Engineering and Applied Science Research January – March 2019;46(1):18-25 Research Article

EASR

Engineering and Applied Science Research

https://www.tci-thaijo.org/index.php/easr/index

Published by the Faculty of Engineering, Khon Kaen University, Thailand

Modified interactive strut-and-tie model for shear strength prediction of RC corbels

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Received 2 March 2018
Revised 16 July 2018
Accepted 17 July 2018

Abstract

On the basis of the modified Kupfer and Gerstle's biaxial failure criterion of concrete, a modified interactive strut-and-tie model (MISTM) to predict the nominal shear strength of RC corbels was derived and is proposed in this paper. The MISTM includes the interaction between the load-carrying capacity of a concrete strut and a tension tie, and the effects of horizontal force. The number of horizontal and vertical stirrups was considered. To validate and calibrate the MISTM, a dataset test results for 302 corbels was collected from the literature. Finally, the predictions obtained from the MISTM were also compared with those computed using the strut-and-tie models in accordance with the ACI 318 code and the state-of-the-art presented in the literature, and are found to be more accurate and reliable.

Keywords: Corbels, Shear strength, Strut-and-Tie, Reinforced concrete

1. Introduction

In precast concrete construction, RC corbels are usually used to transmit loads from beams or girders to columns or other parts of a structure. Additionally, the use of the connections to transfer loads from a steel pedestrian bridge (Figure 1) to concrete columns is another application of RC corbels. Such members should be analyzed and designed to carry not only vertical loads, but also the horizontal forces caused by restrained shrinkage, creep, and temperature change, among other factors. Since there is a relatively low shear span-to-depth ratio, the load-carrying capacity of RC corbels is commonly governed by shear rather than flexure as found in RC deep beams. Thus, RC corbels are considered a discontinuous (D-) region member, where the assumption that plane sections remain planes after bending is not usable, and therefore, conventional beam theory is not applicable [1].

Various research studies have been conducted, both experimentally [2-7] and analytically [8-13], to investigate the strength and behavior of such RC members. Currently, the strut-and-tie models (STMs) available in international codes of practice [14-15] have been applied to predict the shear strength and to design RC corbels. Nevertheless, due to complicated shear-transfer mechanisms in corbels, as exists in other D-region members, there have been inconsistencies and complexities in shear strength predictions found in some methods, as reported by Russo et al. [10], Yang and Ashour [11] and Kassem [12]. Consequently, the development of a more precise and simple STM for calculating the load-carrying capacity failing in shear of such D-region members is still needed.

In the present study, based on a modified Kupfer and Gerstle's bi-axial failure criterion [16], a new method to simply and accurately calculate the shear strength of RC corbels was derived and is presented. Additionally, to verify the proposed STM, the shear strength predictions were compared with 302 corbel test results collected from the literature. Finally, the state-of-the-art STMs [10, 13] and current ACI code [14] were also compared with the method developed in the current study.

Figure 1 A pedestrian bridge supported on RC corbels at Khon Kaen University

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doi: 10.14456/easr.2019.3

2. Developing the proposed STM

2.1 Geometry of the proposed STM

A load-transfer mechanism of RC corbel with a shear span (a), subjected to vertical (V_u) and horizontal forces (N_c), can be represented by an STM consisting of an inclined strut (dotted line) joined between the upper nodal zone (UNZ) and a lower nodal zone (LNZ), as well as a tension tie (solid line), as shown in Figure 2.

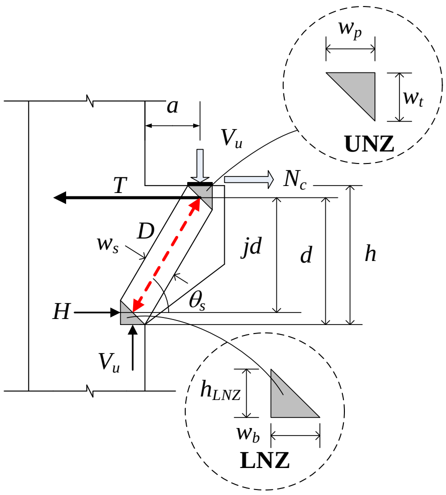

Figure 2 Strut-and-tie models for RC corbels

In general, it can be assumed that the shear strength of a corbel V_n is controlled by the capacity of the inclined concrete strut, reinforced by a number of horizontal and vertical stirrups having the cross-sectional areas A_h and A_v, respectively [8-10, 12-13]. Therefore, V_n can be written as:

$$V_n = V_{nc} + k_v A_v f_{yv} + k_h A_h f_{yh} \tan\theta_s \quad (1)$$

where V_{nc} is the shear strength contributed by the unreinforced concrete strut (without any horizontal and vertical reinforcement), as discussed in Section 2.2. The terms f_{yh} and f_{yv} are the yield strengths of the horizontal and vertical stirrups, respectively. The constants k_h and k_v are factors representing the contributions of such stirrups to the nominal shear strength. Furthermore, θ_s is the angle of the diagonal strut with respect to the horizontal plane. It can be obtained by $\tan\theta_s = jd/(a + w_b/2)$. Since the term, w_b, i.e., the width of LNZ (Figure 2), is not known at this stage, the equation is relatively complicated. However, according to Hwang et al. [8, 13] and Hwang and Lee [9], it can be simply estimated as:

$$\tan\theta_s = \frac{jd}{a} \quad (2)$$

where jd is the vertical distance between the UNZ and LNZ, $j = 1 - k/3$, while k is taken from elastic flexural theory for a singly reinforced concrete section as:

$$k = \sqrt{(n\rho_f)^2 + 2(n\rho_f)} - (n\rho_f) \quad (3)$$

where n represents the modular ratio and ρ_f denotes the main flexural steel ratio affected by the horizontal force, N_c [8], obtained by:

$$\rho_f = \frac{A_s - N_c/f_y}{bd} \quad (4)$$

where b and d are the width and the effective depth of the corbel, respectively. A_s and f_y are the cross-sectional area and the yield strength of the main tension steel of the corbel, respectively. According to Hwang et al. [8] and Russo et al. [10], it can be assumed that the depth of the diagonal concrete strut is equal to the depth of the compression zone of a singly reinforced concrete member (kd). Thus, the cross-sectional area of the inclined concrete strut A_{str} can be computed as:

$$A_{str} = b \times kd \quad (5)$$

2.2 The softening effect using the Kupfer- Gerstle's biaxial failure criterion

Generally, when a cracked RC member is subjected to a compressive force resulting in a reduction in compressive strength caused by transverse tensile strain, this is called the compression softening phenomenon. To take into account this phenomenon in a concrete strut, the softening factor approach based on compression field theory [17-20] is widely applied in most STMs, such as Hwang et al. [8, 13], Hwang and Lee [9], Russo et al. [10] and Chetchotisak et al. [21-22], etc. Alternatively, the failure criterion of concrete is another approach to describe the softening effect on the compressive strength of concrete struts applied in several STMs. For example, Tang and Tan [23], and Zhang and Tan [24] used a modified Mohr-Coulomb failure criterion ($\frac{\sigma_1}{f_{tn}} + \frac{\sigma_2}{f_c'} = 1$), while Wang and Meng [25] and Chetchotisak et al. [26] applied the Kupfer and Gerstle's bi-axial failure criterion [16], i.e., $\frac{\sigma_1}{f_{tn}} + \frac{\sigma_2}{1.25 f_c'} = 1$, where the terms σ_1 and σ_2 represent the principal tensile and compressive stresses, while f_{tn} is the tensile capacity of concrete combined with the amount of web reinforcement (if applicable) in the σ_1 direction. However, as this study attempted to increase the accuracy of the STM, the Kupfer and Gerstle's failure criterion [16] was modified by including a softening factor term ν as:

$$\frac{\sigma_1}{f_{tn}} + \frac{\sigma_2}{1.25\,\nu f_c'} = 1 \quad (6)$$

Here, ν denotes the softening factor taken from one of the selected factors listed in Table 1.

Table 1 Selected softening factors

Researchers	Softening Factor Models
Vecchio & Collins [17]	$\nu = \frac{1}{0.8+170\varepsilon_1} \leq 0.85$
Zhang & Hsu [18]	$\nu = \frac{5.8}{\sqrt{f_c'}} \frac{1}{\sqrt{1+400\varepsilon_1}} \leq \frac{0.9}{\sqrt{1+400\varepsilon_1}}$
Kaufmann & Marti [19]	$\nu = \frac{1}{(0.4+30\varepsilon_1) f_c'^{1/3}}$
Zwicky & Vogel [20]	$\nu = (1.8\text{-}38\varepsilon_1) \cdot (f_c')^{-1/3}$ and $0.85 \cdot (f_c')^{-1/3} \leq \nu \leq 1.6 \cdot (f_c')^{-1/3}$

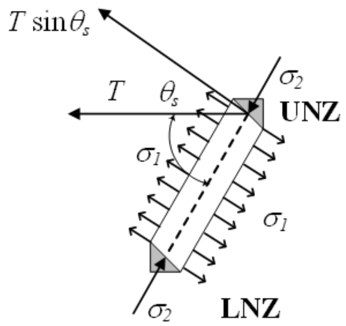

Figure 3 Biaxial state of stresses in a concrete strut

Furthermore, the principal tensile stress σ_1 perpendicular to the inclined concrete strut, shown in Figure 3, was assumed to result from the component of the tension tie force $T \sin\theta_s$ at the UNZ, and had a uniform distribution through the strut, so it can be calculated as:

$$\sigma_1 = \frac{k_1 T \sin\theta_s}{A_{\text{tang}}} \tag{7}$$

where the constant k_1 represents a factor used to simplify the complex stress distribution of σ_1 to a constant value, and the term A_{tang} is the area of the diagonal concrete strut in the tangential plane. T can be computed in terms of the shear strength of a corbel V_n as:

$$T = \frac{V_n}{\tan\theta_s} \tag{8}$$

Substituting Eq. (8) into Eq. (7), σ_1 can be taken in the form of V_n as:

$$\sigma_1 = \frac{k_1 V_n \sin\theta_s}{A_{\text{tang}} \tan\theta_s} \tag{9}$$

Similar to Eq. (9), f_{tn} can be directly obtained as:

$$f_{\text{tn}} = \frac{k_2\left(A_s f_y + F_{ct}\right)\sin\theta_s}{A_{\text{tang}}} \tag{10}$$

where k_2 denotes a factor in the same manner as k_1 and F_{ct} is the tensile strength of the concrete tie obtained from:

$$F_{ct} = f_{ct}\, w_t\, b \tag{11}$$

where f_{ct} is the concrete tensile strength in MPa units, expressed as:

$$f_{ct} = 0.5\sqrt{f_c'} \tag{12}$$

and w_t is the effective depth of the concrete tie written as:

$$w_t = 2(h-d) \tag{13}$$

Additionally, σ_2 is in the direction of the inclined strut and can be calculated as:

$$\sigma_2 = \frac{D}{A_{str}} = \frac{V_n}{A_{str}\sin\theta_s} \tag{14}$$

Substituting Eqs. (9), (10) and (14) into Eq. (6), the nominal shear strength of RC corbel contributed by the concrete strut V_{nc} can be derived as:

$$V_{nc} = \left(\frac{1}{V_{tie}} + \frac{1}{1.25\,V_{str}}\right)^{-1} \tag{15}$$

where

$$V_{str} = v\, f_c' A_{str} \sin\theta_s \tag{16}$$

$$V_{tie} = \alpha\left(A_s f_y + F_{ct}\right)\tan\theta_s \tag{17}$$

where $\alpha = k_2/k_1$. Using a nonlinear optimization approach, e.g., the simplex method [27], the appropriate values for α, k_h and k_v in Eq. (1) as well as v were determined as discussed in the next section. Finally, the STM proposed in this study is entitled "the modified interactive strut-and-tie model (MISTM)".

3. Calibration and verification of the MISTM

3.1 Database of RC corbel test results for calibration and verification of the STMs

An extensive database of 302 corbel test results, collected from the published literature [2-7, 28-42] by Rulak [43], was used in this study. This contains the shear strengths and physical information of RC corbels, as listed in Table 2.

Table 2 Sources of experimental corbel data for verification

Researcher	No. of data points
Abdul-Wahab [28]	1
Alameer [29]	2
Campione et al. [30]	10
Chakrabarti et al. [31]	8
Clifford [32]	18
Cook [33]	1
El-Maaddawy &El-Sayed [34]	2
Fattuhi & Hughes [35]	6
Fattuhi & Hughes [36]	6
Fattuhi [37]	6
Fattuhi [38]	2
Fattuhi [39]	9
Foster et al. [5]	23
Her [40]	23
Kriz & Raths [2]	127
Lu et al. [6]	13
Mattock et al. [3]	24
Nagrodzka-Godycka [41]	3
Tan & Mansur [42]	3
Yang et al. [7]	2
Yong & Balaguru [4]	13
Total	302

The statistics of the important parameters and shear strengths are shown in Figure 4. The specimens had compressive strengths of concrete ranging from 15 to 105 MPa, covering the practical ranges of normal and high strength concrete. Additionally, the shear span-to-depth ratios were from 0.1 to 1.8, and the overall depth varied from 100 to 1,000 mm. Finally, the shear strengths of the specimens were from 250 to 2,800 kN.

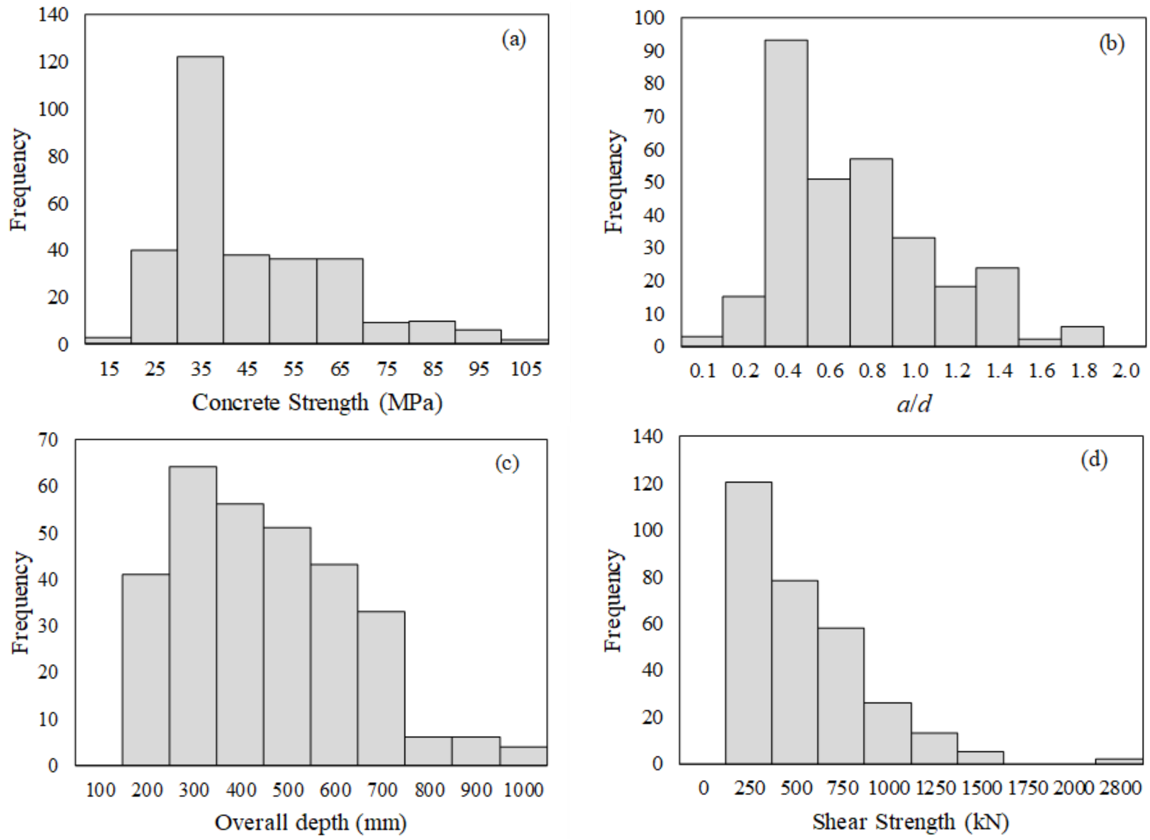

Figure 4 Statistics of the important parameters and shear strengths: (a) concrete strength; (b) *a*/*d* ratio; (c) overall depth, and, (d) shear strength

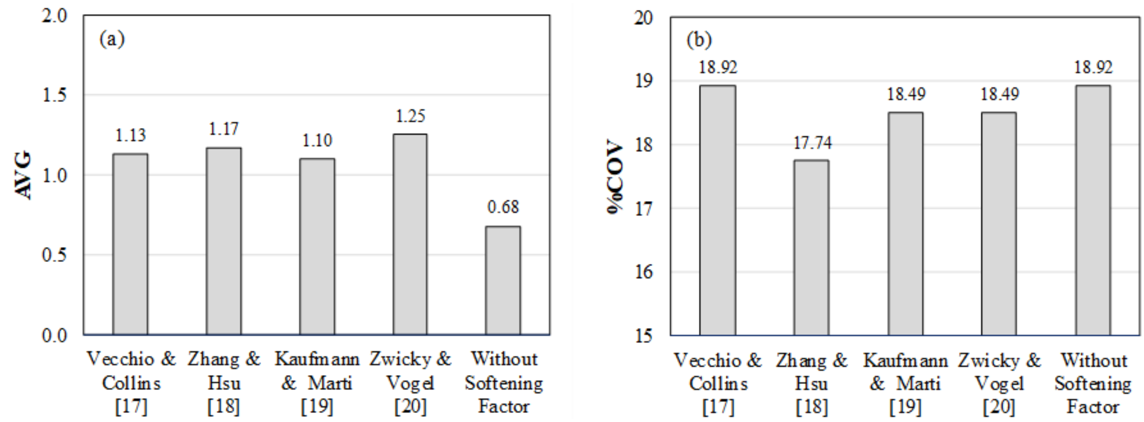

Figure 5 Measurement accuracy of the selected softening factors used in conjunction with MISTM: (a) AVG and (b) COV

3.2 Calibration of the proposed MISTM

To determine the appropriate softening factor and the optimal parameters used in the proposed MISTM, as well as to verify the accuracy of the STMs considered in this study, the average value of the ratio of the test to computed shear strength (AVG) and the related coefficient of variation (COV) are introduced. The AVG value deviated only slightly from unity revealing that the model provides good accuracy, whereas the low values of COV imply a high level of reliability of the model computations. Table 3 shows the appropriate parameters, i.e., the softening factor v, α, k_h and k_v used in conjunction with the MISTM by minimizing the COV. It was found that the α values ranged from 1.94-3.21, while k_h and k_v ranged from 0.25-0.42 and 0.18-0.31, respectively. This may imply that the amount of horizontal web reinforcement has more influence on the shear strength of RC corbels than in the vertical direction.

Table 3 Optimal parameters used in conjunction with the MISTM

Softening Factor v	α	k_h	k_v
Vecchio & Collins [17]	1.94	0.25	0.19
Zhang & Hsu [18]	3.00	0.28	0.18
Kaufmann & Matri [19]	2.47	0.34	0.22
Zwicky & Vogel [20]	2.18	0.30	0.19
Without factor	3.21	0.42	0.31

Additionally, the accuracy of each of the softening factors considered here was investigated, as shown in Figure 5. It can be seen that all softening factors, ν, used in conjunction with the MISTM provided nearly the same level of accuracy. The AVG ranged from 1.10 to 1.25 and %COV from 17 to 18. However, the softening factor proposed by Zhang and Hsu [18] was found to be the most precise, indicated by its lowest value of %COV. Finally, it can also be seen from Figure 5a that the original Kupfer-Gerstle's biaxial failure criterion (without the ν term) clearly overestimates the shear strength (AVG < 1). This can confirm that the use of the modified Kupfer-Gerstle's biaxial failure criterion [16], including the softening factor term ν is reasonable for the MISTM.

4. Comparison with the existing STMs

To compare results of the current study with other prediction models, the STM computed using ACI 318 code (ACI-STM), STM proposed by Russo et al. [10] and the softened strut-and-tie model (SSTM) proposed by Hwang et al. [13] were selected. They are simple STMs that are suitable for practical computations.

4.1 ACI-STM

For the STM computed in accordance with ACI 318-14 code [14], the shear strength of the RC corbel can be written in the form:

$$V_n = 0.85\beta_s f_c' b w_s \sin\theta_s \tag{18}$$

where β_s is the softening factor for strut, i.e., β_s= 0.75 for a bottle-shaped strut with reinforcement satisfying Section 23.5 of ACI 318-14 [14], while β_s= 0.60λ is for a bottle-shaped strut without reinforcement satisfying the same code. The term λ is a factor accounting for the reduction in engineering characteristics of lightweight concrete (LWC), compared to the normal weight concrete (NWC) with the same concrete compression strength. In general, for structural members made from NWC, λ is taken as unity. Additionally, the depth of the inclined strut w_s, following the ACI code, can be written as:

$$w_s = \min\begin{pmatrix} w_t\cos\theta_s + w_p\sin\theta_s \\ h_c\cos\theta_s + w_b\sin\theta_s \end{pmatrix} \tag{19}$$

where w_p , w_b and h_c are the width of the bearing plate, the width and the depth of LNZ, respectively, as shown in Figure 6.

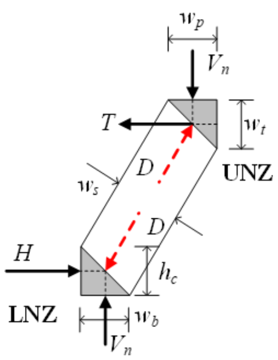

Figure 6 Dimensions of the UNZ and LNZ used in the ACI-STM [14]

4.2 Russo et al. approach

Russo et al. [10] proposed a simple STM using a two-load transfer mechanism to predict corbel shear strengths. The first is the direct strut mechanism representing the portion of concrete in compression, while the second is the horizontal truss mechanism provided by the horizontal web reinforcement. However, the model of Russo et al. [10] is valid for RC corbels having the shear span-to-depth less than unity and was calibrated with a database of 243 test data points. The nominal shear strength can be expressed as:

$$V_u = 0.80\left(k\chi f_c'\cos\phi + 0.65\rho_h f_{yh}\cot\phi\right) \tag{20}$$

where χ represents the simplified Zhang and Hsu [18] softening factor given by:

$$\chi = \left(0.74\left(\frac{f_c'}{105}\right)^3 - 1.28\left(\frac{f_c'}{105}\right)^2 + 0.22\left(\frac{f_c'}{105}\right) + 0.87\right) \tag{21}$$

The term ϕ is the inclination angle of the concrete strut with respect to the vertical axis.

4.3 Hwang et al. approach

Hwang et al. [13] suggested a simplified version of the SSTM previously proposed by Hwang et al. [8] and Hwang and Lee [9], to compute the shear strength of RC corbels as well as other D-region members. In this approach, a geometric simplification of the strut-and-tie index, K, describing the contribution of horizontal and vertical web reinforcement to the whole member strength, was introduced as:

$$K = \tan^A\theta_s + \cot^A\theta_s - 1 + 0.14B \leq 1.64 \tag{22}$$

$$\text{where} \quad A = 12\frac{\rho f_y}{f_c'} \leq 1; \quad B = 30\frac{\rho f_y}{f_c'} \leq 1; \tag{23}$$

The terms ρ and f_y in Eq. (22) can be defined as:

$$\text{for } \theta_s \geq 45 \text{ degrees, } \rho = \rho_h, f_y = f_{yh} \tag{24}$$

$$\text{for } \theta_s < 45 \text{ degrees, } \rho = \rho_v, f_y = f_{yv} \tag{25}$$

Then, the shear strength of RC corbels computed using Hwang et al. [13] can be written as:

$$V_n = K\,v\,f_c' b(kd)\sin\theta_s \tag{26}$$

where v denotes the Zhang and Hsu [18]'s softening factor.

The predicted shear strengths of 302 corbels obtained from the selected the STMs and the MISTM were compared with the test results illustrated in Figure 7. The accuracy measures, i.e. AVG and COV, are also shown. This figure reveals that ACI-STM [14] tends to provide a very large conservative bias and large uncertainty, indicated by high values of AVG and COV. This is due to the relatively small cross-sectional areas of the diagonal concrete struts computed using this method, as discussed by Chetchotisak et al. [26] and Park and Kuchma [44].

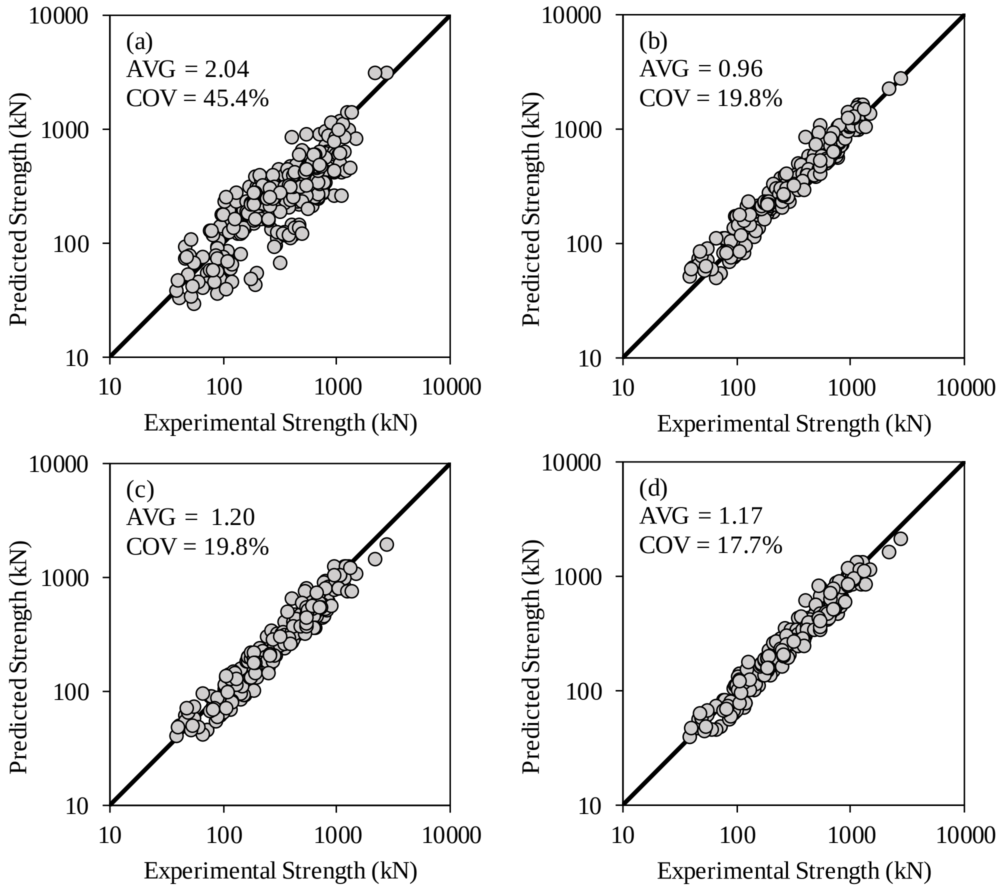

Figure 7 Plots of the predicted versus the experimental strength of corbels: (a) ACI-STM [14]; (b) Russo et al. [10]; (c) Hwang et al. [13] and (d) MISTM

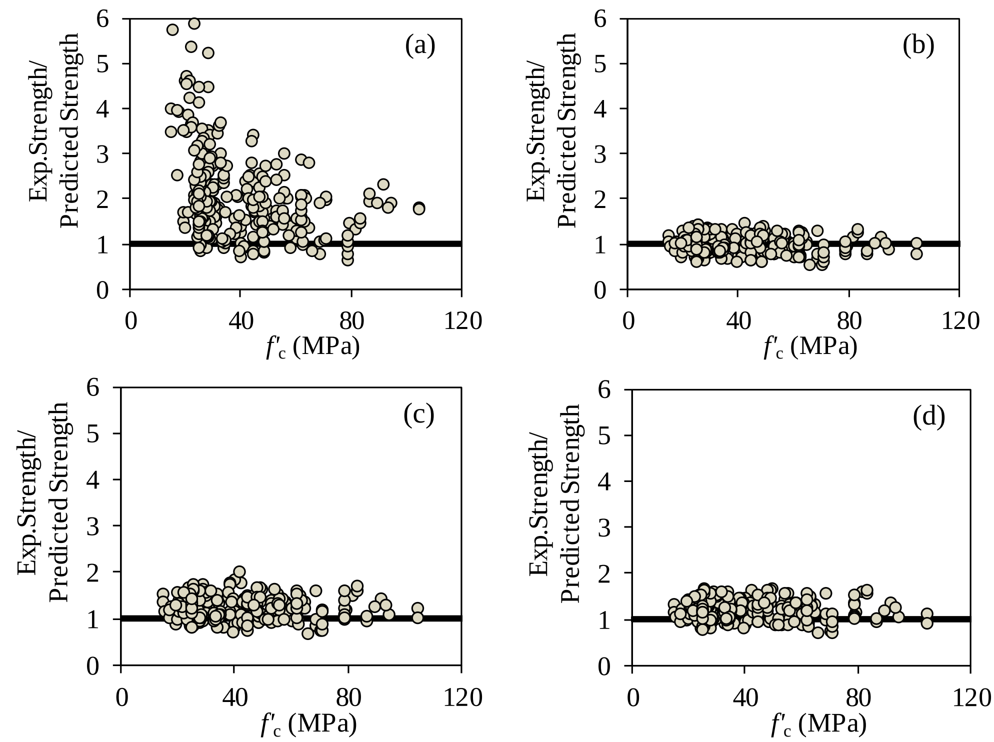

Figure 8 Effect of concrete strength on the shear strength predictions using various STMs: (a) ACI-STM [14]; (b) Russo et al. [10]; (c) Hwang et al. [13] and (d) MISTM

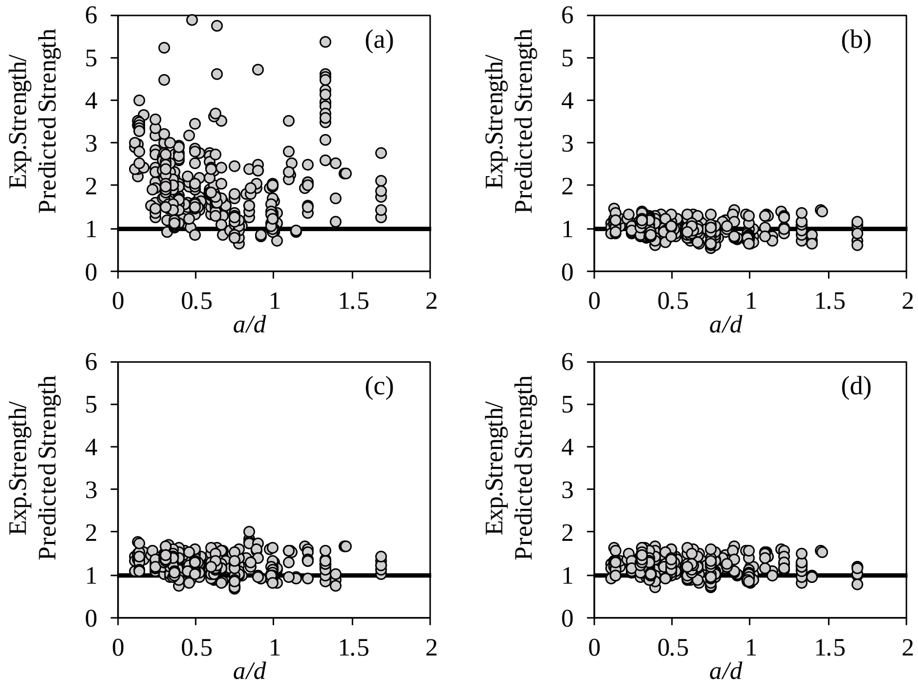

Figure 9 Effect of *a*/*d* on the shear strength predictions using various STMs: (a) ACI-STM [14] and (b) Russo et al. [10] (c) Hwang et al. [13] and (d) MISTM

The remaining STMs all exhibited nearly the same prediction accuracy indicated by the AVG and COV values. One reason for this is that they applied the inclined depth of the compression strut as kd, and the Zhang and Hsu [18] softening factor in conjunction with their STMs. However, the MISTM used a more appropriate approach to consider the softening effect, i.e., the modified Kupfer and Gerstle's bi-axial failure criterion. Thus, the effect of the tension tie strength is included in the MISTM, while the other STMs do not consider this. Consequently, the MISTM provides relatively better consistency as indicated by its lowest COV (Figure 7d).

To investigate the uniformity of the STMS, the shear strength predictions were plotted versus various parameters affecting the shear strength of D-region members [8, 10], i.e., concrete strength and shear span-to-depth ratio, as illustrated in Figures 8 and 9, respectively. These figures indicate that the ACI-STM [14] produces considerably scattered predictions over the range of considered parameters. Especially, one underestimated the shear strength for corbels having a low concrete compressive strength (Figure 8a). In contrast, the prediction obtained from the STMs proposed by Russo et al. [10], Hwang et al. [13] and the MISTM resulted in a good consistency as illustrated in Figures 8b-d and 9b-d, respectively.

5. Conclusions

An alternative approach for predicting the shear strength of RC corbels, developed from a modified Kupfer and Gerstle's biaxial failure criterion for concrete, is proposed in this study. The STMs from ACI 318 code [14], Russo et al. [10] and Hwang et al. [13] were used to benchmark the proposed STM. The main conclusions of this study are:

- By including a softening factor to modify the Kupfer-Gerstle's biaxial failure criterion of concrete, the model was found to be more accurate and reliable than the original failure criterion for developing the MISTM to predict the shear strength of RC corbels.
- The MISTM has a better accuracy and uniformity in shear strength prediction of RC corbels than the STM computed from ACI 318-14 [14], Russo et al. [10] and Hwang et al. [13]. The mean and COV of the tests to predicted strength ratio using the MISTM are 1.17 and 17.7%, respectively.
- The STM computed in accordance with ACI 318 code was found to be very conservative and inconsistent.
- The MISTM may be beneficial for structural design practice owing to its precision, uniformity, and simplicity.

6. Acknowledgement

The authors are grateful to the Sustainable Infrastructure Research and Development Center (SIRDC), Khon Kaen University, Thailand for providing financial support for this research program.

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