

Metamorphic Testing Using Geometric Interrogation Technique and Its Application

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ABSTRACT

Usual techniques used in the automatic test case generation and executions assume that a complete oracle will be available during the testing process. However, in some numerical programs this assumption is not valid because the oracle in form of analytical solution is either not available or too difficult to obtain. To overcome this problem, metamorphic testing proposes to generate follow-up test cases to check important properties of the output function. We study the intrinsic properties of Medial Axes for casting applications. Even though Medial Axes is a geometric function that is independent from the output function of casting simulation, we propose to use its intrinsic properties for metamorphic testing of casting simulation to alleviate the oracle problem.

Keywords: Metamorphic Testing, Medial Axis Transform, Geometric Method.

1. INTRODUCTION

Program testing is an area gaining popularity and momentum in the software engineering community due to its ever-increasing demand placed by the end-users on #software developers [1]. Conventionally, there are two well-know test case selection strategies, namely the black-box method and the white-box method [2]. Both approaches involve in selecting suitable inputs as test cases, executing the program and verifying the test results against some oracles which are the benchmark or known expected results. However, in some cases, the oracle is either not available or too expensive to apply [3].

Recently, Chen *et al.* [4-6] have proposed a method called metamorphic testing (MT) as a new paradigm for program testing. Metamorphic testing was proposed to overcome some of the inherent problems that are present in testing software without test oracle. The test forced the developer to derive the intrinsic characteristics of the test function, rather than the simplistic special value testing, which is oftenly done.

Metamorphic testing also exploits test results from successful test cases in follow up test cases.

When testing numerical programs whose outputs are not easily verifiable, a frequently adopted approach is to use special or simple values as inputs [7]. This method is insufficient for very complex computer programs. In the following, we propose to use the intrinsic properties of the Medial Axis Transform (MAT) as a mean to test the correctness of numerical software written for casting application. The route we approach the problem here is different from those studied by Chen *et al.* [4-6] even though the basic idea is indifferent. Since the programs we are testing are over complicated and large, we abandon the idea of deriving the metamorphic relation from the output function alone. We propose to use the intrinsic properties of a special geometric function G to test the correctness of the of our program p , which gives an output function f . Notice that the properties of G is totally independent of our output function f even though they can be relevant in a very lose sense. Similar to Chen's approach, a test set $T = \{t_1, t_2, \dots, t_n\}$ where $n \in Z^+$ can be easily generated. Thus our approach here is highly biased towards automatic testing of numerical programs in which the analytical solution is not available.

In the following, we briefly review the idea of MAT and proceed to describe the test cases we device for this paper.

2. MEDIAL AXIS AND MEDIAL SURFACE

The MAT is generally attributed to Blum [8]. In two-dimensions the medial axis is the locus of the centre of an inscribed disc of maximal diameter as it rolls around the domain interior expanding and contracting to maintain contact with the domain boundary. The combination of the medial axis and the radius function, which describes the radius of the inscribed disc at any point on the medial axis, is known as the Medial Axis Transform (MAT) [9-12]. A medial axis consists of medial axis edges and medial axis vertices as can be seen in Fig. 1. The MAT is a complete representation of the domain geometry. It provides a reduced-dimensional skeleton of the region, and identifies entities of the object boundary, which are in geometric proximity. At any point on the medial axis a line joining the touching points of the corresponding disc is the shortest cut which can be used to subdivide the model in that vicinity.

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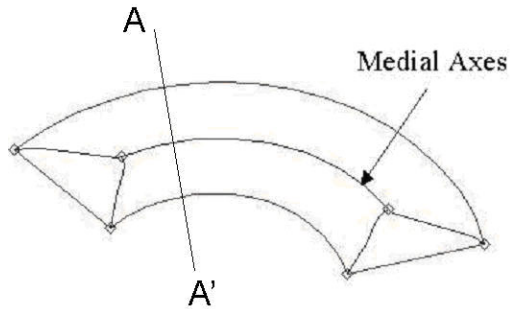


Fig.1: Two-dimensional medial axes

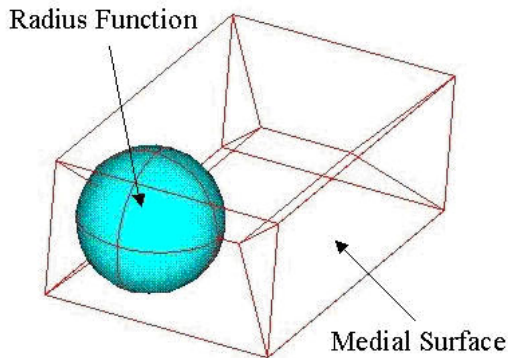


Fig.2: Three-dimensional medial axes

In three-dimension the equivalent construction is the locus of the centres of all inscribed spheres of maximal diameter. This is also known as the medial axis, though perhaps the medial surface would be more appropriate description, since the medial surface is a two-dimensional skeleton of the solid. An example is shown in Fig. 2. A medial surface consists of medial surface faces, medial surface edges and medial surface vertices.

The MAT has been a prime area of study, not only in computer-aided geometric design, but also in such diverse areas as computer graphics, computer vision, pattern recognition, image processing, NC tool path planning, mesh generation and font design. However, we found no literature describing the use of MAT as a tester to test the correctness of a computer program, thus the novelty of this work.

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Mathematically, MAT is a collection of two objects, namely the medial axis and the radius function. The axis is constructed as follows: Given a discrete points $\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_n$ scattered over a plane, their

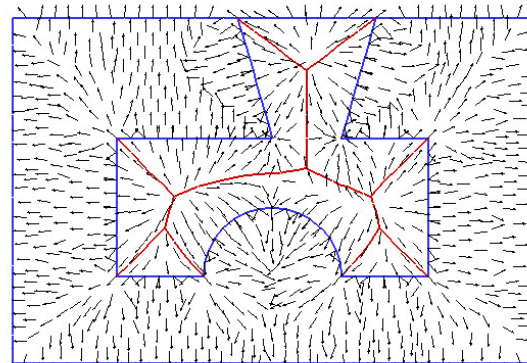


Fig.3: Heat fluxes emanate from medial axes

Voronoi diagram is the locus of points of equal distance to at least two of the given points. The Voronoi diagram partitions the plane into mutually disjoint regions V_1, V_2, \dots, V_n such that points of V_i are closer to \mathbf{p}_i than any other \mathbf{p}_j for $i \neq j$. Thus in a plane, the Voronoi diagram is the medial axis of the region $R^2 - \{\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_n\}$. The radius function is just a consequence of the partition with its value given as $V_i/2$ if the disjoint regions appear to be a circle or sphere.

3. APPLICATION OF MT TO NUMERICAL PROBLEMS

In general, there is no analytical solution exist for casting simulation except in some special cases with very simple geometry. Hence verify the exact correctness of the output of the numerical solution is a formidable task. Nevertheless, because of the physics of the heat transfer mechanism, it is possible to provide some incomplete test which gives valuable insight of the correctness of the solution algorithm. In the following test cases, we assume that the casting is experiencing uniform solidification.

3.1 The Relation Between MAT and Heat Fluxes

Figure 3 depicted a simple casting-mould assembly with the medial axes inscribed within the casting, together with the heat fluxes. Since heat tends to flow from region of high temperature to region of low temperature, it is noted that during the initial cooling period, the majority of the heat fluxes are pointing outward from the medial axes and towards the casting boundaries.

This intrinsic property of medial axes can be used to derive the metamorphic relations that can automatically check the qualitative correctness of the solution at any point within the region of a medial axes. For example, consider *Point a*, P_a , as any point on the medial axes in Fig. 4. The temperature at point

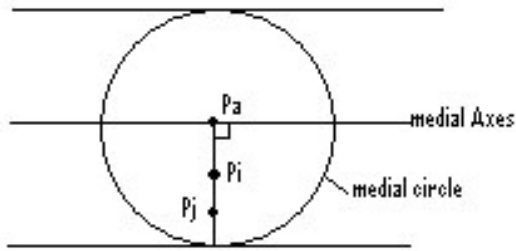


Fig.4: Temperature on the medial axes and along the radius perpendicular to the medial axes at P_a .

A, T_{P_a} will be higher than the temperature at Point i , T_{P_i} which resides along the radius perpendicular to the Medial Axes at P_a . Therefore, a correct coding of casting simulation should produce the numerical solutions that comply with (1).

$$T_{P_a} > T_{P_i} \text{ for } P_i \neq P_a \quad (1)$$

where P_i is any point on the radius perpendicular to the Medial Axes at P_a .

The results obtained for T_{P_a} and T_{P_i} from the test suite generated based on metamorphic relationship in (1) can be utilised for the subsequent test suites. Further test cases can be generated for the second test suites based on the fact that during the initial cooling period, the heat fluxes are pointing outward from the medial axes and towards the casting boundaries. Therefore the metamorphic relationship in (2) is also true.

$$T_{P_i} > T_{P_j} \text{ for } D_{a_i} > D_{a_j} \quad (2)$$

where D_{a_i} is the distance between P_a and P_i and D_{a_j} is the distance between P_a and P_j .

The test cases generated based on metamorphic relation in (1) and (2) can be duplicated and repeated for heat fluxes emanate from medial axes at a different time frame during the initial cooling period of the casting simulation. The temperature on any single point on the medial axes should have a lower temperature as the cooling time increase. Therefore, the numerical solution not only should comply with (1) and (2), but also fulfil the metamorphic relationship in (3).

$$T_{P_a-tm} > T_{P_i-tn} \text{ for } tm > tn \quad (3)$$

Where tm and tn represent two distinct time during the initial cooling period of casting.

With the above three metamorphic relationships, two test suites can be generated and repeated at different time frame to check the qualitative correctness

of the numerical solutions. Additional code or software tools can be developed to generate and executes the test suites to automate the testing without involving human oracle.

3.2 The Radius Function and the Highest Temperature

Figure 5 shows a typical U-shaped casting with its medial axes and inscribed circle. Since the radius is directly proportional to the area/volume of the casting, the bigger the radius, the longer it takes for that region to cool during the solidification. If one plots the medial radius function as colour contour by using HSL colour representation, Fig.e 6 is obtained. One can expect the temperature solution from the casting simulation will be similar to the contour shown in Fig. 6. This is in fact the case as can be seen in Fig. 7.

In the colour contour in Fig. 6, the red colour can be represented by a hue value of 0^0 and the colour gradually changes to yellow as the hue value reaches 60^0 , followed by green for 120^0 and 240^0 for blue. Similarly, for the temperature contour, the value of the luminance can be represented as 0 for black and gradually increases to 1 for white.

Based on the above properties and colour representation, the hue value for any pixel in the colour contour in Fig. 6 and the luminance of any pixel in the finite element solution of the temperature contour in Fig. 7 can be used to establish a metamorphic relationship. Given any two pixels in the colour contour, the relationship between the hue values at those two pixels should be reciprocated in the relationship between the luminance of those two pixels in the temperature contour with an opposite relational operator. For example, if the hue value at *Pixel a* is smaller than *Pixel b* for colour contour, then the luminance at *Pixel a* should be larger than *Pixel b* in the temperature contour. With this simple metamorphic relationship, colour contour derived from the Medial Axes and its inscribed circles can be used to check the correctness of the temperature contour. Test cases can be generated and executed by means of processing the pixel information of the image for colour and temperature contour, hence alleviate the needs of human oracle.

4. CONCLUSIONS

In this paper, we have presented the metamorphic testing of engineering software, namely that of a casting simulation, using medial axis transform. Our analysis and proposed metamorphic relationships showed that it is possible to automatically test the overall correctness of a casting solution algorithm using MAT alone.

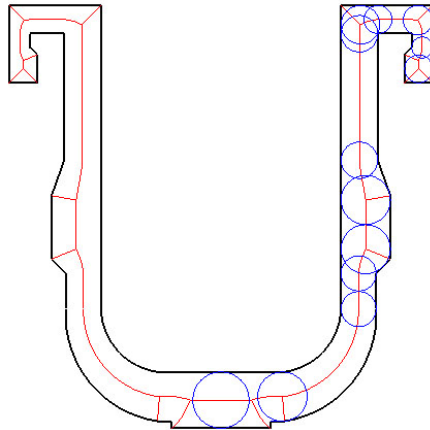


Fig.5: Casting with its medial axes and its inscribed circle

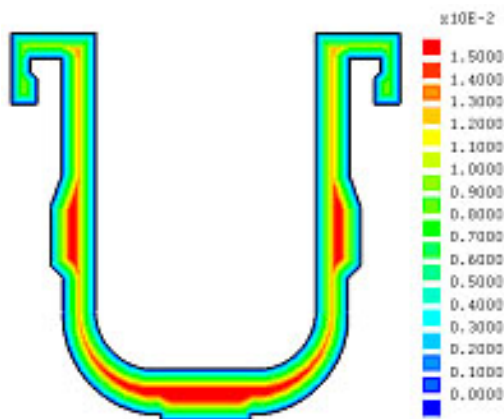


Fig.6: Colour contour of the medial radius function

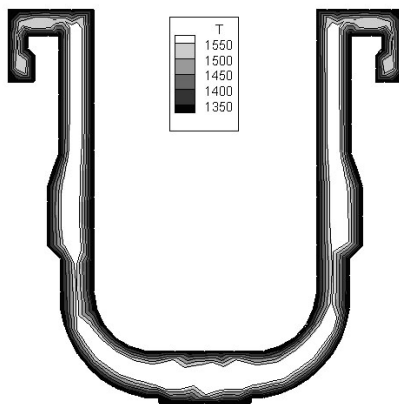


Fig.7: Finite element solution of the temperature contour in casting

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at time of printing.

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