The Parameterization of all Disturbance Observers for Time-Delay Plants with any Input and Output Disturbances

Jie Hu[1], **Kou Yamada**[2], and **Tatsuya Sakanushi**[3], Non-members

ABSTRACT

In this paper, we examine the parameterization of all disturbance observers for time-delay plants with any input and output disturbances. The disturbance observers have been used to estimate the disturbance in the plant. Several papers on design methods of disturbance observers have been published. Recently, the parameterization of all disturbance observers for plants with any input and output disturbances was clarified. However, no paper examines the parameterization of all disturbance observers for time-delay plants with any input and output disturbances. In this paper, we propose parameterizations of all disturbance observers and all linear functional disturbance observers for time-delay plants with any input and output disturbances.

Keywords: Disturbance, Disturbance Observer, Parameterization, Time-delay

1. INTRODUCTION

In this paper, we examine the parameterization of all disturbance observers for time-delay plants with any input and output disturbances. A disturbance observer is used in motion control to cancel the disturbance or to make the closed-loop system robustly stable [1–8]. Generally, a disturbance observer consists of a disturbance signal generator and an observer. The disturbance, which is usually assumed to be a step disturbance, is estimated using the observer. Because disturbance observers have simple structures and are easy to understand, they have been used in many applications [1–6, 8].

However, Mita et al. pointed out that the disturbance observer is nothing more than an alternative design of an integral controller [7]. That is, a control system with a disturbance observer does not guarantee robust stability. In addition, in [7], an extended H_∞ control was proposed as a robust motion control method that cancels disturbances. This implies that, using the method in [7], a control system with a disturbance observer can be designed to guarantee robust stability. From another viewpoint, Kobayashi et al. considered the robust stability of a control system with a disturbance observer and analyzed the parameter variations of the disturbance observer [8]. In this way, robustness analysis of control systems with a disturbance observer has been considered.

Another important control problem is the parameterization problem, that is, the problem of finding all stabilizing controllers for a plant [9–14]. If the parameterization of all disturbance observers for any disturbance could be obtained, we could express results from previous studies of disturbance observers in a uniform manner. In addition, disturbance observers for any disturbance could be designed systematically. From this viewpoint, Yamada et al. examine parameterizations of all disturbance observers and all linear functional disturbance observers for plants with any output disturbance [15] and any input disturbance [16]. Ando et al. examine parameterizations of all disturbance observers and all linear functional disturbance observers for plants with any input and output disturbances [17]. However, no paper examines the parameterization of all disturbance observers for time-delay plants with any input and output disturbances. It has been unsolved until now whether or not, any input and output disturbances can be estimated for time-delay plants.

In this paper, we propose the parameterization of all disturbance observers for time-delay plants with any input and output disturbances and that of all linear functional disturbance observers for time-delay plants with any input and output disturbances. First, the structure and necessary characteristics of disturbance observer for time-delay plants with any input and output disturbances are defined. Next, the parameterization of all disturbance observers for time-delay plants with any input and output disturbances and that of all linear functional disturbance observers for time-delay plants with any input and output disturbances are clarified.

Notation

R	the set of real numbers.
$R(s)$	the set of real rational functions with s.
RH_∞	the set of stable proper real rational functions.

Manuscript received on July 31, 2013 ; revised on November 20, 2013.

Final manuscript received November 30, 2013.

[1,2,3] The authors are with Graduate School of Science and Technology, Gunma University 1-5-1 Tenjincho, Kiryu 376-8515 Japan, E-mail: t13802282@gunma-u.ac.jp, yamada@gunma-u.ac.jp and sakanushi@gunma-u.ac.jp

$\mathcal{U}$ the unimodular procession in RH_∞. That is, $U(s) \in \mathcal{U}$ means that $U(s) \in RH_\infty$ and $U^{-1}(s) \in RH_\infty$.

A^T transpose of A.

$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$ represents the state space description $C(sI - A)^{-1}B + D$.

$\mathcal{L}\{\cdot\}$ the Laplace transformation of $\{\cdot\}$.

$\mathcal{L}^{-1}\{\cdot\}$ the inverse Laplace transformation of $\{\cdot\}$.

2. DISTURBANCE OBSERVER AND PROBLEM FORMULATION

Consider the time-delay plant written by

$$\left\{\begin{array}{rcl} \dot{x}(t) & = & Ax(t) + B\left(u(t-L) + d_1(t-L)\right) \\ y(t) & = & Cx(t) + D\left(u(t-L) + d_1(t-L)\right) \\ & & +d_2(t) \end{array}\right., \quad (1)$$

where $x \in R^n$ is the state variable, $u \in R^p$ is the control input, $y \in R^m$ is the output, $d_1 \in R^p$ is the input disturbance, $d_2 \in R^m$ is the output disturbance, $A \in R^{n\times n}$, $B \in R^{n\times p}$, $C \in R^{m\times n}$ and $L > 0$ is the time-delay. It is assumed that (A, B) is stabilizable, (C, A) is detectable, the input $u(t - L)$ and the output $y(t)$ are available, but the disturbances $d_1(t)$ and $d_2(t)$ are unavailable. The transfer function of the output $y(s)$ in (1) is denoted by

$$y(s) = G(s)e^{-sL}u(s) + G(s)e^{-sL}d_1(s) + d_2(s), \quad (2)$$

where

$$G(s) = C\left(sI - A\right)^{-1}B + D \quad \in R^{m\times p}(s). \quad (3)$$

When the disturbances $d_1(t)$ and $d_2(t)$ are not available, in many cases, the disturbance estimator named the disturbance observer is used. The disturbance observer estimates the disturbances $d_1(t)$ and $d_2(t)$ in (1) by using available measurements the input $u(t - L)$ and the output $y(t)$. Since the available measurements of the plant in (1) are the input $u(t-L)$ and the output $y(t)$, the general form of the disturbance observer $\tilde{d}(s)$ for time-delay plants in (1) is written by

$$\tilde{d}(s) = F_1(s)y(s) + F_2(s)e^{-sL}u(s), \quad (4)$$

where $F_1(s) \in R^{m\times m}(s)$, $F_2(s) \in R^{m\times p}(s)$, $\tilde{d}(s) = \mathcal{L}\{\tilde{d}(t)\}$ and $\tilde{d}(t) \in R^m(t)$. That is, the general form of the disturbance observer $\tilde{d}(s)$ is shown in Fig. 1. In the following, we call the system $\tilde{d}(s)$ in (4) a disturbance observer for time-delay plants with any input and output disturbances, if

$$\lim_{t\to\infty}\left(\mathcal{L}^{-1}\left(G(s)e^{-sL}d_1(s)\right) + d_2(t) - \tilde{d}(t)\right) = 0 \quad (5)$$

is satisfied for any $x(0)$, $u(t)$ and $d(t)$.

The problem considered in this paper is to obtain the parameterization of all disturbance observers $\tilde{d}(s)$ in (4) for time-delay plants with any input and output disturbances.

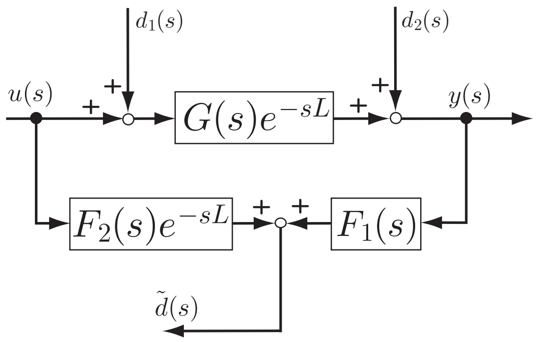

Fig.1: *Structure of a disturbance observer and that of a linear functional disturbance observer*

3. PARAMETERIZATION OF ALL DISTURBANCE OBSERVERS FOR TIME-DELAY PLANTS WITH ANY INPUT AND OUTPUT DISTURBANCES

In this section, we clarify the parameterization of all disturbance observers in (4) for time-delay plants with any input and output disturbances.

The parameterization of all disturbance observers $\tilde{d}(s)$ in (4) for time-delay plants with any input and output disturbances is summarized in the following theorem.

Theorem 1: The system $\tilde{d}(s)$ in (4) is a disturbance observer for time-delay plants with any input and output disturbances if and only if $F_1(s)$ and $F_2(s)$ are defined by

$$F_1(s) \quad = \quad I \in RH_\infty^{m\times m} \quad (6)$$

and

$$F_2(s) \quad = \quad -G(s) \in RH_\infty^{m\times p}, \quad (7)$$

respectively.

Proof: First, necessity is shown. That is, we show that if the system $\tilde{d}(s)$ in (4) satisfies (5), then $F_1(s)$ and $F_2(s)$ in (4) are written by (6) and (7), respectively. The control input $u(s)$ is written by

$$u(s)e^{-sL} = D(s)\xi(s), \quad (8)$$

where $\xi(s)$ is the pseudo state variable, $D(s) \in RH_\infty^{p\times p}$ and $N(s) \in RH_\infty^{m\times p}$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = N(s)D^{-1}(s). \quad (9)$$

From (8) and (9), (2) is rewritten by

$$y(s) \quad = \quad N(s)\xi(s)+G(s)e^{-sL}d_1(s)+d_2(s). \quad (10)$$

Substituting (8) and (10) into (4), we have

$$\tilde{d}(s) = (F_1(s)N(s)+F_2(s)D(s))\,\xi(s) + F_1(s)G(s)e^{-sL}d_1(s)+F_1(s)d_2(s). \quad (11)$$

From (11),

$$\begin{aligned} & G(s)e^{-sL}d_1(s) + d_2(s) - \tilde{d}(s) \\ = \; & (I - F_1(s))\,G(s)e^{-sL}d_1(s) + (I - F_1(s))\,d_2(s) \\ & - (F_1(s)N(s) + F_2(s)D(s))\,\xi(s), \end{aligned} \quad (12)$$

is satisfied. From the assumption that (5) is satisfied, $F_1(s) \in RH_\infty^{m\times m}$ and $F_2(s) \in RH_\infty^{m\times p}$ are written by (6) and (7). In this way, necessity has been proved.

Next, sufficiency is shown. That is, we show that if $F_1(s)$ and $F_2(s)$ are written by (6) and (7), then the system $\tilde{d}(s)$ in (4) satisfies (5). Substituting (6) and (7) for (4), $\tilde{d}(s)$ is written by

$$\tilde{d}(s) = y(s) - G(s)e^{-sL}u(s). \quad (13)$$

From (13) and (2), $d(s) - \tilde{d}(s)$ satisfies

$$\begin{aligned} & G(s)e^{-sL}d_1(s) + d_2(s) - \tilde{d}(s) \\ = \; & y(s) - G(s)e^{-sL}u(s) - \big(y(s) - G(s)e^{-sL}u(s)\big) \\ = \; & 0. \end{aligned} \quad (14)$$

From (14), we have

$$\lim_{t\to\infty}\Big(\mathcal{L}^{-1}\left(G(s)e^{-sL}d_1(s)\right) + d_2(t) - \tilde{d}(t)\Big) = 0. \quad (15)$$

In this way, sufficiency has been proved.

We have thus proved Theorem 1. ■

Note that from Theorem 1, if the plant $G(s)e^{-sL}$ is unstable, there exists no disturbance observer for time-delay plants with any input and output disturbances satisfying (5). Most plants in the motion-control field are unstable, so this is a problem for the disturbance observer for time-delay plants with any input and output disturbances to be solved. When a disturbance observer for time-delay plants with any input and output disturbances is used to attenuate disturbances such as in [1–6], even if the system $\tilde{d}(s)$ in (4) satisfying (5) cannot be designed, the control system can be designed to attenuate disturbances effectively. That is, to attenuate disturbances, it is enough to estimate $(I-F(s))(G(s)e^{-sL}d_1(s)+d_2(s))$, where $F(s) \in RH_\infty^{m\times m}$ and $(I-F(s))G(s) \in RH_\infty^{m\times p}$. From this point of view, in the next section, when $G(s)e^{-sL}$ is unstable, we define a linear functional disturbance observer for time-delay plants with any input and output disturbances and clarify the parameterization of all linear functional disturbance observers for time-delay plants with any input and output disturbances.

4. PARAMETERIZATION OF ALL LINEAR FUNCTIONAL DISTURBANCE OBSERVERS FOR TIME-DELAY PLANTS WITH ANY INPUT AND OUTPUT DISTURBANCES

In this section, we define a linear functional disturbance observer for time-delay plants with any input and output disturbances and clarify the parameterization of all linear functional disturbance observers for time-delay plants with any input and output disturbances.

For any $d_1(s)$, $d_2(s)$, initial state $x(0)$ and $u(t-L)$, we call $\tilde{d}(s)$ the linear functional disturbance observer for time-delay plants with any input and output disturbances if

$$\begin{aligned} & G(s)e^{-sL}d_1(s) + d_2(s) - \tilde{d}(s) \\ = \; & F(s)\left(G(s)e^{-sL}d_1(s) + d_2(s)\right) \end{aligned} \quad (16)$$

is satisfied, where $F(s) \in RH_\infty^{m\times m}$ and $(I - F(s))G(s) \in RH_\infty^{m\times p}$. Because the available measurements of the plant in (1) are the control input $u(t-L)$ and the output $y(t)$, the general form of the linear functional disturbance observer for time-delay plants with any disturbance is written as (4), where $F_1(s) \in R^{m\times m}(s)$ and $F_2(s) \in R^{m\times p}(s)$. That is, the general form of the linear functional disturbance observer $\tilde{d}(s)$ is shown in Fig. 1.

Next, we clarify the parameterization of all linear functional disturbance observers for time-delay plants with any input and output disturbances, which is summarized in the following theorem.

Theorem 2: The system $\tilde{d}(s)$ in (4) is a linear functional disturbance observer for time-delay plants with any input and output disturbances if and only if $F_1(s)$, $F_2(s)$ and $F(s)$ are described by

$$F_1(s) = \tilde{D}(s) + Q(s)\tilde{D}(s), \quad (17)$$

$$F_2(s) = -\tilde{N}(s) - Q(s)\tilde{N}(s) \quad (18)$$

and

$$F(s) = I - F_1(s), \quad (19)$$

respectively, where $N(s) \in RH_\infty^{m\times p}$, $D(s) \in RH_\infty^{p\times p}$, $\tilde{N}(s) \in RH_\infty^{m\times p}$ and $\tilde{D}(s) \in RH_\infty^{m\times m}$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = \tilde{D}^{-1}(s)\tilde{N}(s) = N(s)D^{-1}(s) \quad (20)$$

and

$$\tilde{D}(s)N(s) - \tilde{N}(s)D(s) = 0. \quad (21)$$

$Q(s) \in RH_\infty^{m\times m}$ is any function.

Proof of this theorem requires following lemma.

Lemma 1: Suppose that $A(s) \in RH_\infty^{n\times m}$, $B(s) \in RH_\infty^{q\times m}$, $C(s) \in RH_\infty^{p\times m}$, rank $\begin{bmatrix} A^T(s) & B^T(s) \end{bmatrix}^T = r$. The equation written as

$$X(s)A(s) + Y(s)B(s) = C(s) \tag{22}$$

has a solution $X(s)$ and $Y(s)$ if and only if there exists $U(s) \in \mathcal{U}$ satisfying

$$\begin{bmatrix} A(s) \\ B(s) \\ C(s) \end{bmatrix} = U(s) \begin{bmatrix} A(s) \\ B(s) \\ 0 \end{bmatrix}. \tag{23}$$

When a pair of $X_0(s)$ and $Y_0(s)$ is a solution to (22), all solutions are given by

$$\begin{aligned} &\begin{bmatrix} X(s) & Y(s) \end{bmatrix} \\ &= \begin{bmatrix} X_0(s) & Y_0(s) \end{bmatrix} + Q(s) \begin{bmatrix} W_1(s) & W_2(s) \end{bmatrix}, \end{aligned} \tag{24}$$

where $W_1(s) \in RH_\infty^{p\times n}$ and $W_2(s) \in RH_\infty^{p\times q}$ are functions satisfying

$$W_1(s)A(s) + W_2(s)B(s) = 0 \tag{25}$$

and

$$\text{rank} \begin{bmatrix} W_1(s) & W_2(s) \end{bmatrix} = n + q - r \tag{26}$$

and $Q(s) \in RH_\infty^{p\times(n+q-r)}$ is any function [12].

Using above-mentioned Lemma 1, we shall show the proof of Theorem 2.

Proof: First, necessity is shown. That is, we show that if the system $\tilde{d}(s)$ in (4) satisfies (16), then $F_1(s)$, $F_2(s)$ in (4) and $F(s)$ are written by (17), (18) and (19), respectively.

From (20), the control input $u(s)e^{-sL}$ is written as

$$u(s)e^{-sL} = D(s)\xi(s), \tag{27}$$

where $\xi(s)$ is the pseudo state variable. Using the pseudo state variable $\xi(s)$, (4) is rewritten as

$$\begin{aligned} \tilde{d}(s) &= (F_1(s)N(s) + F_2(s)D(s))\,\xi(s) \\ &\quad + F_1(s)G(s)e^{-sL}d_1(s) + F_1(s)d_2(s). \end{aligned} \tag{28}$$

Then, $G(s)e^{-sL}d_1(s) + d_2(s) - \tilde{d}(s)$ is written as

$$\begin{aligned} &G(s)e^{-sL}d_1(s) + d_2(s) - \tilde{d}(s) \\ &= (I - F_1(s))\,G(s)e^{-sL}d_1(s) + (I - F_1(s))\,d_2(s) \\ &\quad - (F_1(s)N(s) + F_2(s)D(s))\,\xi(s). \end{aligned} \tag{29}$$

From the assumption that (16) is satisfied,

$$I - F_1(s) = F(s) \tag{30}$$

and

$$F_1(s)N(s) + F_2(s)D(s) = 0 \tag{31}$$

hold true. Equation (30) is corresponding to (19).

Since $(I-F(s))G(s) \in RH_\infty^{m\times p}$, from (20), $I-F(s)$ is necessary to have the form

$$I - F(s) = \tilde{Q}(s)\tilde{D}(s), \tag{32}$$

where $\tilde{Q}(s) \in RH_\infty^{m\times m}$. From (32) and (30), $F_1(s)$ is necessary to have the form

$$F_1(s) = \tilde{Q}(s)\tilde{D}(s). \tag{33}$$

Substituting (33) into (31), we have

$$\tilde{Q}(s)\tilde{D}(s)N(s) + F_2(s)D(s) = 0. \tag{34}$$

Next, using Lemma 1, we obtain all solutions of $\tilde{Q}(s)$ and $F_2(s)$ satisfying (34). From (21), a pair of solution to (34) is given by

$$\tilde{Q}(s) = I \tag{35}$$

and

$$F_2(s) = -\tilde{N}(s). \tag{36}$$

Since $N(s)$ and $D(s)$ are right coprime,

$$\text{rank} \begin{bmatrix} N(s) \\ D(s) \end{bmatrix} = p \tag{37}$$

holds true. Therefore, we have

$$\begin{aligned} &\text{rank} \begin{bmatrix} \tilde{D}(s)N(s) \\ D(s) \end{bmatrix} \\ &= \text{rank} \begin{bmatrix} \tilde{D}(s) & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} N(s) \\ D(s) \end{bmatrix} \\ &= \text{rank} \begin{bmatrix} N(s) \\ D(s) \end{bmatrix} \\ &= p. \end{aligned} \tag{38}$$

In addition, from (38) and (21), a pair of $W_1(s)$ and $W_2(s)$ satisfying

$$W_1(s)\tilde{D}(s)N(s) + W_2(s)D(s) = 0 \tag{39}$$

and

$$\begin{aligned} &\text{rank} \begin{bmatrix} W_1(s) & W_2(s) \end{bmatrix} \\ &= m + p - \text{rank} \begin{bmatrix} \tilde{D}(s)N(s) \\ D(s) \end{bmatrix} \\ &= m \end{aligned} \tag{40}$$

is

$$W_1(s) = I \tag{41}$$

and

$$W_2(s) = -\tilde{N}(s). \tag{42}$$

From Lemma 1, all solutions $F_1(s)$ and $F_2(s)$ to (34) are written by

$$\tilde{Q}(s) \quad = \quad I + Q(s) \tag{43}$$

and

$$F_2(s) \quad = \quad -\tilde{N}(s) - Q(s)\tilde{N}(s), \tag{44}$$

respectively, where $Q(s) \in RH_\infty^{m\times m}$ is any function. Substituting (43) for (33), we have (17). In this way, necessity has been proved.

Next, sufficiency is shown. That is, we show that if $F_1(s)$, $F_2(s)$ and $F(s)$ are written by (17), (18) and (19), respectively, then (4) satisfies (16). Substituting (17) and (18) for (4), we have

$$\begin{aligned}
&\tilde{d}(s)\\
&= \left(\tilde{D}(s) + Q(s)\tilde{D}(s)\right)\left(G(s)e^{-sL}d_1(s) + d_2(s)\right)\\
&= F_1(s)\left(G(s)e^{-sL}d_1(s) + d_2(s)\right). \qquad (45)
\end{aligned}$$

From (45), $G(s)e^{-sL}d_1(s)+d_2(s)-\tilde{d}(s)$ is written by

$$\begin{aligned}
&G(s)e^{-sL}d_1(s) + d_2(s) - \tilde{d}(s)\\
&\quad = \left(I - F_1(s)\right)\left(G(s)e^{-sL}d_1(s) + d_2(s)\right)\\
&\quad = F(s)\left(G(s)e^{-sL}d_1(s) + d_2(s)\right). \qquad (46)
\end{aligned}$$

In this way, sufficiency has been proved.

We have thus proved Theorem 2. ■

5. NUMERICAL EXAMPLE

In this section, numerical examples are shown to illustrate the effectiveness of the proposed parameterizations.

5.1 Numerical example for disturbance observer

Consider the problem to obtain the parameterization of all disturbance observers for stable time-delay plant $G(s)e^{-sL}$ written by

$$\begin{aligned}
&G(s)e^{-sL}\\
&= \begin{bmatrix} \dfrac{2}{(s+1)(s+2)} & \dfrac{s-3}{(s+1)(s+2)} \\ \dfrac{s-6}{(s+1)(s+2)} & \dfrac{-1}{(s+1)(s+2)} \end{bmatrix} e^{-s}\\
&= \left[\begin{array}{cccc|cc} -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 & 0 & 1 \\ \hline 2 & -4 & -2 & 5 & 0 & 0 \\ -7 & -1 & 8 & 1 & 0 & 0 \end{array}\right] e^{-s}, \qquad (47)
\end{aligned}$$

where $L = 1$[sec]. From Theorem 1, the parameterization of all disturbance observers for time-delay plants $G(s)e^{-sL}$ in (47) with any input and output disturbances is given by (4) with (6) and (7).

When the control input $u(t)$, the input disturbance $d_1(t)$, the output disturbance $d_2(t)$ and the initial state $x(0)$ are given by

$$u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \tag{48}$$

$$d_1(t) = \begin{bmatrix} \sin 2\pi t \\ 2\sin 2\pi t \end{bmatrix}, \tag{49}$$

$$d_2(t) = \begin{bmatrix} \sin \pi t \\ 2\sin \pi t \end{bmatrix} \tag{50}$$

and

$$x(0) = \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 \end{bmatrix}^T, \tag{51}$$

respectively, the response of the error

$$\begin{aligned}
e(t) \quad &= \quad \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix}\\
&= \quad \mathcal{L}^{-1}\{(G(s)e^{-sL}d_1(s) + d_2(s))\} - \tilde{d}(t) \qquad (52)
\end{aligned}$$

is shown in Fig. 2. Here, the solid line shows the response of $e_1(t)$ and the dotted line shows that of $e_2(t)$.

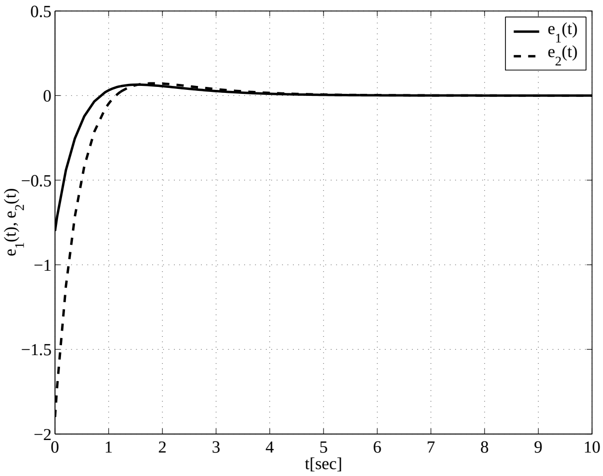

Fig.2: *The response of the error* $e(t) = \mathcal{L}^{-1}\{(G(s)e^{-sL}d_1(s) + d_2(s))\} - \tilde{d}(t)$

Figure 2 shows that the disturbance observer $\tilde{d}(s)$ in (4) for time-delay plants with any input and output disturbances can estimate $\mathcal{L}^{-1}\{(G(s)e^{-sL}d_1(s)+d_2(s))\}$ effectively.

In this way, it is shown that using the obtained parameterization of all disturbance observers for time-delay plants with any input and output disturbances, we can easily design the disturbance observer for time-delay plants with any input and output disturbances.

5.2 Numerical example for linear functional disturbance observer

Consider the problem of obtaining the parameterization of all linear functional disturbance observers for any input and output disturbances for unstable time-delay plant $G(s)e^{-sL}$ written by

$$
\begin{aligned}
&G(s)e^{-sL}\\
&= \begin{bmatrix} \dfrac{2}{(s-1)(s-2)} & \dfrac{s-3}{(s-1)(s-2)} \\ \dfrac{s-6}{(s-1)(s-2)} & \dfrac{-1}{(s-1)(s-2)} \end{bmatrix} e^{-s}\\
&= \left[\begin{array}{cccc|cc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 & 1 \\ \hline -2 & 2 & 2 & -1 & 0 & 0 \\ 5 & 1 & -4 & -1 & 0 & 0 \end{array}\right] e^{-s}, \qquad (53)
\end{aligned}
$$

where $L = 1$[sec].

Using the method in [18], state space descriptions of $\tilde{N}(s)$ and $\tilde{D}(s)$ satisfying (20) and (21) are given by

$$
\tilde{N}(s) = \left[\begin{array}{cccc|cc} 12.42 & 3.623 & -8.916 & -3.065 & 1 & 0 \\ 3.378 & 12.58 & -0.8861 & -7.035 & 0 & 1 \\ 21.92 & 2.180 & -15.91 & -3.099 & 1 & 0 \\ 13.70 & 34.20 & -5.713 & -19.09 & 0 & 1 \\ \hline -2 & 2 & 2 & -1 & 0 & 0 \\ 5 & 1 & -4 & -1 & 0 & 0 \end{array}\right] \quad (54)
$$

and

$$
\tilde{D}(s) = \left[\begin{array}{cccc|cc} 12.42 & 3.623 & -8.916 & -3.065 & -0.5575 & -2.508 \\ 3.379 & 12.58 & -0.8861 & -7.035 & -4.542 & -2.493 \\ 21.92 & 2.180 & -15.91 & -3.099 & 0.9186 & -4.017 \\ 13.70 & 34.20 & -5.713 & -19.09 & -13.11 & -7.983 \\ \hline 2 & -2 & -2 & 1 & 1 & 0 \\ -5 & -1 & 4 & 1 & 0 & 1 \end{array}\right], \quad (55)
$$

respectively. From Theorem 2, the parameterization of all linear functional disturbance observers for time-delay plants $G(s)e^{-sL}$ in (53) with any input and output disturbances is given by (4) with (17), (18) and (19).

$Q(s)$ in (17) and (18) is chosen as

$$
Q(s) = \begin{bmatrix} \dfrac{10}{s+10} & 0 \\ 0 & \dfrac{20}{s+20} \end{bmatrix} = \left[\begin{array}{cc|cc} -10 & 0 & 1 & 0 \\ 0 & -20 & 0 & 1 \\ \hline 10 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 \end{array}\right]. \quad (56)
$$

Substituting above-mentioned parameters for (17) and (18), a linear functional disturbance observer $\tilde{d}(s)$ for time-delay plants $G(s)e^{-sL}$ in (53) is designed as (4), where

$$
F_1(s) = \left[\begin{array}{cccccc|cc} -10 & 0 & 2 & -2 & -2 & 1 & 1 & 0 \\ 0 & -20 & -5 & -1 & 4 & 1 & 0 & 1 \\ 0 & 0 & 12.42 & 3.623 & -8.916 & -3.065 & -0.5575 & -2.508 \\ 0 & 0 & 3.379 & 12.58 & -0.8861 & -7.035 & -4.542 & -2.493 \\ 0 & 0 & 21.92 & 2.180 & -15.91 & -3.099 & 0.9186 & -4.017 \\ 0 & 0 & 13.70 & 34.20 & -5.713 & -19.09 & -13.11 & -7.983 \\ \hline 10 & 0 & 2 & -2 & -2 & 1 & 1 & 0 \\ 0 & 20 & -5 & -1 & 4 & 1 & 0 & 1 \end{array}\right], \quad (57)
$$

$$
F_2(s) = \left[\begin{array}{cccccc|cc} -10 & 0 & -2 & 2 & 2 & -1 & 0 & 0 \\ 0 & -20 & 5 & 1 & -4 & -1 & 0 & 0 \\ 0 & 0 & 12.42 & 3.623 & -8.916 & -3.065 & 1 & 0 \\ 0 & 0 & 3.378 & 12.58 & -0.8861 & -7.035 & 0 & 1 \\ 0 & 0 & 21.92 & 2.180 & -15.91 & -3.099 & 1 & 0 \\ 0 & 0 & 13.70 & 34.20 & -5.713 & -19.09 & 0 & 1 \\ \hline -10 & 0 & 2 & -2 & -2 & 1 & 0 & 0 \\ 0 & -20 & -5 & -1 & 4 & 1 & 0 & 0 \end{array}\right] \quad (58)
$$

and

$$
F(s) = \left[\begin{array}{cccccc|cc} -10 & 0 & 2 & -2 & -2 & 1 & 1 & 0 \\ 0 & -20 & -5 & -1 & 4 & 1 & 0 & 1 \\ 0 & 0 & 12.42 & 3.623 & -8.916 & -3.065 & -0.5575 & -2.508 \\ 0 & 0 & 3.379 & 12.58 & -0.8861 & -7.035 & -4.542 & -2.493 \\ 0 & 0 & 21.92 & 2.180 & -15.91 & -3.099 & 0.9186 & -4.017 \\ 0 & 0 & 13.70 & 34.20 & -5.713 & -19.09 & -13.11 & -7.983 \\ \hline -10 & 0 & -2 & 2 & 2 & -1 & 0 & 0 \\ 0 & -20 & 5 & 1 & -4 & -1 & 0 & 0 \end{array}\right]. \quad (59)
$$

Note that the designed disturbance observer $\tilde{d}(s)$ is the system to estimate $\mathcal{L}^{-1}\{(I-F(s))(G(s)e^{-sL}d_1(s)+d_2(s))\}$.

When the control input $u(t)$, the input disturbance $d_1(t)$, the output disturbance $d_2(t)$ and the initial state $x(0)$ are given by

$$
u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad (60)
$$

$$
d_1(t) = \begin{bmatrix} \sin 2\pi t \\ 2\sin 2\pi t \end{bmatrix}, \quad (61)
$$

$$d_2(t) = \begin{bmatrix} \sin \pi t \\ 2\sin \pi t \end{bmatrix} \quad (62)$$

and

$$x(0) = \begin{bmatrix} 0.1 & 0.5 & -0.5 & -0.1 \end{bmatrix}^T, \quad (63)$$

respectively, the response of the error

$$\begin{aligned} e(t) &= \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} \\ &= \mathcal{L}^{-1}\{(I - F(s))(G(s)e^{-sL}d_1(s) + d_2(s))\} \\ &\quad -\tilde{d}(t) \end{aligned} \quad (64)$$

is shown in Fig. 3. Here, the solid line shows the response of $e_1(t)$ and the dotted line shows that of $e_2(t)$. Figure 3 shows that the linear functional disturbance observer $\tilde{d}(s)$ in (4) for time-delay plants with any input and output disturbances can estimate $\mathcal{L}^{-1}\{(I - F(s))(G(s)e^{-sL}d_1(s) + d_2(s))\}$ effectively.

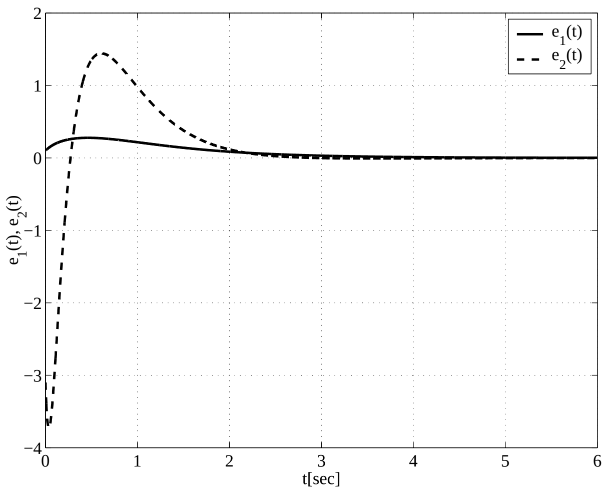

Fig.3: *The response of the error* $e(t) = \mathcal{L}^{-1}\{(I - F(s))(G(s)e^{-sL}d_1(s) + d_2(s))\} - \tilde{d}(t)$

In this way, it is shown that using the obtained parameterization of all linear functional disturbance observers for time-delay plants with any input and output disturbances, we can easily design a linear functional disturbance observer for time-delay plants with any input and output disturbances.

6. CONCLUSIONS

In this paper, we proposed parameterizations of all disturbance observers for time-delay plants with any input and output disturbances and all linear functional disturbance observers for time-delay plants with any input and output disturbances. The results in this paper are summarized as follows:

1. We clarified that for stable time-delay plants, exact disturbance observer for time-delay plants with any input and output disturbances, which can estimate disturbance exactly, can be designed.
2. The parameterization of all disturbance observers for time-delay plants with any input and output disturbances was proposed.
3. The linear functional disturbance observer for time-delay plants with any input and output disturbances was defined.
4. The parameterization of all linear functional disturbance observers for time-delay plants with any input and output disturbances was proposed.
5. Numerical examples were shown to illustrate the effectiveness of the proposed parameterizations of all disturbance observers for time-delay plants with any input and output disturbances and all linear functional disturbance observers for time-delay plants with any input and output disturbances.

A design method for control system using obtained parameterizations of all disturbance observers for time-delay plants with any input and output disturbances and all linear functional disturbance observers for time-delay plants with any input and output disturbances will be described in another article. Applications of the parameterizations in this paper for real plants will also be presented in another article.

References

[1] K. Ohishi, K. Ohnishi and K. Miyachi, "Torque-speed regulation of DC motor based on load torque estimation," *Proc. IEEJ IPEC-TOKYO*, Vol.2, pp.1209-1216, 1983.

[2] S. Komada and K. Ohnishi, "Force feedback control of robot manipulator by the acceleration tracing orientation method," *IEEE Transactions on Industrial Electronics*, Vol.37, No.1, pp.6-12, 1990.

[3] T. Umeno and Y. Hori, "Robust speed control of DC servomotors using modern two degrees-of-freedom controller design", *IEEE Transactions on Industrial Electronics*, Vol.38, No.5, pp.363-368, 1991.

[4] M. Tomizuka, "On the design of digital tracking controllers," *Transactions of the ASME Journal of Dynamic Systems, Measurement, and Control*, Vol.115, pp.412-418, 1993.

[5] K. Ohnishi, M. Shibata and T. Murakami, "Motion control for advanced mechatronics", *IEEE/ASME Transaction on Mechatronics*, Vol.1, No.1, pp.56-67, 1996.

[6] H. S. Lee and M. Tomizuka, "Robust motion controller design for high-accuracy positioning systems," *IEEE Transactions on Industrial Electronics*, Vol.43, No.1, pp.48-55, 1996.

[7] T. Mita, M. Hirata, K. Murata and H. Zhang, "H_∞ control versus disturbance-observer-based control," *IEEE Transactions on Industrial Electronics*, Vol.45, No.3, pp.488-495, 1998.

[8] H. Kobayashi, S. Katsura and K. Ohnishi, "An analysis of parameter variations of disturbance observer for motion control," *IEEE Transactions*

on Industrial Electronics, Vol.54, No.6, pp.3413-3421, 2007.

[9] G. Zames, "Feedback and optimal sensitivity: model reference transformations, multiplicative seminorms and approximate inverse," *IEEE Transactions on Automatic Control*, Vol.26, pp.301-320, 1981.

[10] D. C. Youla, J. J. Bongiorno and H. Jabr, "Modern Wiener–Hopf design of optimal controllers. Part I: The single-input-output case," *IEEE Trans. Automatic Control*, Vol.AC-21, No.3, pp.3-13, 1976.

[11] C. A. Desoer, R. W. Liu, J. Murray and R. Saeks, "Feedback system design: The fractional representation approach to analysis and synthesis," *IEEE Trans. Automatic Control*, Vol.AC-25, No.3, pp.399-412, 1980.

[12] M. Vidyasagar, "Control System Synthesis - A factorization approach -", *MIT Press*, 1985.

[13] M. Morari and E. Zafiriou, "Robust Process Control," *Prentice-Hall*, 1989.

[14] J. J. Glaria and G. C. Goodwin, "A parameterization for the class of all stabilizing controllers for linear minimum phase systems," *IEEE Transactions on Automatic Control*, Vol.39, pp.433-434, 1994.

[15] K. Yamada, I. Murakami, Y. Ando, T. Hagiwara, Y. Imai and M. Kobayashi, "The parametrization of all disturbance observers," *ICIC Express Letters*, Vol.2, pp.421-426, 2008.

[16] K. Yamada, I. Murakami, Y. Ando, T. Hagiwara, Y. Imai, D. Z. Gong and M. Kobayashi, "The parametrization of all disturbance observers for plants with input disturbance," *The 4th IEEE Conference on Industrial Electronics and Applications*, pp.41-46, 2009.

[17] Y. Ando, K. Yamada, T. Sakanushi, T. Hagiwara, N. T. Mai, I. Murakami, Y. Nakui and H. Z. Lin, "The parameterization of all disturbance observers for plants with any input and output disturbances," *ICIC Express Letters*, Vol.5, No.4(A), pp.953-958, 2011.

[18] C. N. Nett, C. A. Jacobson and M. J. Balas, "A connection between state-space and doubly coprime fractional representation," *IEEE Transactions on Automatic Control*, Vol.29, pp.831-832, 1984.

Jie Hu was born in Beijing, China, in 1986. She received a B.S. degree in Civil Engineering from North China University of Technology, Beijing, China, in 2008, and M.S. degree in Mechanical System Engineering from Gunma University, Gunma, Japan, in 2013. She is currently a doctoral student in Mechanical Science and Technology at Gunma University. Her research interests include observer and repetitive control. She received the 2013 Electrical Engineering/Electronics, Computer, Telecommunication, and Information Technology International Conference (ECTI-CON2013) Best Paper Award.

Kou Yamada was born in Akita, Japan, in 1964. He received B.S. and M.S. degrees from Yamagata University, Yamagata, Japan, in 1987 and 1989, respectively, and the Dr. Eng. degree from Osaka University, Osaka, Japan in 1997. From 1991 to 2000, he was with the Department of Electrical and Information Engineering, Yamagata University, Yamagata, Japan, as a research associate. From 2000 to 2008, he was an associate professor in the Department of Mechanical System Engineering, Gunma University, Gunma, Japan. From 2008 to 2013, he was a professor in the Department of Mechanical System Engineering, Gunma University, Gunma, Japan. Since 2013, he has been a professor in the Division of Mechanical Science and Technology, Gunma University, Gunma, Japan. His research interests include robust control, repetitive control, process control and control theory for inverse systems and infinite-dimensional systems. Dr. Yamada received the 2005 Yokoyama Award in Science and Technology, the 2005 Electrical Engineering/Electronics, Computer, Telecommunication, and Information Technology International Conference (ECTI-CON2005) Best Paper Award, the Japanese Ergonomics Society Encouragement Award for Academic Paper in 2007, the 2008 Electrical Engineering/Electronics, Computer, Telecommunication, and Information Technology International Conference (ECTI-CON2008) Best Paper Award, Fourth International Conference on Innovative Computing, Information and Control Best Paper Award in 2009 and the 2013 Electrical Engineering/Electronics, Computer, Telecommunication, and Information Technology International Conference (ECTI-CON2013) Best Paper Award.

Tatsuya Sakanushi was born in Hokkaido, Japan, in 1987. He received B.S. and M.S. degrees in Mechanical System Engineering, and the Dr. Eng. degree from Gunma University, Gunma, Japan, in 2009, 2011 and 2013, respectively. Since 2013, he has been an assistant professor in the Division of Mechanical Science and Technology, Gunma University, Gunma, Japan. His research interests include PID control, observer and repetitive control. Dr. Sakanushi received Fourth International Conference on Innovative Computing, Information and Control Best Paper Award in 2009 and the 2013 Electrical Engineering/Electronics, Computer, Telecommunication, and Information Technology International Conference (ECTI-CON2013) Best Paper Award.