}$ in the gain of the scaling factor for S_H of CIE94, T, to improve the prediction of the hue difference by direct increasing S_H to decrease $\Delta E_{00}^{'}$.

3. COLOUR SPACE TRANSFORMATION

The transformation recommended by the chromacity model of CIE 1931 (CIE31) [1] was through the specification of corresponding tristimulus values of the norm X, Y, Z primaries (XYZ) chromaticity coordinate (x, y, z) of four stimuli of well-defined spectral distributions-red colour of 700.0 nm, green colour of 546.1 nm, blue colour of 435.8 nm, and source colour of tristimulus values of the norm Red, Green, Blue primaries (RGB) chromaticity coordinate (r, g, b).

3.1 Colour Space Matching

The tristimulus values of the norm RGB chromaticity coordinate (r, g, b) can be written in (x, y, z) [1] as

$$\begin{aligned} x &= \frac{0.49000r + 0.31000g + 0.20000b}{0.66697r + 1.13240g + 1.20063b} \\ y &= \frac{0.17697r + 0.81240g + 0.01063b}{0.66697r + 1.13240g + 1.20063b} \\ z &= \frac{0.00000r + 0.01000g + 0.99000b}{0.66697r + 1.13240g + 1.20063b} \end{aligned} \tag{1}$$

where parameters can be denoted below:

(r,g,b) are the tristimulus values of the norm RGB chromaticity coordinate, in fact, tristimulus values of the norm RGB chromaticity coordinate for 2° observer in the environment of the wavelength λ, $(r(\lambda), g(\lambda), b(\lambda))$, and

(x,y,z) are the tristimulus values of the norm XYZ chromaticity coordinate, in fact, tristimulus values of the norm XYZ chromaticity coordinate for 2° observer in the environment of the wavelength λ, $(x(\lambda), y(\lambda), z(\lambda))$.

The chromacity model of CIE 1964 (CIE64) supplementary standard colorimetric 10° observer adopted the transform matrix derived by the CIE31 supplementary standard colorimetric 10° observer. To transform the original tristimulus values of the norm RGB chromaticity coordinate (r,g,b) to the corresponding tristimulus values of the norm XYZ chromaticity coordinate (x,y,z), the CIE64 colour matching matrix can be operated as

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = N \begin{bmatrix} 0.341080 & 0.189145 & 0.387529 \\ 0.139058 & 0.837460 & 0.073316 \\ 0.000000 & 0.039553 & 2.026200 \end{bmatrix} \begin{bmatrix} r \\ g \\ b \end{bmatrix} \quad (2)$$

where parameters can be denoted below:

(r,g,b) are the tristimulus values of the norm RGB chromaticity coordinate for 10° observer, in fact, the average tristimulus values of the norm RGB chromaticity coordinate for 10° observer $\bar{r}_{10}, \bar{g}_{10}, \bar{b}_{10}$,

(x,y,z) are the tristimulus values of the norm XYZ chromaticity coordinate for 10° observer, in fact, the average tristimulus values of the norm XYZ chromaticity coordinate for 10° observer $\bar{x}_{10}, \bar{y}_{10}, \bar{z}_{10}$, and

N is the norm factor, $\dfrac{X+Y+Z}{R+G+B} = 1$.

Regarding Table 1 formulas of the lightness of CIELAB or CIE94 L^*, the a chroma of CIELAB or CIE94 a^*, the b chroma of CIELAB or CIE94 b^*, the total colour difference of CIELAB ΔE^*_{ab}, the total colour difference of CIE94 $\Delta^{E^*_{94}}$, and other parameters of CIELAB and CIE94 can be compared with those of the lightness of CIEDE2000 L, the a chroma of CIEDE2000 $a^{'}$, the b chroma of CIEDE2000 $b^{'}$, the total colour difference of CIE00 ΔE^*_{00}, and other parameters of CIEDE2000, respectively. Actually, (2) can match (r,g,b) and corresponding $(x,y,)$, tristimulus values of the RGB chromaticity coordinate (R,G,B) and corresponding tristimulus values of the XYZ chromaticity coordinate X,Y,Z appeared in various formulas in Table 1.

neutral white point X_n, Y_n, Z_n, the CIE XYZ tristimulus values of the reference white point, depends on syandard illuminants [11, 12], for example, CIE standard illuminat D55 (Asia), CIE standard illuminat D65 (Western Europe/Northern Europe), CIE standard illuminat E, etc. The standard illuminants were defined for theoretical source of visible light with a profile of the spectral power distribution. In addition, the standard illuminants provide a basis for comparing images or colours under different lighting. CIE31 introduced CIE standard illuminants A, B, and C, with the intention of representing average incandescent light, direct sunlight, and average daylight, respectively. And, CIE standard illuminant D represents phases of daylight, CIE standard illuminant E represents the equal-energy illuminant, and CIE stanadrd illuminant F represents fluorescent lamps of various composition. However, CIE standard illuminant E has a constant spectral power distribution inside the visible spectrum. It is useful as a theoretical reference because this illuminant gives equal weight to all wavelengths. CIE standard illuminant E also has equal tristimulus values of the XYZ chromaticity coordinate (X,Y,Z), thus tristimulus values of the norm XYZ chromaticity coordinate (x,y,z) is equal to $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. This reference neutral white point is by design and suitable for the equal-energy illuminant and application designs.

Typically, the CIE standard illuminant E is not a black body and does not have a colour temperature. Approximated by a CIE standard illuminants D, D series, with a correlated colour temperature of 5455 K, manufacturers sometimes compare light sources against CIE standard illuminant E to calculate the excitation purity [12, 14]. The CIE standard illuminant E is beneath the Planckian locus of the conventional CIE31 colour space chromaticity diagram [15, 16] as depicted in Fig. 2, and closed isothermal with CIE standard illuminant D55 giving lower colour temperature [12, 17, 18] or warmer colours of red/yellow than CIE standard illuminant D65.

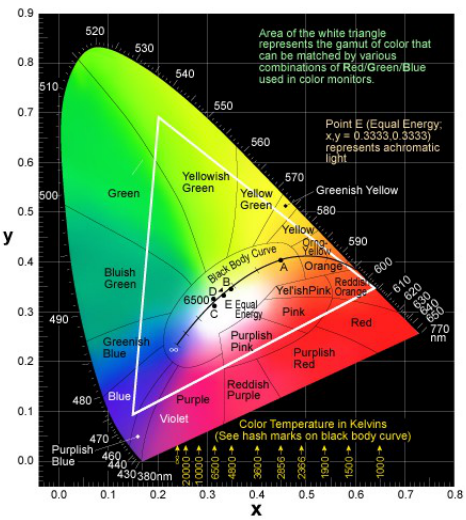

Fig.2: CIE31 colour space chromaticity diagram with CIE standard illuminants A, B, C, D, and E [15].

3.2 Lightness Difference

This subsection describes and proposes the lightness difference effective term $\Delta L_F^{'}$ of the total colour difference of CIEDE2000 $\Delta E_{00}^{'}$ listed in Table 1 as

$$\Delta L_F^{'} = \left(\frac{\Delta L^{'}}{A}\right)^2 \qquad (3)$$

where parameters can be denoted below:

$\Delta L^{'}$ is the lightness difference of CIEDE2000, $\Delta L^{'} = \Delta L_2^{'} - \Delta L_1^{'}$,

A is the denominator of $\frac{\Delta L^{'}}{A}$, $k_L S_L = k_L\left(1 + \frac{0.015(\bar{L}-50)^2}{\sqrt{20+(\bar{L}-50)^2}}\right)$

k_L is the weighting factor of the compensation for lightness used as the defined reference constant for lightness [1, 9]

$\overline{L^{'}}$ is the average value of the variable $L^{'}$, $\overline{L^{'}} = 0.5(L_1^{'} + L_2^{'})$,

$L^{'}$ is the lightness of CIEDE2000, $L^{'} 116f(t) - 16, \quad t = \frac{Y}{Y_n}$, the tristimulus values of the reference neutral white point $(X_n, Y_n, Z_n) = \frac{1}{3}\frac{1}{3}\frac{1}{3}$ were selected under CIE standard illuminant E [12] as described above, and

$f(t)$ is the forward transform function,

$$f(t) = \begin{cases} t^{\frac{1}{3}}, \ t > \left(\frac{6}{29}\right)^3 \approx 0.008856, \\ \alpha t + \beta, \ t \le 0.008856, \\ \alpha = \frac{1}{3}\left(\frac{116}{24}\right)^2, \beta\frac{16}{116}, \end{cases}$$

note: $\alpha \approx 7.787$ and $\beta \approx 0.1379$.

When $k_L = k_C = k_H = 1$, CIE94 and CIEDE2000 formulas can be applied for general industrial uses [1, 8], for example, graphic works $K_1 = 0.045$ and $K_2 = 0.015$. And in sensitive case, when $k_L = 2, k_c = k_H = 1, K_1 = 0.048$ and $K_2 = 0.014$, CIE94 and CIEDE2000 formulas can be applied for allowable colour differences in the textile industry.

The proposed term, the lightness difference effective term $\Delta L_F^{'}$ of the total colour difference of CIEDE2000 $\Delta E_{00}^{'}$, (3), can be rewritten into only $L_1^{'}$ and $L_2^{'}$ terms as

$$\Delta L_F^{'} = \frac{(L_1^{'} - L_2^{'})^2}{\left(k_L\left(1 + \frac{0.015\left(\frac{L_1^{'}+L_2^{'}}{2}-50\right)^2}{\left(20+\left(\frac{L_1^{'}+L_2^{'}}{2}-50\right)^2\right)^{0.5}}\right)\right)^2}. \qquad (4)$$

Consequently, (4) can be rewritten into four categories classified by t_1 and t_2, respectively replacing $L_1^{'}$ and $L_2^{'}$,with their forward transform functions $f(t_1)$ and $f(t_2)$. In first category of $t_1 > 0.008856$ and $t_2 > 0.008856$, expressed in (5), sensitively, the impact term is $(t_1^3 - t_2^3)^2$ which has the exponent of 2 of t^3 giving the 6th power.

$$\Delta L_F^{'} = \begin{cases} \frac{1}{k_L}\left(\frac{116(t_1^3 - t_2^3)}{\left(1 + \frac{0.015[58(t_1^3+t_2^3)-66]^2}{(20+[58(t_1^3+t_2^3)-66]^2)^{0.5}}\right)}\right)^2, \\ t_1 > 0.008856, \ t_2 > 0.008856, \\ \frac{1}{k_L}\left(\frac{116(t_1^3 - \alpha t_2 - \beta)}{\left(1 + \frac{0.015[58(t_1^3+\alpha t_2+\beta)-66]^2}{(20+[58(t_1^3+\alpha t_2+\beta)-66]^2)^{0.5}}\right)}\right)^2, \\ t_1 > 0.008856, \ t_2 \le 0.008856, \\ \frac{1}{k_L}\left(\frac{116\alpha(t_1 - t_2)}{\left(1 + \frac{0.015[58(\alpha(t_1+t_2)+2\beta)-66]^2}{(20+[58(\alpha(t_1+t_2)+2\beta)-66]^2)^{0.5}}\right)}\right)^2, \\ t_1 \le 0.008856, \ t_2 \le 0.008856, \\ \frac{1}{k_L}\left(\frac{116(\alpha t_1 + \beta - t_2^3)}{\left(1 + \frac{0.015[58(\alpha t_1+\beta+t_2^3)-66]^2}{(20+[58(\alpha t_1+\beta+t_2^3)-66]^2)^{0.5}}\right)}\right)^2, \\ t_1 \le 0.008856, \ t_2 > 0.008856, \end{cases} \qquad (5)$$

In second category of $t_1 > 0.008856$ and $t_2 > 0.008856$, due to low value of t_2, the impact term is $(t_1^3 - \alpha t_2 - \beta)^2$ which also has the exponent of 2 of t^3 giving the 6th power. In third category of $t_1 \le 0.008856$ and $t_2 \le 0.008856$, sensitively, the impact term is $(t_1 - t_2)$ which has the exponent of 2 of t giving the 2th power.

In addition, in fourth category of $t_1 > 0.008856$ and $t_2 > 0.008856$, in fact, similar to the third category by switching $L_1^{'}$ and $L_2^{'}$ accompanied with $f(t_1)$ and $f(t_2)$. Thus, due to low value of t_1, the impact term is the denominator which can close to zero. As a result, the proposed term, the lightness difference effective term $\Delta L_F^{'}$ of the total colour difference of CIEDE2000 $\Delta E_{00}^{'}$, (3), can neglect the problem of sign because $\Delta L_F^{'}$ has only positive value by the exponent of 2. In addition the exponent of 2 can magnify the lightness difference of CIEDE2000 $\Delta L^{'}$ as the 2th power.

4. EXPERIMENTAL RESULTS AND ANALYSIS

Regarding (5), the lightness difference effective term $\Delta L_F^{'}$ of the total colour difference of CIEDE2000 $\Delta E_{00}^{'}$ can be categorized into four categories by t_1 and t_2, more described details in next subsection. In addition, this section also illustrates their probability masses of $\Delta L_F^{'}$.

4.1 $\Delta L_F^{'}$ Surfaces

In first category of $t_1 > 0.008856$ and $t_2 > 0.008856$, the lightness difference effective term $\Delta L_F^{'}$ surface can be illustrated in Fig. 3. The values of t_1 and t_2 below around 0.5, the first interval starting values having the impact term of $(t_1^3 - t_2^3)^2$, give very low $\Delta L_F^{'}$. These first interval values of t gives very small light differences between two colour samples, experimental image colour pixels, due to effects of white colour in tint colours around the the reference neutral white point (X_n, Y_n, Z_n). However, values of $\Delta L_F^{'}$ in first interval values of t can be optionally immensely magnified by the logarithm function.

The $\Delta L_F^{'}$ surface has an even function property around $t_1 = t_2$ asymptote of equal two compared colours. In addition, values of t_1(low t_2) or t_2 (low t_1) above around $\frac{66}{58} = 1.1379 \approx 1.0$, are affected by $(20+[58(t_1^3+t_2^3)-66]^2)^{0.5}$. And, $t \approx 1.0$, $t = \frac{Y}{Y_n}$, gives Y closed to Y_n in tristimulus values of (X_n, Y_n, Z_n) of CIE standard illuminant E, $(X_n, Y_n, Z_n) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ as described in lightness difference subsection. As a result, values of t_1 (low t_2) or t_2 (low t_1) above around 1.0 still give very high $\Delta L_F^{'}$ due to the impact term of $(t_1^3-t_2^3)^2$. Notably, in Fig. 2, values of t above around 1.0 relevant to Y above around $\frac{1}{3}$ includes high values of bluish green, high values of bluish green, green, yellowish green, yellow green, yellow, orange yellow, orange, high values of yellowish pink, and high values of reddish orange. In addition, the maximum y in tristimulus values of the norm XYZ chromaticity coordinate (x, y, z) was recorded [19] and processed in this experiment equal to 0.8341, thus, the maximum t is 0.8341·3=2.5023 for the wavelength λ between 520-522 nm.

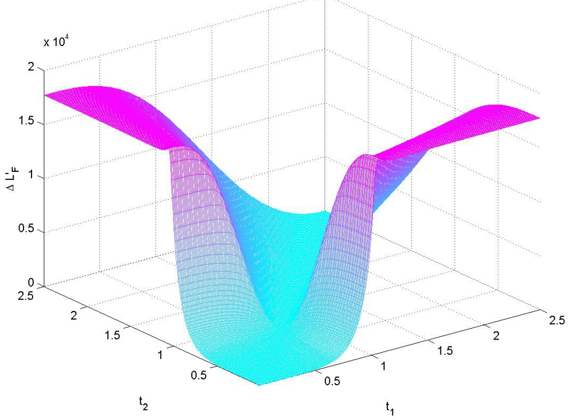

Fig.3: *Values of $\Delta L_F^{'}$ of first category of t_1 and t_2.*

Additionally, $t_1(t_2)$ between around 0.5 and 1.0 can give sharply very low to very high $\Delta L_F^{'}$ due to the effect of $(0.015[58(t_1^3+t_2^3)-66]^2/20+[58(t_1^3+t_2^3)-66]^2)^{0.5} = 0$. In addition, values of t_1 and t_2 around between the $t_1 = t_2$ locus of closed two compared colour samples and the values of $t_1(t_2)$ above 1.0 can give sensitively very low to very high $\Delta L_F^{'}$. Notably, resemble colours around the centroid expressed by the colour discrimination ellipses, on the CIE31 colour space chromaticity diagram can be depicted in the Brown and MacAdam ellipses diagram [20, 21]. In addition, there are wide differences in the discrimination based on adjustments taking place within context by context and the formula predicting sizes and orientations of those various sizes' ellipses depending on the chromaticity point of a colour has not been proposed. In fact, the total colour difference of CIELAB ΔE_{ab}^* generally can be represented by a perfect circle [22] for every couple saturation and hue on the CIELAB colour space. More precisely, the shape of the colour discrimination threshold of the human eye is bounded by an ellipse [22], on the a^*b^* plane perpendicular to the lightness axis.

In second category of $t_1 \leq 0.008856$ and $t_2 \leq 0.008856$, the lightness difference effective term $\Delta L_F^{'}$ surface can be illustrated in Fig. 4. The values of t_1 below around 0.5, the first interval starting values having the impact term of $(t_1^3 - \alpha t_2 - \beta)^2$, give very low $\Delta L_F^{'}$ due to effects of white colour in tint colours around the the reference neutral white point (X_n, Y_n, Z_n). And, values of t_1 above around $\frac{66}{58} = 1.1379 \approx 1.0$, are affected by $(20 + [58(t_1^3 + \alpha t_2 + \beta) - 66]^2)^{05}$. And, values of t_1 above around 1.0 still give very high $\Delta L_F^{'}$ due to the impact term of $(t_1^3 - \alpha t_2 - \beta)^2$ especially 3rd power of t_1^3 and very low values of t_2. In addition, higher values of t_2 can give smaller values of $\Delta L_F^{'}$. Additionally, $t_1(t_2)$ between around 0.5 and 1.0 can give sharply very low to very high $\Delta L_F^{'}$ due to the effect of $(0.015[58(t_1^3+t_2^3)-66]^2/20+[58(t_1^3+t_2^3)-66]^2)^{0.5} = 0$.

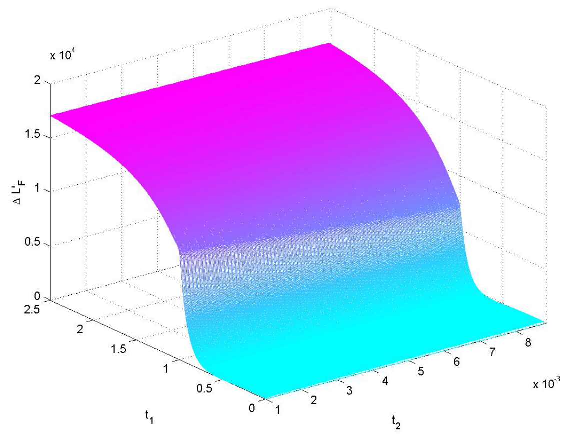

Fig.4: *Values of $\Delta L_F^{'}$ of second category of t_1 and t_2.*

In third category of $t_1 \leq 0.008856$ and $t_2 \leq 0.008856$, the lightness difference effective term $\Delta L_F^{'}$ surface can be illustrated in Fig. 5. The $\Delta L_F^{'}$ surface has an even function property around $t_1 = t_2$ asymptote of equal two compared colours, obtained from the impact term $(t_1 - t_2)$ and its 2nd power, certainly with very low positive $\Delta L_F^{'}$. And, values of t_1 and t_2 around between the $t_1 = t_2$ locus of closed two compared colour samples and the values of $t_1(t_2)$ can give very low to high $\Delta L_F^{'}$. Notably, the denominator of third category in (5) always gives smaller values along t_1 or t_2. In addition, in Fig. 2, values of $t = \frac{Y}{Y_n}$ below 0.008856 relevant to Y below 0.0266 includes high values of blue, high values of purplish blue, high values of violet, and high values of purple.

In addition, in fourth category of $t_1 \leq 0.008856$ and $t_2 \leq 0.008856$, $\Delta L_F^{'}$ surface is resemble to Fig. 4 due to the effect of the 2nd power of its category in (5) except replacing t_1 with t_2 and vice versa as previously explained about this category of (5).

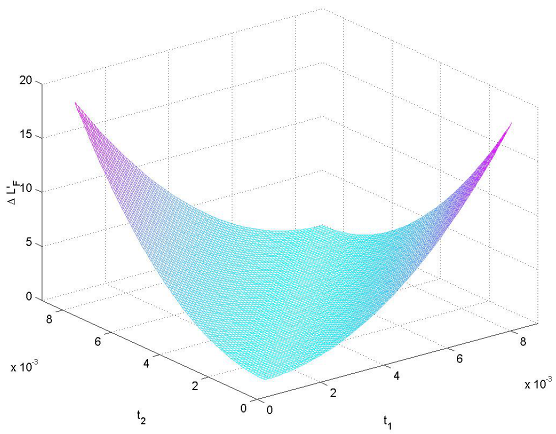

Fig.5: *Values of $\Delta L_F^{'}$ of third category of t_1 and t_2.*

4.2 Probability Mass Functions

All values of the lightness difference effective term $\Delta L_F^{'}$ surfaces of first to third categories in Fig. 3 to Fig. 5 can respectively reveal their probability mass functions [23] in Fig. 6 (a) to Fig. 6 (c) to express thier possible events.

In first category of $t_1 > 0.008856$ and $t_2 > 0.008856$, the probability mass function illustrated in Fig. 6 (a) expresses its values in a domain of 0.00 to 1.80×10^4 of $\Delta L_F^{'}$. The maximum value of a probability mass function is closed to 0.30 while the minimum value is closed to 0.05. The most two maximum values of 0.293 and 0.250 are respectively on the left side and the right side of possible events depicted with the probability mass function. Moreover, the least minimum value of 0.042 similarly are of the 4th and 5th bars of a 10-bar normalized histogram of alike uniform distribution between left and right bars, represented in the probability mass function. Respectively, mean and standard deviation of the probability mass function in first category are 0.85×10^4 and 0.66×10^4.

In second category of $t_1 > 0.008856$ and $t_2 > 0.008856$, (fourth category of $t_1 \leq 0.008856$ and $t_2 \leq 0.008856$) the probability mass illustrated in Fig. 6 (b) expresses its values in a domain of 0.00

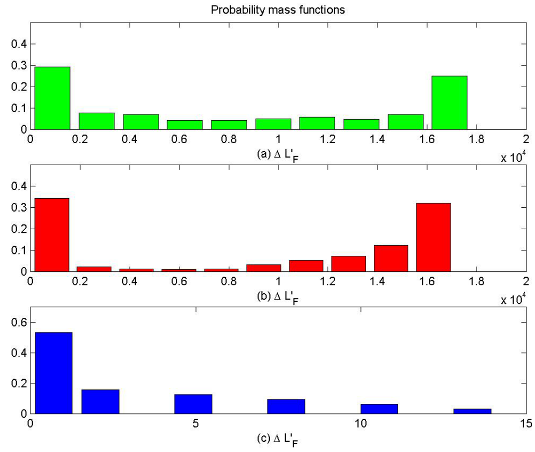

Fig.6: *Probability mass functions of $\Delta L_F^{'}$ of (a) first category, (b) second (fourth) category, and (c) third category.*

to 1.70×10^4 of $\Delta L_F^{'}$. The maximum value of the probability mass function is closed to 0.35 while the minimum value is above 0.022. The most two maximum values 0.342 and 0.320 are respectively on the left side and the right side of possible events depicted with the probability mass function. Moreover, the least minimum value 0.022 is on the 4th bar of a 10-bar normalized histogram increasing their values, conformed to two dimemsional summations of the second category of (5), from 2nd to 10th bars represented in the probability mass function. Respectively, mean and standard deviation of the probability mass function in second (fourth) category are 0.94×10^4 and 0.68×10^4.

In third category of $t_1 \leq 0.008856$ and $t_2 \leq 0.008856$, the probability mass function illustrated in Fig. 6 (c) expresses its values in a domain of 0.00 to 15.00 of $\Delta L_F^{'}$. The maximum value of the probability mass function is 0.53 on the left side while the minimum value is 0.03 on the right side of the probability mass function. Moreover, values of the probability mass function gradually decreases from 2nd to 10th bars of a 10-bar normalized histogram represented in the probability mass function. Respectively, mean and standard deviation of $\Delta L_F^{'}$ in third category are 3.12 and 3.52 units in magnified scale of $0 - 10^5$ relevant to the lightness scale of $0 - 100$ in HSL. Nevertheless, exact means and standard deviations of all categories should exclude the colour region out of the gamut of tristimulus values of the norm XYZ chromaticity coordinate (x, y, z) with dependent t_1 and t_2, in fact, only two comparative image colour pixels.

5. CONCLUSION

Among four categories of compared colours categorized by t_1 and t_2, domains of the forward transform function $f(t)$ transform tristimulus values of the

XYZ chormaticity coordinate (X, Y, Z) to tristimulus values of the L^*, a^*, b^* chormaticity coordinate (L^*, a^*, b^*). The lightness difference effective term $\Delta L_F^{'}$ surfaces of first category and the third category have an even function around $t_1 = t_2$ asymptote. And, there are insensitive zone and sensitive zone of very low to very high $\Delta L_F^{'}$, in addition, saturated zone only in the first category. The $\Delta L_F^{'}$ surfaces of second (fourth) category has sharply sensitive values from very low to very high $\Delta L_F^{'}$ including along $t_1(t_2)$. Among four categories of related probability mass functions, the maximum probabilities of four categories are normally on the left side of thier probability mass function, very low $\Delta L_F^{'}$. However, first and second (fourth) categories give the maximum probabilities on the right side of their probability mass function, very high $\Delta L_F^{'}$. As a result, the third category gives very low $\Delta L_F^{'}$.

In future work, the complicated lightness difference transform in three dimensional colour space with helpful equations retrieved from boundary chromaticity coordinates for observer in the environment of wavelengths will be more investigated to improve the lightness difference formula.

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Bongkarn Homnan received the B.Eng., M.Eng, and Ph.D. degrees in electrical engineering from Chulalongkorn Universi-ty, Bangkok, Thailand, in 1994, 1998, and 2003, respectively. He currently is a Researcher in the Research Service Center and an Associate Professor in the Department of Computer and Telecom-munication Engineering, Dhurakij Pundit University, Bangkok, Thailand. His current research interests include mobile com-munication systems and the applications of artificial intelligence and image processing to communication systems.

Watit Benjapolakul received the Ph.D. degree in electronic engineering from the University of Tokyo, Tokyo, Japan, in 1989. He now is an Associated Professor in the Department of Electrical Engineering, Chulalongkorn University, Bangkok Thai-land. His current research is in the fields of mobile communication system, broad-band networks, and application artificial intel-ligence to communication systems.

Table 1: *Comparison of parameters among 3 colour spaces*

Parameters	CIE 1976(L^*, a^*, b^*)	CIE 1994 (CIE94)	CIEDE2000
L^*	$L^* 116 f(Y/Y_n) - 16$	L^*	$L^{'} = L^*$
a^*	$a^* = 500[f(X/X_n) - f(Y/Y_n)]$	a^*	$a^{'} = (1+G)a^*$
b^*	$b^* = 200[f(X/X_n) - f(Y/Y_n)]$	b^*	b^*
C^*_{ab}	$C^*_{ab} = (a^* + b^{*2})^{0.5}$	C^*_{ab}	$C^{'}_{ab} = (a^* + b^{*2})^{0.5}$
h^*_{ab}	$h^*_{ab} = \arctan \frac{b^*}{a^*}$	h^*_{ab}	$h^*_{ab} = \frac{180}{\pi} \arctan \frac{b^*}{a^*}$
ΔL^*	$\Delta L^* = \Delta L^*_2 - \Delta L^*_1$	$-\Delta L^*$	$\Delta L^{'} = \Delta L^*$
ΔC^*_{ab}		$\Delta C^*_{ab} = -(\Delta C^*_{ab,1} - \Delta C^*_{ab,2})$	$\Delta C^{'}_{ab} = -(\Delta C^{'}_{ab,1} - \Delta C^{'}_{ab,2})$
ΔH^*_{ab}		$\Delta H^*_{ab} = ((\Delta E^*_{ab})^2 - (\Delta L^*)^2 + (\Delta C^*_{ab})^2)^{0.5}$	$\Delta H^{'}_{ab} = 2(C^{'}_{ab,1} C^{'}_{ab,2})^{0.5} \sin \frac{\Delta h^{'}_{ab}}{2}$
ΔE^*_{ab}	$\Delta E^*_{ab} = ((\Delta L^*)^2 - (\Delta a^*)^2 + (\Delta b^*)^2)^{0.5}$	$\Delta E^*_{94} = \left(\left(\frac{\Delta L^*}{A}\right)^2 + \left(\frac{\Delta C^*_{ab}}{B}\right)^2 + \left(\frac{\Delta H^*_{ab}}{C}\right)^2\right)^{0.5}$	$\Delta E^{'}_{00} = \left(\left(\frac{\Delta L^{'}}{A}\right)^2 + \left(\frac{\Delta C^{'}_{ab}}{B}\right)^2 + \left(\frac{\Delta H^{'}_{ab}}{C}\right)^2 + D\right)^{0.5}$
$\overline{L^{'}}$			$\overline{L^{'}} = 0.5(L^{'}_1 + L^{'}_2)$
$\overline{C^*_{ab}}$			$\overline{C_{ab}} = 0.5(C^*_{ab,1} + C^*_{ab,2})$
$\overline{C^{'}_{ab}}$			$\overline{C^{'}_{ab}} = 0.5(C^{'}_{ab,1} + C^{'}_{ab,2})$
$\Delta\theta$			$\Delta\theta = 20e^{-(\frac{\overline{h^{'}_{ab}} - 275}{25})^2}$

Parameters	CIE 1976(L^*, a^*, b^*)	CIE 1994 (CIE94)	CIEDE2000
$\Delta h'_{ab}$			$\Delta h'_{ab} = \begin{cases} 0, & C'_{ab,1}C'_{ab,2} = 0, \\ h'_{ab,2} - h'_{ab,1}, & C'_{ab,1}C'_{ab,2} \neq 0, \lvert h'_{ab,2} - h'_{ab,1} \rvert \leq -180, \\ h'_{ab,2} - h'_{ab,1} + 360, & C'_{ab,1}C'_{ab,2} \neq 0, h'_{ab,2} - h'_{ab,1} < -180, \\ h'_{ab,2} - h'_{ab,1} - 360, & C'_{ab,1}C'_{ab,2} \neq 0, h'_{ab,2} - h'_{ab,1} > 180, \end{cases}$
A		$A = k_L S_L$	A
B		$B = k_C S_C$	B
C		$C = k_H S_H$	C
D			$D = R_T \dfrac{\Delta C'_{ab}}{B} \dfrac{\Delta H'_{ab}}{C}$
$f(t)$	$f(t) = \begin{cases} t^{\frac{1}{3}}, \ t > 0.008856 \\ 7.787t + 16/116, t \leq 0.008856 \end{cases}$	$f(t)$	$f(t)$
G			$G = 0.5 \left(1 - \left(\dfrac{(\overline{C^*_{ab}})^7}{(\overline{C^*_{ab}})^7 + 25^7} \right)^{0.5} \right)$

Parameters	CIE 1976(L^*, a^*, b^*)	CIE 1994 (CIE94)	CIEDE2000
$\overline{\Delta h'_{ab}}$			$\overline{\Delta h'_{ab}} = \begin{cases} 0.5(h'_{ab,1} + h'_{ab,2}), \\ \quad C'_{ab,1}C'_{ab,2} \neq 0, \\ \quad \lvert h'_{ab,1} - h'_{ab,2} \rvert \leq 180, \\ 0.5(h'_{ab,1} - h'_{ab,2} + 360), \\ \quad C'_{ab,1}C'_{ab,2} \neq 0, \\ \quad \lvert h'_{ab,1} - h'_{ab,2} \rvert > 180, h'_{ab,1} - h'_{ab,2} < 360, \\ 0.5(h'_{ab,1} - h'_{ab,2} - 360), \\ \quad C'_{ab,1}C'_{ab,2} \neq 0, \\ \quad \lvert h'_{ab,1} - h'_{ab,2} \rvert > 180, h'_{ab,1} - h'_{ab,2} \leq 360 \\ h'_{ab,1} + h'_{ab,2}, \\ \quad C'_{ab,1}C'_{ab,2} = 0 \end{cases}$
R_C			$R_C = 2\left(1 - \left(\frac{(\overline{C^*_{ab}})^7}{(\overline{C^*_{ab}})^7 + 25^7}\right)^{0.5}\right)$
R_T			$R_T = -2R_C \sin(2\Delta\theta)$
S_C		$S_C = 1 + K_1 C^*_{ab}$	$S_C 1 + 0.045\overline{C'_{ab}}$
S_H		$S_H = 1 + K_2 C^*_{ab}$	$S_H 1 + 0.015\overline{C'_{ab}}T$
S_L		$S_L = 1$	$S_L = 1 + \frac{0.015(\overline{L'} - 50)^2}{(20 + (\overline{L'} - 50)^2)^{0.5}}$
T			$T = 1 - 0.17\cos(\overline{h'_{ab}} - 30) + 0.24\cos(2\overline{h'_{ab}}) + 0.32\cos(3\overline{h'_{ab}} + 6) - 0.20\cos(4\overline{h'_{ab}} - 63)$

Parameters	CIE 1976(L^*, a^*, b^*)	CIE 1994 (CIE94)	CIEDE2000
$\left.\begin{matrix} X_n \\ Y_n \\ Z_n \end{matrix}\right\}$	the tristimulus of the reference neutral white point		