

## Developing Instruction Based on Open Approach and Its Impact on Levels of Geometric Thinking and Geometric Achievement of Eighth - Grade Students

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### ABSTRACT

*This research studied the impact of the instruction based on the open approach on van Hiele geometric thinking levels and on the geometric achievement of eight-grade students. The researcher also traced changes in geometric thinking level according to the four constructs: recognition, definition, classification and proof. Quantitative and qualitative evidence revealed that (a) there was the increasing number of students who exhibited Level 3 of geometric thinking and the corresponding decrease in the number of students exhibiting Level 2 of geometric thinking, (b) there were no significant differences on geometric achievement between the experimental and the control group, (c) target students in Level 2 of geometric thinking made progress to Level 3 of geometric thinking in some constructs: definition and classification and (d) there were two sociomathematical norms. The first sociomathematical norm was what counted as a valid way of showing that the triangles were congruent. The classroom mathematical practices which were compatible with this norm were the use of measurement, the use of fit on top and the use of reasoning. The second sociomathematical norm was what constituted a valid proof. The classroom mathematical practices which were compatible with this norm were the use of drawing and examples and chain of reasoning.*

**Key words:** van Hiele Geometric Thinking Level, Open approach, Open-ended problems, Middle School, Reasoning

### INTRODUCTION

Geometry plays an important role in school mathematics curriculum. Many students in various parts of the world have been facing difficulties in learning geometry. Pierre van Hiele and Dian van Hiele-Geldof (1959/1985) formulated a model to explain why students had those difficulties. They proposed five levels of geometric thinking which were visualization, analysis, informal deduction, deduction and rigor (Crowley, 1987). After their discovery, various studies have been conducted to verify and elaborate the theory including that of Gutierrez and Jaime (1998). They analyzed the list of descriptors of van Hiele Theory from many publications. They proposed a framework of geometric thinking with different key constructs across the levels : recognition, definition, classification and proof. According to this model, students' thinking in geometry progresses sequentially through levels. Several studies (Burger and Shaughnessy, 1986; Fuys et al., 1988) and also a study in Thailand (Chaiyasang, 1987) found that most middle school students were functioning at lower levels of thinking than they should be. In fact, they should be capable of logical deduction which is consistent with van Hiele Level 3 of geometric thinking, i.e., informal deduction. However,

they have not reached that level. The possible cause may arise from ineffective teaching. In Thailand, teachers spend much time on teaching students to remember facts, definitions, theorems rather than to solve problems. Nodha (2000) stated that students' mathematical thinking evolves through problem solving. He suggested a teaching method called the "open approach" which engages students to solve non-routine open-ended problems, problems with various ways to solve or correct answers. This approach provides an opportunity for students to solve problems by their own mathematical thinking as well as to see a variety of solution from other students.

Participating in mathematical community help students develop their thinking (Baroody and Caslick, 1998). While engaging students to work in small groups and discussing with the whole class, students are able to explain their reasoning and make sense of other students' reasoning. In this essence, the negotiation between teacher and students or students and students creates sociomathematical norms and classroom mathematical practices which helps students develop their thinking (Cobb, 1999).

### PURPOSES OF THE STUDY

This study was designed to address the following purposes:

1. Develop the instruction based on the open approach and to study its impact on levels of geometric thinking of eight-grade students.
2. Evaluate the effect of the instruction based on the open approach on geometric achievement of eight-grade students.
3. Trace changes in target students' levels of geometric thinking.
4. Examine sociomathematical norms and classroom mathematical practices emerged during the instruction.

### THEORETICAL CONSIDERATION

In conceptualizing this study, the researcher has drawn on three theoretical perspectives. First, the researcher adopts van Hiele geometric thinking level and Gutierrez and Jaime (1998) framework which characterizes students' thinking across four key constructs. Second, the instruction is informed by the open approach (Nodha, 2000). Finally, to study the evolution of students' collective thinking, the researcher adopts the emergent perspective of Cobb (1999) to examine growth in students' geometric thinking from both the psychological and the social perspectives.

#### van Hiele Geometric Thinking

van Hiele's proposed five levels of geometric thinking are as follow:

*Level 1 (Visualization).* Students identify and operate on shapes and other geometric configurations according to their appearances. Students say that a given figure is a rectangle, for instance, because "it looks like a door". Students at this level include imprecise visual qualities and irrelevant attributes, such as orientation, in describing the shapes while omitting relevant attributes.

*Level 2 (Analysis).* At this level, students recognize and can characterize shapes by their properties. For instance, a student may think that a square is a figure that has four equal sides and four right angles. Students establish properties experimentally by observing, measuring, drawing and model making.

*Level 3 (Informal deduction).* Students can form abstract definitions, distinguish between necessary and sufficient sets of conditions for a concept and understand and some-

times even provide logical arguments in the geometric domain. They can classify figures hierarchically (by ordering their properties) and give informal arguments to justify their classifications; a square, for example, is identified as a rhombus because it can be thought of as a “rhombus with some extra properties”.

*Level 4 (Deduction).* Students can establish theorems within an axiomatic system. They recognize the difference among undefined terms, definitions, axioms and theorems. They are capable of constructing original proofs.

*Level 5 (Rigor).* At this level, students reason formally about mathematical systems. They can study geometry in the absence of reference models and they can reason by formally manipulating geometric statements such as axioms, definitions and theorems.

*Gutierrez and Jaime Framework*

Gutierrez and Jaime (1998) stated that the level of van Hiele geometric thinking is integrated by several key thinking processes. They proposed a framework for assessing the van Hiele geometric thinking levels by regarding how a student considers and uses the following thinking constructs:

**1. Recognition.** Recognition of types and families of geometric figures, identification of components and properties of the figures.

**2. Definition.** Definition of a geometric concept. This construct can be viewed in two ways: as the students use a given definition read in a textbook, heard from the teacher or other students, and as the students formulate definition of the concept they are learning.

**3. Classification.** Classification of geometric figures or concepts into different families or classes.

**4. Proof.** Proof of properties or statements by explaining in some convincing way why such a property or statement is true.

They summarized the main characteristics of each construct, used to distinguish among students at the different van Hiele levels in Table 1.

**Table 1.** The main characteristics of each construct across van Hiele levels.

	<b>Level 1</b>	<b>Level 2</b>	<b>Level 3</b>	<b>Level 4</b>
<b>Recognition</b>	Physical attributes	mathematical properties	-----	-----
<b>Use of definitions</b>	-----	Only definitions with simple structure	Any definition	Accept several equivalent definitions
<b>Formulation of definitions</b>	List of physical properties	List of mathematical properties	Set of necessary and sufficient properties	Can prove the equivalence of definition
<b>Classification</b>	Exclusive, based on physical attributes	Exclusive, based on mathematical attributes	Can move among inclusive and exclusive	-----
<b>Proof</b>	-----	Verification with examples	Informal logical proofs	Formal mathematical proofs

## The open approach

The origin of open approach was the research on evaluation of higher-order thinking in mathematics education, using open-ended problems as a theme in the early of 1970s in Japan. The open approach of Nodha is broader than that in the early one, that is, the problems used include problem situations, process problems, problems with multiple ways to solve; and open-ended problems, problems with multiple correct answers; and procedures for using these problems including classroom conditions and teaching objectives (Inprasitha, 1997).

### *Open-ended problems*

The problems used in the open approach are non-routine problems that are classified into three types:

1. Process is open. This type of problem has multiple correct ways of solving the original problem. All mathematical problems are inherently open in this sense.
2. End products are open. This type of problem has multiple correct answers.
3. Ways to develop the problem are open. After students have solved the problem, they can develop new problems by changing the conditions or attributes of the original problem.

### *Teaching objectives of the open approach*

The teacher who uses the open approach needs to employ the following processes:

1. understand students' ideas as much as possible.
2. enrich the ideas during mathematics activities by means of students negotiations with others and/or teachers' advice.
3. encourage their self- determination in elaborating the activity mathematically.

### *Pattern of teaching*

Stigler and Hiebert (1999) analyzed the pattern of teaching mathematics in Japan that is consistent with the style of teaching in the open approach. The Japanese lessons often follow a sequence of five activities:

- Reviewing the previous lesson
- Presenting the problem for the day
- Students working individually or in groups
- Discussing solution methods
- Highlighting and summarizing the major points

Hashimoto and Becker (1999) (cited by Conway, 1996) suggested that teachers should prepare detailed lesson plans organized as follow:

- presenting the problems or topics and directions,
- understanding the problems,
- problem solving by students using their own natural mathematical thinking ability,
- comparing and discussing students' solutions and,
- presenting a summary of the lesson.

In this study, the researcher modified the teaching style of Japanese lessons by adding 'working individually' to the instructional sequence as follow:

- Reviewing the previous lesson
- Presenting and understanding the problems
- Solving the problems
- Presenting and discussing the solutions
- Summarizing the lessons
- Working individually

**The emergent perspective**

The emergent perspective proposed by Cobb (1999) is based on the assumption that learning can be characterized as both the process of active individual construction and a process of mathematical enculturation. Therefore, students’ mathematical developments - increasingly-sophisticated ways of reasoning- are seen to be related to their participation in particular communities of practices. Cobb developed the interpretive framework which correlated the social perspective and psychological perspective as shown in Table 2.

**Table 2.** The interpretive framework.

Social Perspective	Psychological Perspective
Classroom social norms	Beliefs about our own role, others’ role, and the general nature of mathematical activity
Sociomathematical norms	Specific mathematical beliefs and values
Classroom mathematical practices	Mathematical conceptions and activity

**METHODOLOGY**

**Instrumentation**

**The van Hiele Geometric Thinking Test.** This test was adapted from Usiskin (1982) from the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) Project. It comprised of 20 multiple- choice items with five alternatives in each item. It was discriminated into four levels with five items in each level. The test was used to assess levels of geometric thinking of students in the experimental group before and after the instruction.

**The Geometric Achievement Test.** This test was constructed by the researcher. It comprised of 20 multiple-choice and 3 written items in congruent triangle, parallel lines and parallelograms. The reliability of the test was 0.72. The difficulty indices were between 0.47 –0.78 and the discrimination indices were between 0.20–0.36.

**The instructional program: lesson plans constructed based on the open approach.** The researcher constructed the lesson plans with twelve 100-min sessions with over 8 weeks. The lesson plans comprised the learning objectives, instructional activities and the expected responses from the students. In typical session, the researcher opens with reviewing the previous lesson, presenting the open-ended problems, students solving the problems and presenting their solutions and discussion. The open-ended problem is informed according to the topic of congruent triangle, parallel lines and parallelograms and also from the constructs: recognition, definition, classification and proof.

**Participants**

The participants of this study were eight-grade students and a witness. The students were randomly selected from two classes among five classes with mixed ability of eight-grade students in the Demonstration School of Chiang Mai University, Thailand. One class with 40 students was randomly assigned to be the experimental group and another class with 42 students was randomly assigned to be the control group. The experimental group was taught by the instruction constructed by the researcher while the control group was taught by another teacher with the conventional approach. Both classes used the same contents and exercises. One witness, a mathematics teacher, helped the researcher to observe the experimental class.

In the experimental group, six case-study students: three from Level 2 and three from Level 3 of geometric thinking were chosen purposively to study changes in their level of geometric thinking according to the constructs.

### Procedure

Two weeks before the teaching started, all students in the experimental group were administered the van Hiele Geometric Test to assess the level of geometric thinking. Most students were at Level 2 and 3. The researcher selected purposively one group which all three students were at Level 2 of geometric thinking and another group which all three students were at Level 3 of geometric thinking to be case-study students. Six case-study students were called the target students.

During the instruction, the researcher followed the lesson plans. Two video cameras and two audio recorders were used to record target students' problem solving behaviors. The witness observed the classes.

After the instruction, the van Hiele Geometric Test was administered again to the experimental group. The Geometric Achievement Test was administered to both the experimental and control groups.

### Data analysis

Data of this study were from the following sources: (a) before and after the van Hiele Geometric Test and the Geometric Achievement Test after the instruction; (b) videotapes of target students and classroom events; (c) students' written work; and (d) witness' reflections. There were two parts for data analysis. The first part was concerned with students' responses in both tests. The second part was concerned with analysis of target students' thinking according to the constructs in Gutierrez and Jaime Framework during the instruction. This analysis focused on students' individual and collective thinking and also identified the classroom sociomathematical norms and classroom practices emerged during the instruction.

## RESULTS

The results were provided according to the purposes of this study:

1. The impact of the instruction on students' levels of geometric thinking; The result was shown in Table 3.

**Table 3.** Frequencies of geometric thinking levels (n=40).

	Levels			Total
	1	2	3	
Before the instruction	1	16	23	40
After the instruction	1	7	32	40

Table 3 presented the frequencies of geometric thinking levels for all students (n=40) prior to and following the implementation of the instruction based on the open approach. The result showed that there was a significant difference on levels of geometric thinking and the instruction based on the open approach ( $\chi^2_{df=2} = 14.10, p < .05$ ). This showed that after the instruction, there was an increase in the number of students exhibiting Level 3 of geometric thinking and the corresponding decrease in the number of students exhibiting Level 2 of geometric thinking.

2. The effect of the instruction on students' geometric achievement; there was no significant difference on geometric achievement between the experimental and control group ( $p = .432$ ).

3. Changes in target students' levels of geometric thinking; only target students in Level 2 made progress to Level 3 of geometric thinking. Thus, the researcher studied in depth only in this group. The results of changes in each construct of target students were as follow:

**Recognition.** All students coherently and consistently recognized figures from their physical attributes which indicated Level 1 of geometric thinking to the use of mathematical properties which indicated Level 2 of geometric thinking before, after and during the instruction.

**Definition.** Students showed that they were able to know sets of properties associated with definitions of shapes or the necessary conditions for the congruent triangles which indicated Level 2 of geometric thinking in this construct before and during the instruction. They were able to exhibit Level 3 of geometric thinking: interrelate the properties between classes of figures and understand necessary and sufficient conditions for the congruent triangle after the instruction.

**Classification.** All students showed that they had improved to Level 3 of geometric thinking in this construct. Students also exhibited the idea of inclusive relationships among classes of triangles and classes of quadrilaterals.

**Proof.** Students exhibited at Level 2 thinking for proof. They often used several examples or measurements to verify the truth of statements. They also used the patty paper to verify the congruence of triangles. However, students were able to show some aspects of proof in Level 3, for example, they were able to understand if-then statements, apply the properties to solve problem, follow the proof but not be able to construct their informal proof.

4. The emergence of sociomathematical norms and classroom mathematical practices: two sociomathematical norms emerged, accompanied with their classrooms mathematical practices. The first sociomathematical norm was "what counts as a valid way of showing that triangles are congruent". The classrooms mathematical practices accompanied with this sociomathematical norm were the use of measurements, the use of fit on top and the use of reasoning. The second sociomathematical norm was "what constitutes a valid proof" accompanied with the classroom mathematical practices: the use of drawing and examples as well as chain of reasoning.

## DISCUSSION

Regarding to students' geometric thinking performance, the findings of this study revealed that in the experimental group, most students who made changes in level, including target students, developed their levels of geometric thinking to the higher level, i.e., from Level 2 to Level 3. As Gutierrez and Jaime (1998) stated that the van Hiele levels of thinking were integrated by several processes: recognition, definition, classification and proof, thus, it is possible that students may progress in the mastering of some processes but not of others, so he or she progresses in some process on the higher level. In accordance to this study, the researcher examined each process and found that target students mastered on some processes: definition and classification but not on proof in Level 3 thinking.

Another factor that possibly made students change to the higher level is the processes of the instruction based on the open approach. After students solved the open-ended problems, they have to present and discuss with the whole class. This kind of processes helps students develop their thinking. For example, students were asked to construct the triangle with three given segments and then compare their triangle with those constructed by their friends whether the triangles were congruent. After that, they were asked to construct the

triangles with the three given angles as well as were asked to compare them. When students had presentation, in case of the triangles which were constructed by the three given segments, they agreed to conclude that the triangles were congruent, that was, by Side-Side-Side Theorem. On the other hand, in case of the triangles which were constructed by the three given angles, some group showed that their triangles were congruent but some groups did not. From this point, students discussed why the results were not the same. After discussion with the teacher in whole class, students were able to conclude that when they had three angles congruent, the triangles might or might not be congruent.

In addition, the factor which is possible to help students progress to the higher level is the social factors, sociomathematical norms. From the environment provided by the researcher, students had an opportunity to discuss what counts as a valid way to show the triangles congruent and what counts as a valid proof. Followed from their discussion, the classroom mathematical practices which was compatible to this sociomathematical norm had evolved to the sophisticated level, that was, from using measurement to the use of reasoning. This might be because students have opportunity to negotiate with each other during the discussion that leads to the evolution of sophisticated level.

However, there were a few students who decreased to the lower level and the majority of students were at the same level before and after the instruction. The possible factors which impeded students' progress are students' previous experiences and the classroom culture (Nodha, 2000). There are two factors that influence student's previous experiences : their learning style and teaching strategies and the use of mathematics textbooks (Inprasitha, 1997).

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