A Design Method for Smith Predictors for Minimum-Phase Time-Delay Plants

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ABSTRACT

In this paper, we examine a design method for a modified Smith predictor for minimum-phase time-delay plants. The modified Smith predictor is well known as an effective time-delay compensator for a plant with large time delays, and several papers on the modified Smith predictor have been published. However, the parameterization of all stabilizing modified Smith predictors has not been obtained. If this can be obtained, we can express existing proposals for modified Smith predictors in a uniform manner, and the modified Smith predictor can be designed systematically. The purpose of this paper is to propose the parameterization of all stabilizing modified Smith predictors for minimum-phase time-delay plants. The control characteristics of the control system using the parameterization of all stabilizing modified Smith predictors are also given. Finally, numerical examples for stable plants and unstable plants are illustrated to show the effectiveness of the proposed parameterization of all stabilizing modified Smith predictors.

Keywords: Minimum-Phase System, Time-Delay System, Smith Predictor, Parameterization

1. INTRODUCTION

In this paper, we examine a design method for Smith predictors for minimum-phase time-delay plants. Proposed by Smith to overcome time delays [1], it is well known as an effective time-delay compensator for a stable plant with large time delays [1–12]. The Smith predictor in [1] cannot be used for plants having an integral mode, because a step disturbance will result in a steady state error [2–4]. To overcome this problem, Watanabe and Ito [4], Astrom, Hang and Lim [9], and Matusek and Micic [10] proposed a design method for a modified Smith predictor for time-delay plants with an integrator. Watanabe and Sato expanded the result in [4] and proposed a design method for modified Smith predictors for multivariable systems with multiple delays in inputs and outputs [5].

Because the modified Smith predictor cannot be used for unstable plants [2–11], De Paor [6], De Paor and Egan [8] and Kwak, Sung, Lee and Park [12] proposed a design method for modified Smith predictors for unstable plants. Thus, several design methods of modified Smith predictors have been published.

On the other hand, another important control problem is the parameterization problem, the problem of finding all stabilizing controllers for a plant [13–21]. This problem was considered in [20, 21]. However, the parameterization of all stabilizing modified Smith predictors has not been obtained. If this result could be obtained, we could express previous studies of modified Smith predictors in a uniform manner. In addition, modified Smith predictors could be designed systematically.

The purpose of this paper is to propose the parameterization of all stabilizing modified Smith predictors for minimum-phase time-delay plants. First, the structure and necessary characteristics of modified Smith predictors described in past studies in [1–12] are defined. Next, the parameterization of all stabilizing modified Smith predictors for minimum-phase time-delay plants is proposed, for both stable and unstable plants. The control characteristics of the control systems using this parameterization are also given. Finally, a numerical example is presented to show the effectiveness of the proposed parameterization.

This paper is organized as follows: In Section 2. the Smith predictor is introduced briefly and the problem considered in this paper is explained. In Section 3. and Section 4., the parameterizations of all stabilizing modified Smith predictors for stable and unstable plants are given, respectively. In Section 3. and Section 4., we clarify the control characteristics using the parameterization of all stabilizing modified Smith predictors. Simple numerical examples are illustrated in Section 5.

Notation

R	The set of real numbers.
$R(s)$	The set of real rational functions with s.
RH_∞	The set of stable proper real rational functions.
H_∞	The set of stable causal functions.
$\mathcal{U}$	The set of unimodular functions on RH_∞. That is, $U(s) \in \mathcal{U}$ implies both $U(s) \in RH_\infty$ and $U^{-1}(s) \in RH_\infty$.
$\Re\{\cdot\}$	The real part of $\{\cdot\}$.

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2. MODIFIED SMITH PREDICTOR

Consider the control system:

$$\left\{ \begin{array}{rcl} y & = & G(s)e^{-sT}u + d \\ u & = & C(s)\left(r - y\right) \end{array} \right. , \tag{1}$$

where $G(s)e^{-sT}$ is the single-input/single-output time-delay plant with time-delay $T > 0$, $C(s)$ is the controller, $y \in R$ is the output, $u \in R$ is the input, $d \in R$ is the disturbance and $r \in R$ is the reference input. $G(s)$ is assumed to be coprime and of minimum phase, that is, $G(s)$ has no zeros in the closed right half plane.

According to [1–12], the modified Smith predictor $C(s)$ is decided by the form:

$$C(s) = \frac{C_1(s)}{1 + C_2(s)e^{-sT}}, \tag{2}$$

where $C_1(s) \in R(s)$ and $C_2(s) \in R(s)$. In addition, using the modified Smith predictor in [1–12], the transfer function from r to y of the control system in (1), written as

$$y = \frac{C(s)G(s)e^{-sT}}{1 + C(s)G(s)e^{-sT}} r \tag{3}$$

has a finite number of poles. That is, the transfer function from r to y of the control system in (1) is written as

$$y = \bar{G}(s)e^{-sT}r, \tag{4}$$

where $\bar{G}(s) \in RH_\infty$. Therefore, we call $C(s)$ the modified Smith predictor if $C(s)$ takes the form of (2) and the transfer function from r to y of the control system in (1) has a finite number of poles.

The problem considered in this paper is to obtain the parameterization of all modified Smith predictors $C(s)$ that make the control system in (1) stable. In Section 3., we propose the parameterization of all stabilizing modified Smith predictors $C(s)$ for stable plants. In Section 4., we expand the result in Section 3. and propose the parameterization of all stabilizing modified Smith predictors $C(s)$ for unstable plants.

3. THE PARAMETERIZATION OF ALL STABILIZING MODIFIED SMITH PREDICTORS FOR STABLE PLANTS

The parameterization of all stabilizing modified Smith predictors for the stable plant $G(s)e^{-sT}$ is summarized in the following theorem.

Theorem 1: $G(s)e^{-sT}$ is assumed to be stable. The parameterization of all stabilizing modified Smith predictors $C(s)$ takes the form

$$C(s) = \frac{Q(s)}{1 - Q(s)G(s)e^{-sT}}, \tag{5}$$

where $Q(s) \in RH_\infty$ is any function.

Proof: First, the necessity is shown. If the controller $C(s)$ in (2) makes the control system in (1) stable and makes the transfer function from r to y of the control system in (1) have a finite number of poles, then $C(s)$ takes the form of (5). From the assumption that the controller $C(s)$ in (2) makes the transfer function from r to y of the control system in (1) have a finite number of poles,

$$\frac{C(s)G(s)e^{-sT}}{1 + C(s)G(s)e^{-sT}} = \frac{C_1(s)G(s)e^{-sT}}{1 + \left(C_2(s) + C_1(s)G(s)\right)e^{-sT}} \tag{6}$$

has a finite number of poles. This implies that

$$C_2(s) = -C_1(s)G(s) \tag{7}$$

is necessary, that is:

$$C(s) = \frac{C_1(s)}{1 - C_1(s)G(s)e^{-sT}}. \tag{8}$$

From the assumption that $C(s)$ in (2) makes the control system in (1) stable, $C(s)G(s)e^{-sT}/(1 + C(s)G(s)e^{-sT})$, $C(s)/(1+C(s)G(s)e^{-sT})$, $G(s)e^{-sT}/(1+C(s)G(s)e^{-sT})$ and $1/(1+C(s)G(s)e^{-sT})$ are stable. From simple manipulation and (7), we have

$$\frac{C(s)G(s)e^{-sT}}{1 + C(s)G(s)e^{-sT}} = C_1(s)G(s)e^{-sT}, \tag{9}$$

$$\frac{C(s)}{1 + C(s)G(s)e^{-sT}} = C_1(s), \tag{10}$$

$$\frac{G(s)e^{-sT}}{1 + C(s)G(s)e^{-sT}} = \left(1 - C_1(s)G(s)e^{-sT}\right)G(s)e^{-sT} \tag{11}$$

and

$$\frac{1}{1 + C(s)G(s)e^{-sT}} = 1 - C_1(s)G(s)e^{-sT}. \tag{12}$$

It is obvious that the necessary condition for all the transfer functions in (9), (10), (11) and (12) to be stable is $C_1(s) \in RH_\infty$. Using $Q(s) \in RH_\infty$, let $C_1(s)$ be

$$C_1(s) = Q(s), \tag{13}$$

we find that $C(s)$ takes the form of (5). Thus, the necessity has been shown.

Next, the sufficiency is shown. If $C(s)$ takes the form of (5), then the controller $C(s)$ makes the control system in (1) stable and makes the transfer function from r to y of the control system in (1) have a finite number of poles. From simple manipulation, we have

$$\frac{C(s)G(s)e^{-sT}}{1 + C(s)G(s)e^{-sT}} = Q(s)G(s)e^{-sT}, \tag{14}$$

$$\frac{C(s)}{1+C(s)G(s)e^{-sT}}=Q(s), \tag{15}$$

$$\frac{G(s)e^{-sT}}{1+C(s)G(s)e^{-sT}} = \left(1-Q(s)G(s)e^{-sT}\right)G(s)e^{-sT} \tag{16}$$

and

$$\frac{1}{1+C(s)G(s)e^{-sT}}=1-Q(s)G(s)e^{-sT}. \tag{17}$$

From the assumption that $G(s)e^{-sT}$ is stable and $Q(s) \in RH_\infty$, (14), (15), (16) and (17) are all stable. In addition, because the transfer function from r to y of the control system in (1) takes the form (14) and $Q \in RH_\infty$, the transfer function from r to y of the control system in (1) has a finite number of poles.

We have thus proved Theorem 1. ■

Note 1: Note that because the proof of Theorem 1 does not require the assumption that $G(s)$ is of minimum phase, even if $G(s)e^{-sT}$ is of non-minimum phase, the parameterization of all stabilizing modified Smith predictors is given by Theorem 1.

Next, we explain the control characteristics of the control system using the parameterization of all stabilizing modified Smith predictors in (5). The transfer function from the reference input r to the output y of the control system in (1) takes the form

$$y = Q(s)G(s)e^{-sT}r. \tag{18}$$

Therefore, for the output y to follow the step reference input $r = 1/s$ without steady state error,

$$Q(0)G(0) = 1 \tag{19}$$

must be satisfied. $Q(s)$ is chosen by

$$Q(s)=\frac{q(s)}{G(s)}, \tag{20}$$

where

$$q(s)=\frac{1}{(1+s\tau)^\alpha}, \tag{21}$$

where $\tau > 0$ and α is a positive integer that makes $Q(s)$ in (20) proper.

The disturbance attenuation characteristics are as follows. The transfer function from the disturbance d to the output y of the control system in (1) is given by

$$y=\left(1-Q(s)G(s)e^{-sT}\right)d. \tag{22}$$

Therefore, to attenuate the step disturbance $d = 1/s$ effectively, $Q(s)$ must satisfy

$$Q(0)G(0) = 1. \tag{23}$$

That is, when $Q(s)$ is chosen according to (20), the control system in (20) can attenuate not only the step disturbance d effectively but also a disturbance with frequency component ω satisfying

$$1-q(j\omega)e^{-j\omega T} \simeq 0. \tag{24}$$

4. THE PARAMETERIZATION OF ALL STABILIZING MODIFIED SMITH PREDICTORS FOR UNSTABLE PLANTS

In this section, we expand the result in Section 3. and propose the parameterization of all stabilizing modified Smith predictors $C(s)$ for unstable minimum phase plants.

This parameterization is summarized in the following theorem.

Theorem 2: $G(s)e^{-sT}$ is assumed to be unstable and to be of minimum phase. For simplicity, the unstable poles of $G(s)e^{-sT}$ are assumed to be distinct. That is, when $s_i(i = 1, \cdots, n)$ denote unstable poles of $G(s)$, $s_i \neq s_j (i \neq j; i = 1, \cdots, n; j = 1, \cdots, n)$. Under these assumptions, there exists $\bar{G}_u(s) \in \mathcal{U}$ satisfying

$$\bar{G}_u(s_i)=\frac{1}{G_s(s_i)e^{-s_iT}}, \tag{25}$$

where $G_s(s)$ is a stable minimum-phase function of $G(s)$, that is, when $G(s)$ is factorized as

$$G(s) = G_u(s)G_s(s), \tag{26}$$

$G_u(s)$ is the unstable biproper minimum phase function and $G_s(s)$ is the stable minimum phase function. Using these functions, the parameterization of all stabilizing modified Smith predictors $C(s)$ is written as

$$C(s)=\frac{C_f(s)}{1-C_f(s)G(s)e^{-sT}}, \tag{27}$$

where $C_f(s)$ is given by

$$C_f(s)=\frac{\bar{G}_u(s)}{G_u(s)}\left(1+\frac{Q(s)}{G_u(s)}\right) \tag{28}$$

and $Q(s) \in RH_\infty$ is any function.

The proof of Theorem 2 requires the following Lemma.

Lemma 1: $G(s)$ is assumed to be unstable and to be of minimum phase. For simplicity, the unstable poles $s_i(i = 1, \cdots, n)$ of $G(s)e^{-sT}$ are assumed to be distinct. Under these assumptions, there exists $\bar{G}_u(s) \in \mathcal{U}$ satisfying (25), where $G_s(s)$ is the stable function of $G(s)$.

Proof: From the assumption that $G(s)$ is of minimum phase, $G_s(s)$ is also of minimum phase. Therefore, for all s_i on the real axis, $1/\left(G_s(s_i)e^{-s_iT}\right)$ are all the same sign. From Theorem 2.3.3 in [16], there exists $\bar{G}_u(s) \in \mathcal{U}$ satisfying (25).

We have thus proved Lemma 1. ■

Using this Lemma, we shall show the proof of Theorem 2.

Proof: First, the necessity is shown. If the controller $C(s)$ in (2) makes the control system in (1) stable and makes the transfer function from r to y of the control system in (1) have a finite number of poles, then $C(s)$ takes the form (27). From the same assumption,

$$\frac{C(s)G(s)e^{-sT}}{1+C(s)G(s)e^{-sT}} = \frac{C_1(s)G(s)e^{-sT}}{1+(C_2(s)+C_1(s)G(s))\,e^{-sT}} \quad (29)$$

has a finite number of poles. This implies that

$$C_2(s) = -C_1(s)G(s) \quad (30)$$

is satisfied, that is, $C(s)$ is necessarily

$$C(s) = \frac{C_1(s)}{1-C_1(s)G(s)e^{-sT}}. \quad (31)$$

From the assumption that $C(s)$ in (2) makes the control system in (1) stable, $C(s)G(s)e^{-sT}/(1+C(s)G(s)e^{-sT})$, $C(s)/(1+C(s)G(s)e^{-sT})$, $G(s)e^{-sT}/(1+C(s)G(s)e^{-sT})$ and $1/(1+C(s)G(s)e^{-sT})$ are stable. From simple manipulation and (30), we have

$$\frac{C(s)G(s)e^{-sT}}{1+C(s)G(s)e^{-sT}} = C_1(s)G(s)e^{-sT}, \quad (32)$$

$$\frac{C(s)}{1+C(s)G(s)e^{-sT}} = C_1(s), \quad (33)$$

$$\frac{G(s)e^{-sT}}{1+C(s)G(s)e^{-sT}} = \left(1-C_1(s)G(s)e^{-sT}\right)G(s)e^{-sT} \quad (34)$$

and

$$\frac{1}{1+C(s)G(s)e^{-sT}} = 1-C_1(s)G(s)e^{-sT}. \quad (35)$$

It is obvious that the necessary condition for all the transfer functions in (32), (33) and (35) to be stable is that $C_1(s)G(s)$ is stable. This implies that $C_1(s)$ must take the form

$$C_1(s) = \frac{\bar{C}_1(s)}{G_u(s)}, \quad (36)$$

where $\bar{C}_1(s) \in RH_\infty$. From the assumption that the transfer function in (34) is stable and from (36), for $s_i (i = 1, \cdots, n)$, which are the unstable poles of $G(s)$,

$$\begin{aligned} 1-C_1(s_i)G(s_i)e^{-s_iT} &= 1-\bar{C}_1(s_i)G_s(s_i)e^{-s_iT} \\ &= 0 \quad (i = 1, \cdots, n) \end{aligned} \quad (37)$$

must be satisfied. From Lemma 1, there exists $\bar{G}_u(s) \in \mathcal{U}$ satisfying

$$1-\bar{G}_u(s_i)G_s(s_i)e^{-s_iT} = 0 \quad (i = 1, \cdots, n). \quad (38)$$

Note that the condition in (38) is equivalent to (25), because $G_s(s)$ the stable minimum phase function, that is $G_s(s_i) \neq 0$. Using $\bar{G}_u(s) \in \mathcal{U}$ satisfying (38), $\bar{C}_1(s)$ is rewritten as

$$\bar{C}_1(s) = \bar{G}_u(s)\left(1+\frac{\bar{C}_1(s)-\bar{G}_u(s)}{\bar{G}_u(s)}\right). \quad (39)$$

Because $\bar{G}_u(s) \in \mathcal{U}$ and $\bar{C}_1(s) \in RH_\infty$, $(\bar{C}_1(s)-\bar{G}_u(s))/\bar{G}_u(s)$ is stable. In addition, because $(\bar{C}_1(s)-\bar{G}_u(s))/\bar{G}_u(s)$ takes the form

$$\frac{\bar{C}_1(s)-\bar{G}_u(s)}{\bar{G}_u(s)} = \frac{\bar{C}_1(s)}{\bar{G}_u(s)} - 1 \quad (40)$$

and both $\bar{C}_1(s)$ and $1/\bar{G}_u(s)$ are proper, $(\bar{C}_1(s)-\bar{G}_u(s))/\bar{G}_u(s)$ is proper. Therefore, $(\bar{C}_1(s)-\bar{G}_u(s))/\bar{G}_u(s) \in RH_\infty$.

From (37) and (38),

$$\bar{C}_1(s_i)-\bar{G}_u(s_i) = 0 \quad (i = 1, \cdots, n) \quad (41)$$

holds. This implies that $s_i (i = 1, \cdots, n)$, which are zeros of $G_u(s)$, are zeros of $\bar{C}_1(s)-\bar{G}_u(s)$, because $\bar{G}_u \in \mathcal{U}$ and $\bar{C}_1(s) \in RH_\infty$. When we rewrite $(\bar{C}_1(s)-\bar{G}_u(s))/\bar{G}_u(s)$ as

$$\frac{\bar{C}_1(s)-\bar{G}_u(s)}{\bar{G}_u(s)} = \frac{Q(s)}{G_u(s)}, \quad (42)$$

then $Q(s) \in RH_\infty$, because $1/G_u(s) \in RH_\infty$. In this way, it is shown that if the controller $C(s)$ in (2) makes the control system in (1) stable and makes the transfer function from r to y of the control system in (1) have a finite number of poles, then $C(s)$ is written as (27).

Next, the sufficiency is shown. If $C(s)$ takes the form (27), then the controller $C(s)$ makes the control system in (1) stable and makes the transfer function from r to y of the control system in (1) have a finite number of poles. After simple manipulation, we have

$$\frac{C(s)G(s)e^{-sT}}{1+C(s)G(s)e^{-sT}} = \bar{G}_u(s)\left(1+\frac{Q(s)}{G_u(s)}\right)G_s(s)e^{-sT}, \quad (43)$$

$$\frac{C(s)}{1+C(s)G(s)e^{-sT}} = \frac{\bar{G}_u(s)}{G_u(s)}\left(1+\frac{Q(s)}{G_u(s)}\right), \quad (44)$$

$$\frac{G(s)e^{-sT}}{1+C(s)G(s)e^{-sT}} = \left\{1-\bar{G}_u(s)\left(1+\frac{Q(s)}{G_u(s)}\right)G_s(s)e^{-sT}\right\}G(s)e^{-sT} \quad (45)$$

and

$$\frac{1}{1+C(s)G(s)e^{-sT}} = 1-\bar{G}_u(s)\left(1+\frac{Q(s)}{G_u(s)}\right)G_s(s)e^{-sT}. \quad (46)$$

Because $\bar{G}_u(s) \in \mathcal{U}$, $Q(s) \in RH_\infty$, $G_s(s) \in RH_\infty$, the transfer functions in (43), (44) and (46) are stable. If the transfer function in (45) is unstable, the unstable poles of the transfer function in (45) are unstable poles of $G(s)$. From the assumption that $\bar{G}_u(s)$ satisfies (25), the unstable poles of $G(s)$ are not the poles of $\left\{1 - \bar{G}_u(s)\left(1 + \frac{Q(s)}{G_u(s)}\right) G_s(s)e^{-sT}\right\} G(s)e^{-sT}$. Therefore, the transfer function in (45) is stable. In addition, because the transfer function from r to y of the control system in (1) is given by (43) and $\bar{G}_u(s)$, $Q(s)$, $1/G_u(s) \in RH_\infty$, the transfer function from r to y of the control system in (1) has a finite number of poles.

We have thus proved Theorem 2. ■

Note 2: $\bar{G}_u(s)$ satisfying (25) is obtained using the method given in the proof of Theorem 2.3.3 in [16].

The modified Smith predictor in (27) is explained based on the frequency domain. In the time domain, using the modified Smith predictor in (27), the control input $u(t)$ is given as follows. From the assumption that the unstable poles of $G(s)e^{-sT}$ are distinct, and from (25), $G_u(s)$ and $\bar{G}_u(s)(1+Q(s)/G_u(s))G(s)$ can be rewritten as

$$G_u(s) = \sum_{i=1}^{n} \frac{c_i}{s - s_i} + \delta \tag{47}$$

and

$$\bar{G}_u(s)\left(1 + \frac{Q(s)}{G_u(s)}\right) G(s) = \sum_{i=1}^{n} \frac{c_i e^{s_i T}}{s - s_i} + \bar{Q}(s), \tag{48}$$

respectively, where

$$c_i = \left.(s - s_i)\, G_u(s)\right|_{s=s_i} \quad (i = 1, \ldots, n). \tag{49}$$

$\bar{Q}(s) \in RH_\infty$ holds true because the unstable poles of $\bar{G}_u(s)(1 + Q(s)/G_u(s))G(s)$ are equal to $s_i(i = 1, \ldots, n)$. $\delta \neq 0 \in R$ is satisfied, because $G_u(s)$ is biproper. Using δ, $\bar{Q}(s)$ and $c_i(i = 1, \ldots, n)$ in (47), (48) and (49), the control input $u(t)$ is given by

$$\begin{aligned} u(t) & \\ = & \frac{1}{\delta}\bar{Q}(p)u(t-T) - \frac{1}{\delta}\int_{-T}^{0}\sum_{i=1}^{n} c_i e^{-s_i\tau}u(t+\tau)d\tau \\ & + \frac{1}{\delta}\bar{G}_u(p)\left(1 + \frac{Q(p)}{G_u(p)}\right)(r(t) - y(t)), \end{aligned} \tag{50}$$

where p is the differential operator, i.e. $py(t) = \frac{dy(t)}{dt}$.

The control input $u(t)$ in (50) is obviously realizable, because $\bar{Q}(s) \in RH_\infty$, $\Re\{s_i\} \geq 0 (i = 1, \ldots, n)$, $\bar{G}_u(s)(1 + Q(s)/G_u(s)) \in RH_\infty$ and $u(t + \tau)$, $-T \leq \tau \leq 0$, is the past history of the control input over the finite interval T. The fact that the control input $u(t)$ in (50) is written in the frequency domain as $C(s)$ in (27) is confirmed as follows: Taking the Laplace transformation of (50) yields

$$\begin{aligned} \delta u(s) & \\ = & \bar{Q}(s)e^{-sT}u(s) - \int_{-T}^{0}\sum_{i=1}^{n} c_i e^{-s_i\tau}u(s)e^{\tau s}d\tau \\ & + \bar{G}_u(s)\left(1 + \frac{Q(s)}{G_u(s)}\right)(r(s) - y(s)) \\ = & \bar{Q}(s)e^{-sT}u(s) - \left(\sum_{i=1}^{n}\frac{c_i}{s - s_i}\right. \\ & \left. - \sum_{i=1}^{n}\frac{c_i e^{s_i T}}{s - s_i}e^{-sT}\right)u(s) + \bar{G}_u(s)\left(1 + \frac{Q(s)}{G_u(s)}\right) \\ & \cdot(r(s) - y(s)). \end{aligned} \tag{51}$$

From (47), (48), (51) and simple manipulation, we have

$$\begin{aligned} u(s) &= \frac{\bar{G}_u(s)\left(1 + \frac{Q(s)}{G_u(s)}\right)(r(s) - y(s))}{G_u(s) - \bar{G}_u(s)\left(1 + \frac{Q(s)}{G_u(s)}\right)G(s)e^{-sT}} \\ &= \frac{C_f(s)}{1 - C_f(s)G(s)e^{-sT}}(r(s) - y(s)). \end{aligned} \tag{52}$$

From the above equation, we find that the control input $u(t)$ in (50) is written in the frequency domain as $C(s)$ in (27).

Next, we explain the control characteristics of the control system using the parameterization of all stabilizing modified Smith predictors in (27). The transfer function from the reference input r to the output y of the control system in (1) is written as

$$y = \bar{G}_u(s)\left(1 + \frac{Q(s)}{G_u(s)}\right)G_s(s)e^{-sT}r. \tag{53}$$

Therefore, when $G(s)$ has a pole at the origin, for the output y to follow the step reference input $r = 1/s$ without steady state error,

$$\bar{G}_u(0)G_s(0) = 1 \tag{54}$$

must be satisfied. Because $\bar{G}_u(s) \in \mathcal{U}$ satisfies (38), (54) holds true. This implies that when $G(s)$ has a pole at the origin, the output y follows the step reference input r without steady state error, independent of $Q(s) \in RH_\infty$ in (27). On the other hand, when $G(s)$ has no pole at the origin, for the output y to follow the step reference input $r = 1/s$ without steady state error,

$$\bar{G}_u(0)\left(1 + \frac{Q(0)}{G_u(0)}\right)G_s(0) = 1 \tag{55}$$

must hold. From simple manipulation, if $Q(s)$ is chosen satisfying

$$Q(0) = G_u(0)\left(\frac{1}{\bar{G}_u(0)G_s(0)} - 1\right), \tag{56}$$

then the output y follows the step reference input without steady state error.

The disturbance attenuation characteristics are as follows. The transfer function from the disturbance d to the output y of the control system in (1) is given by

$$y = \left\{1 - \bar{G}_u(s)\left(1 + \frac{Q(s)}{G_u(s)}\right)G_s(s)e^{-sT}\right\}d. \quad (57)$$

Therefore, to attenuate the step disturbance $d = 1/s$ effectively, $Q(s)$ must satisfy

$$\bar{G}_u(0)\left(1 + \frac{Q(0)}{G_u(0)}\right)G_s(0) = 1. \quad (58)$$

That is, when $Q(s)$ is chosen satisfying (56), the step disturbance d is attenuated effectively.

5. NUMERICAL EXAMPLE

In this section, numerical examples for stable and unstable plants are presented to show the effectiveness of the proposed parameterization of all stabilizing modified Smith predictors.

Let us consider the problem of finding the parameterization of all stabilizing modified Smith predictors for the stable plant $G(s)e^{-sT}$ written as

$$G(s)e^{-sT} = \frac{1}{s^2 + 3s + 4}e^{-4s}, \quad (59)$$

where

$$G(s) = \frac{s+1}{s^2 + 3s + 4} \quad (60)$$

and $T = 4$[sec]. From Theorem 1, the parameterization of all stabilizing modified Smith predictors for $G(s)e^{-sT}$ in (59) is given by (5). For the output y to follow the step reference input without steady state error, $Q(s) \in RH_\infty$ is chosen using (20), where $q(s)$ is

$$q(s) = \frac{1}{(1 + 0.1s)^2}. \quad (61)$$

The response of the output y for the step reference input r is shown in Fig. 1 . Figure 1 shows that the control system is stable and the output y follows the step reference input r without steady state error.

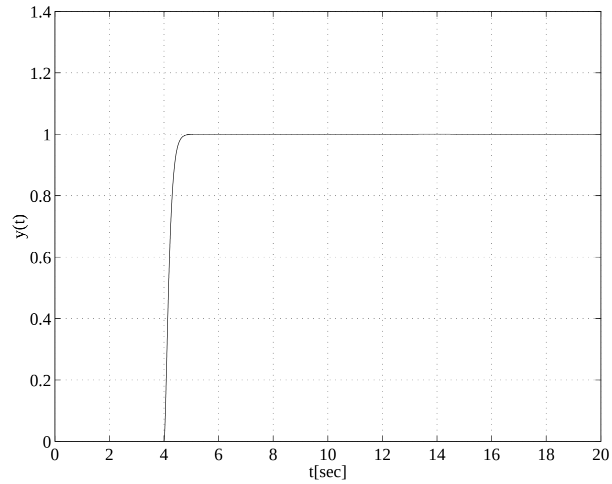

Fig.1: *Step response*

Next we show a numerical example for an unstable plant. Let us consider the problem of finding the parameterization of all stabilizing modified Smith predictors for the unstable plant $G(s)e^{-sT}$ given by

$$G(s)e^{-sT} = \frac{s+1}{s^2 + 3s}e^{-s}, \quad (62)$$

where

$$G(s) = \frac{s+1}{s^2 + 3s} \quad (63)$$

and $T = 1$[sec].

$G(s)$ is factorized by (26) as

$$G_u(s) = \frac{s+2}{s} \quad (64)$$

and

$$G_s(s) = \frac{s+1}{(s+2)(s+3)}. \quad (65)$$

One $\bar{G}_u(s)$ in (28) satisfying (25) is given by

$$\bar{G}_u(s) = \frac{6(2s+1)}{3s+1}. \quad (66)$$

From Theorem 2, the parameterization of all stabilizing modified Smith predictors for $G(s)e^{-sT}$ in (62) is given by (27), where

$$C_f(s) = \frac{6s(2s+1)}{(3s+1)(s+2)}\left(1 + \frac{s}{s+2}Q(s)\right) \quad (67)$$

and $Q(s) \in RH_\infty$.

choosing $Q(s) \in RH_\infty$ as

$$Q(s) = \frac{s+2}{s+3}, \quad (68)$$

we have

$$C_f(s) = \frac{6s(2s+1)(2s+3)}{(3s+1)(s+2)(s+3)}. \quad (69)$$

Because $G(s)$ in (63) has a pole at the origin, from the discussion in the preceding section, the output y follows the step reference input r without steady state error. The step response of the control system in (1) using $C_f(s)$ in (69) is shown in Fig. 2 . Figure 2 shows that the control system in (1) is stable and the output y follows the step reference input r without steady state error.

In this way, we find that by using the results in this paper, we can easily obtain the parameterization of all stabilizing modified Smith predictors for time-delay plants.

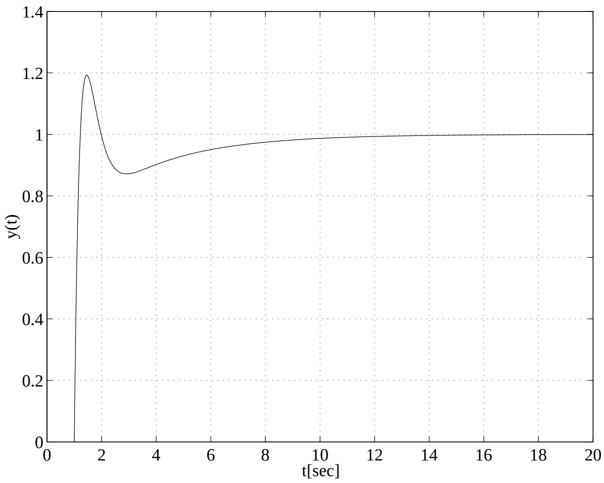

Fig.2: *Step response*

6. CONCLUSION

In this paper, we proposed the parameterization of all stabilizing modified Smith predictors for minimum-phase time-delay plants. First, the parameterization of all stabilizing modified Smith predictors for stable plants, which are not necessarily of minimum phase, was proposed. Next, we expanded the result of the parameterization for stable plants and proposed the parameterization of all stabilizing modified Smith predictors for unstable plants. The control characteristics of the control system using the parameterization of all stabilizing modified Smith predictors and design method of $Q(s)$ in (5) and (27) were also given. Finally, numerical examples were presented to show the effectiveness of the proposed parameterization.

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