Chiang Mai J. Sci. 2016; 43(1) : 257-269
http://epg.science.cmu.ac.th/ejournal/
Contributed Paper

# Improved Modified Ratio Estimators of Population Mean Based on Deciles

**Muhammad Abid*[a,b], Nasir Abbas [c] and Muhammad Riaz [d]**

[a] Department of Mathematics, Institute of Statistics, Zhejiang University, Hangzhou 310027, China.

[b,c] Department of Statistics, Government College University, Faisalabad, Pakistan.

[d] Department of Mathematics and Statistics, King Fahad University of Petroleum and Minerals, Saudi Arabia.

*Author for correspondence; e-mail: mhmmd_abid@yahoo.com

Received: 9 November 2014

Accepted: 9 March 2015

## ABSTARCT

Different population characteristics of the auxiliary variables have so far been employed to develop ratio estimators for estimating population mean of the study variable. This article comprises developing some new modified ratio estimators using the linear combinations of coefficient of variation, population correlation coefficient and deciles of the auxiliary variable. The mean square errors of all the proposed ratio estimators and the efficiency conditions are also derived. Numerical illustrations have been made to support the findings of the study. From theoretical and numerical findings, it is noted that the proposed estimators are more efficient as compared to all the existing estimators used in this study.

**Keywords:** bias, coefficient of variation, correlation coefficient, deciles, mean squared error, population

## 1. INTRODUCTION

A process used in statistical analysis in which a predetermined number of observations are taken from a larger population is called sampling. The purpose is to reduce the cost and/or the amount of work that it would take to survey the entire target population. In survey research, there are situations when the information is available on every unit in the population. If a variable that is known for every unit of the population is not a variable of interest but is instead employed to improve the sampling plan or to enhance estimation of the variables of interest, then it is called an auxiliary variable. The term auxiliary variable is most commonly associated with the use of such variables which are available for all the units in the population, in ratio, product and regression estimation. Auxiliary information is often used to improve the efficiency of estimators in survey sampling. The ratio estimator is most effective for estimating population mean when there is linear relationship between study variable and auxiliary variable in the form of positive correlation. The variable of interest or the variable about which we want to draw some inference is called a study variable.

Consider a finite population $U = \{U_1, U_2, U_3, \dots, U_N\}$ of $N$distinct and identifiable units. Let $Y$ be the study variable with value $Y_i$ measured of $U_i, i = 1,2,\dots,N$ giving a vector $Y = \{Y_1, Y_2, Y_3, \dots, Y_N\}$. The objective is to estimate population mean $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$ on the basis of a random sample. If the population parameters of the auxiliary variable $X$ such as population mean, co-efficient of variation, co-efficient of kurtosis, co-efficient of skewness, median, quartiles, population correlation coefficient, deciles etc., are known. We provide here the complete list of notations to be used in this paper:

## NOMENCLATURE

**Roman**

| | |
|---|---|
| $N$ | Population size |
| $n$ | Sample size |
| $f = {}^{n}/_{N}$ | Sampling fraction |
| $X$ | Auxiliary variable |
| $Y$ | Study variable |
| $X$ | Auxiliary variable |
| $\bar{X}, \bar{Y}$ | Population means |
| $\bar{x}, \bar{y}$ | Sample means |
| $x, y$ | Sample totals |
| $S_x = \sqrt{\frac{\sum_{i=1}^{N}(X_i-\bar{X})^2}{N}}, S_y = \sqrt{\frac{\sum_{i=1}^{N}(Y_i-\bar{Y})^2}{N}}$ | Population standard deviations for $X$ and $Y$ |
| $S_{xy}$ | Population covariance between $X$ and $Y$ |
| $C_x = \frac{S_x}{\bar{X}}, C_y = \frac{S_y}{\bar{Y}}$ | Coefficient of variation for $X$ and $Y$ |
| $B(.)$ | Bias of the Estimator |
| $MSE(.)$ | Mean square error of the estimator |
| $D_s = s \times \frac{((N+1))}{10}$ th value | Decile of auxiliary variable |
| $\hat{\bar{Y}}_i$ | Existing estimators of Kadilar and Cingi [2, 3], Yan and Tian [14] |
| $\hat{\bar{Y}}_{Dj}$ | Existing estimators Subramani and Kumarapandiyan [12] |
| $\hat{\bar{Y}}_{pk}$ | Proposed estimator of class-I |
| $\hat{\bar{Y}}_{pl}$ | Proposed estimator of class-II |

**Subscript**

| | |
|---|---|
| $i$ | For existing estimator of Kadilar and Cingi [2, 3], Yan and Tian [14] |
| $j$ | For existing estimator of Subramani and Kumarapandiyan [12] |
| $k$ | For proposed estimator of class-I |
| $l$ | For proposed estimator of class-II |

**Greek**

| | |
|---|---|
| $\rho = \frac{S_{xy}}{S_x S_y}$ | Coefficient of correlation |
| $\beta_1 = \frac{N\sum_{i=1}^{N}(X_i-\bar{X})^3}{(N-1)(N-2)S^3}$ | Coefficient of skewness of auxiliary variable |
| $\beta_2 = \frac{N(N+1)\sum_{i=1}^{N}(X_i-\bar{X})^4}{(N-1)(N-2)(N-3)S^4} - \frac{3(N-1)^2}{(N-2)(N-3)}$ | Coefficient of kurtosis of auxiliary variable |

Based on the above mentioned notations, the mean ratio estimator for estimating the population mean, $\bar{Y}$, of the study variable $Y$ is defined as

$$\hat{\bar{Y}}_r = \frac{\bar{y}}{\bar{x}}\bar{X} \tag{1}$$

The bias, constant and the mean square error (MSE) of the ratio estimator are respectively given by

$$B\left(\hat{\bar{Y}}_r\right) = \frac{(1-f)}{n}\frac{1}{\bar{X}}\left(RS_x^2 - \rho S_x S_y\right), R = \frac{\bar{Y}}{\bar{X}} MSE\left(\hat{\bar{Y}}_r\right) = \frac{(1-f)}{n}\left(S_y^2 + R^2 S_x^2 - 2R\rho S_x S_y\right)$$

The ratio estimator given in (1) is used for improving the precision of the estimate of the population mean compared to simple random sampling whenever a positive correlation exist between the study variable and the auxiliary variable. Cochran [1] suggested a classical ratio type estimator for the estimation of finite population mean using one auxiliary variable under simple random sampling scheme. Murthy [4] proposed a product type estimator by considering dual property like ratio method of estimation where $\bar{x}$and $\bar{y}$are unbiased estimators of the population means $\bar{X}$ and respectively. He suggested that the one should used product estimator to estimatethe population mean or total of study variable *y* by using auxiliary information when coefficient of correlation is negative. Prasad [5] proposed ratio type estimator when values of coefficient of variation of study variable, coefficient of variation of auxiliary variable and the population correlation coefficient $\rho$ is at hand. Rao [6] suggested difference type estimator that performs better than conventional linear regression estimator and get an improvement as compared to ratio and regression estimators. Singh and Tailor [8] proposed a family of estimators using known values of some parameters by using SRSWOR for estimation of population mean of the study variable. Singh et.al [9], Sisodia and Dwivedi [10] utilized coefficient of variation of the auxiliary variate. Upadhyaya and Singh [13] derived ratio type estimators using coefficient of variation and coefficient of kurtosis of the auxiliary variate. Further improvements are achieved by introducing a large number of modified ratio estimators with the use of known coefficient of variation, coefficient of kurtosis, coefficient of skewness, deciles, etc. In addition, you can also see the work of Rana et al. [11] which developed a new measure of central tendency based on deciles.

The organization of this article is as follows: In Section 2, we describe about the existing estimators. In Section 3, we provide the structure of our proposed modified ratio estimator and the efficiency comparison of the proposed estimator with the existing estimator. In Section 4, we provide an empirical study of our proposed estimator. Finally, we close with a summary conclusion in the last section. In the next section, we give the biases, constants and the mean squared errors of the existing modified ratio estimator.

## 2. MATERIALS AND METHODS

Kadilar and Cingi [2] suggested the following ratio estimators for the population mean $\bar{Y}$ of the variate of interest *y* in simple random sampling using some auxiliary information.

$$\hat{\bar{Y}}_1 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{\bar{x}}\bar{X}, \hat{\bar{Y}}_2 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+C_x)}(\bar{X}+C_x),$$

$$\hat{\bar{Y}}_3 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\beta_2)}(\bar{X}+\beta_2), \hat{\bar{Y}}_4 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+C_x)}(\bar{X}\beta_2+C_x),$$

$$\hat{\bar{Y}}_5 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\beta_2)}(\bar{X}C_x+\beta_2).$$

The biases, constants and the mean squared errors of the Kadilar and Cingi [2] are given below:

$$B\left(\hat{\bar{Y}}_1\right)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_1^2,\ B\left(\hat{\bar{Y}}_2\right)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_2^2,\ B\left(\hat{\bar{Y}}_3\right)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_3^2,\ B\left(\hat{\bar{Y}}_4\right)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_4^2,$$

$$B\left(\hat{\bar{Y}}_5\right)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_5^2.$$

$$R_1=\frac{\bar{Y}}{\bar{X}},\ R_2=\frac{\bar{Y}}{(\bar{X}+C_x)},\ R_3=\frac{\bar{Y}}{(\bar{X}+\beta_2)},\ R_4=\frac{\bar{Y}\beta_2}{(\bar{X}\beta_2+C_x)},\ R_5=\frac{\bar{Y}C_x}{(\bar{X}C_x+\beta_2)}.$$

$$MSE\left(\hat{\bar{Y}}_1\right)=\frac{(1-f)}{n}\left(R_1^2S_x^2+S_y^2(1-\rho^2)\right),\ MSE\left(\hat{\bar{Y}}_2\right)=\frac{(1-f)}{n}\left(R_2^2S_x^2+S_y^2(1-\rho^2)\right),$$

$$MSE\left(\hat{\bar{Y}}_3\right)=\frac{(1-f)}{n}\left(R_3^2S_x^2+S_y^2(1-\rho^2)\right),\ MSE\left(\hat{\bar{Y}}_4\right)=\frac{(1-f)}{n}\left(R_4^2S_x^2+S_y^2(1-\rho^2)\right),$$

$$MSE\left(\hat{\bar{Y}}_5\right)=\frac{(1-f)}{n}\left(R_5^2S_x^2+S_y^2(1-\rho^2)\right).$$

Kadilar and Cingi [3] also developed some modified ratio estimators using coefficient of correlation which are shown below:

$$\hat{\bar{Y}}_6=\frac{\bar{y}+b(\bar{X}-\bar{x})}{\bar{x}}(\bar{X}+\rho),\ \hat{\bar{Y}}_7=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\rho)}(\bar{X}C_x+\rho),\ \hat{\bar{Y}}_8=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+C_x)}(\bar{X}\rho+C_x),$$

$$\hat{\bar{Y}}_9=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+\rho)}(\bar{X}\beta_2+\rho),\ \hat{\bar{Y}}_{10}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\beta_2)}(\bar{X}\rho+\beta_2).$$

The biases, constants and the mean squared errors of Kadilar and Cingi [3] are specified below:

$$B\left(\hat{\bar{Y}}_6\right)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_6^2,\ B\left(\hat{\bar{Y}}_7\right)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_7^2,\ B\left(\hat{\bar{Y}}_8\right)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_8^2,\ B\left(\hat{\bar{Y}}_9\right)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_9^2,$$

$$B\left(\hat{\bar{Y}}_{10}\right)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{10}^2.$$

$$R_6=\frac{\bar{Y}}{\bar{X}+\rho},\ R_7=\frac{\bar{Y}C_x}{(\bar{X}C_x+\rho)},\ R_8=\frac{\bar{Y}\rho}{(\bar{X}\rho+C_x)},\ R_9=\frac{\bar{Y}\beta_2}{(\bar{X}\beta_2+\rho)},\ R_{10}=\frac{\bar{Y}\rho}{(\bar{X}\rho+\beta_2)}.$$

$$MSE\left(\hat{\bar{Y}}_6\right)=\frac{(1-f)}{n}\left(R_6^2S_x^2+S_y^2(1-\rho^2)\right),\ MSE\left(\hat{\bar{Y}}_7\right)=\frac{(1-f)}{n}\left(R_7^2S_x^2+S_y^2(1-\rho^2)\right),$$

$$MSE\left(\hat{\bar{Y}}_8\right)=\frac{(1-f)}{n}\left(R_8^2S_x^2+S_y^2(1-\rho^2)\right),\ MSE\left(\hat{\bar{Y}}_9\right)=\frac{(1-f)}{n}\left(R_9^2S_x^2+S_y^2(1-\rho^2)\right),$$

$$MSE\left(\hat{\bar{Y}}_{10}\right)=\frac{(1-f)}{n}\left(R_{10}^2S_x^2+S_y^2(1-\rho^2)\right).$$

Yan and Tian [14] proposed the modified ratio estimators using coefficient of skewness and kurtosis;

$$\hat{\bar{Y}}_{11}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\beta_1)}(\bar{X}+\beta_1),\ \hat{\bar{Y}}_{12}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_1+\beta_2)}(\bar{X}\beta_1+\beta_2).$$

The biases, constants and the mean squared errors of Yan and Tian [14] are given below:

$$B\left(\hat{\bar{Y}}_{11}\right)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{11}^2,\ B\left(\hat{\bar{Y}}_{12}\right)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{12}^2.$$

$$R_{11}=\frac{\bar{Y}}{(\bar{X}+\beta_1)},\ R_{12}=\frac{\bar{Y}\beta_1}{(\bar{X}\beta_1+\beta_2)}.$$

$$MSE\left(\hat{\bar{Y}}_{11}\right)=\frac{(1-f)}{n}\left(R_{11}^2S_x^2+S_y^2(1-\rho^2)\right),\ MSE\left(\hat{\bar{Y}}_{12}\right)=\frac{(1-f)}{n}\left(R_{12}^2S_x^2+S_y^2(1-\rho^2)\right).$$

Subramani and Kumarapandiyan [12] suggested a class of estimators with the use of population deciles of auxiliary information in simple random sampling for estimation of population mean.

$\hat{\bar{Y}}_{Dj} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+D_j)}\left(\bar{X} + D_j\right)$, where $j = 1,2, \dots 10$.

Whereas the bias, constant and the mean square error is as under;

$B\left(\hat{\bar{Y}}_j\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{Dj}^2$ . $R_{Dj} = \frac{\bar{Y}}{(\bar{X}+D_j)}$ . $MSE\left(\hat{\bar{Y}}_j\right) = \frac{(1-f)}{n}\left(R_{Dj}^2 S_x^2 + S_y^2(1-\rho^2)\right)$.

where $j = 1,2, \dots 10$.

Motivated by the estimators in Subramani and Kumarapandiyan [12], we propose two classes of modified ratio estimators using the known value of the population deciles, coefficient of variation and population correlation coefficient of the auxiliary variable. A decile is any of the nine values that divide the sorted data into ten equal parts, so that each part represents 1/10 of the sample or population.

## 3. RESULIS

### 3.1 The Suggested Estimator

In this section, we have suggested two classes of modified ratio type estimators using the population deciles, coefficient of variation and population correlation coefficient.

#### 3.1.1 Proposed Estimator of Class – I.

The proposed estimators by using the linear combination of coefficient of correlation and the deciles in a general form are as under;

$\hat{\bar{Y}}_{pk} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+D_k)}(\bar{X}\rho + D_k)$, where $k = 1,2, \dots 10$.

The biases, constants and mean squared errors of the new modified ratio estimators using a linear combination of a population correlation coefficient and the deciles are specified below:

$B\left(\hat{\bar{Y}}_{pk}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{pk}^2$ . $R_{pk} = \frac{\bar{Y}\rho}{(\bar{X}\rho+D_{pk})}$ . $MSE\left(\hat{\bar{Y}}_{pk}\right) = \frac{(1-f)}{n}\left(R_{pk}^2 S_x^2 + S_y^2(1-\rho^2)\right)$.

where$k = 1,2, \dots 10$.

We have derived the conditions for which the proposed estimators of class-I is more efficient than the existing modified ratio estimators

$MSE\left(\hat{\bar{Y}}_{pk}\right) < MSE\left(\hat{\bar{Y}}_{Dj}\right) < MSE\left(\hat{\bar{Y}}_i\right)$ If $R_{pk} < R_{Dj} < R_i$ where $k = 1,2, \dots 10$,

where $j = 1,2, \dots 10$ and $i = 1,2, \dots 12$.

**Theorem 3.1.1:** The proposed estimator of class-I i.e. $\hat{\bar{Y}}_{pk}$ perform better than Subramani and ANDKumarapandiyan [12] estimator i.e. $\hat{\bar{Y}}_{Dj}$ if $\rho < 1$. where $k = 1,2, \dots 10$ and $j = 1,2, \dots 10$.

**Proof:** $R_{pk} < R_{Dj} \Rightarrow \frac{\bar{Y}\rho}{\bar{X}\rho+D_k} < \frac{\bar{Y}}{\bar{X}+D_j} \Rightarrow \frac{\rho}{\bar{X}\rho+D_k} < \frac{1}{\bar{X}+D_j} \Rightarrow \rho\bar{X} + \rho D_j < \bar{X}\rho + D_k$

$\Rightarrow \rho D_j < D_k \Rightarrow \rho < \frac{D_k}{D_j}$

where $k = j \Rightarrow \rho < 1$

**3.1.2 Proposed Estimator of Class – II**

The proposed estimators of class-II by using the linear combination of coefficient of variation and the deciles are specified below;

$\hat{\hat{Y}}_{pl} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+D_k)}(\bar{X}C_x + D_l)$. where $l = 1,2, \dots 10.$

The biases, constants and mean squared errors of the new modified ratio estimators using a linear combination of a population coefficient of variation and the deciles are mentioned below:

$B\left(\hat{\hat{Y}}_{pl}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{pl}^2 \,.\, R_{pk} = \frac{\bar{Y}C_x}{(\bar{X}C_x+D_{pl})} \,.\, MSE\left(\hat{\hat{Y}}_{pl}\right) = \frac{(1-f)}{n}\left(R_{pl}^2 S_x^2 + S_y^2(1-\rho^2)\right).$
where $l = 1,2, \dots 10.$

We have derived the conditions for which the proposed estimators of class-II is more efficient than the existing modified ratio estimators

$MSE\left(\hat{\hat{Y}}_{pl}\right) < MSE\left(\hat{\hat{Y}}_{Dj}\right) < MSE\left(\hat{\hat{Y}}_i\right)$ If $R_{pl} < R_{Dj} < R_i$ where $l = 1,2, \dots 10,$
where $j = 1,2, \dots 10$ and $i = 1,2, \dots 12.$

**Theorem 3.1.2:** The proposed estimator of class-II i.e. $\hat{\hat{Y}}_{pl}$ perform better than Subramani and Kumarapandiyan [12] estimator i.e. $\hat{\hat{Y}}_{Dj}$ if$C_x < 1$. where $l = 1, 2, \dots 10$ and $j = 1, 2, \dots 10.$

**Proof:** $R_{pl} < R_{Dj} \Rightarrow \frac{\bar{Y}C_x}{\bar{X}C_x+D_l} < \frac{\bar{Y}}{\bar{X}+D_j} \Rightarrow \frac{C_x}{\bar{X}C_x+D_l} < \frac{1}{\bar{X}+D_j} \Rightarrow C_x\bar{X} + C_xD_j < \bar{X}C_x + D_l$

$\Rightarrow C_xD_j < D_l \Rightarrow C_x < \frac{D_k}{D_j}$ where $l = j \Rightarrow C_x < 1$

We have also derived the conditions in which the proposed estimators of class-I and II are more efficient than the usual ratio estimator.

**Theorem 3.1.3:** The proposed estimator of class-I i.e. $\hat{\hat{Y}}_{pk}$ perform better than the usual ratio estimator i.e. $\hat{\hat{Y}}_r$

If $\left(\frac{\rho S_y - RS_x}{S_x}\right) \leq R_{pk} \leq \left(\frac{RS_x - \rho S_y}{S_x}\right)$ or $\left(\frac{RS_x - \rho S_y}{S_x}\right) \leq R_{pk} \leq \left(\frac{\rho S_y - RS_x}{S_x}\right)$. where $k = 1,2, \dots 10.$

**Proof:** $MSE\left(\hat{\hat{Y}}_{pj}\right) \leq MSE\left(\hat{\hat{Y}}_r\right) \Rightarrow \frac{(1-f)}{n}\left(R_{pj}^2 S_x^2 + S_y^2(1-\rho^2)\right)$

$$\leq \frac{(1-f)}{n}\left(S_y^2 + R^2S_x^2 - 2R\rho S_xS_y\right)$$

$$\Rightarrow R_{pk}^2 S_x^2 - \rho^2 S_y^2 - R^2 S_x^2 + 2R\rho S_xS_y \leq 0$$

$$\Rightarrow \left(\rho S_y - RS_x\right)^2 - R_{pk}^2 S_x^2 \geq 0$$

$$\Rightarrow \left(\rho S_y - RS_x + R_{pk}S_x\right)\left(\rho S_y - RS_x - R_{pk}S_x\right) \geq 0.$$

Condition I: $\left(\rho S_y - RS_x + R_{pk}S_x\right) \leq 0$ and $\left(\rho S_y - RS_x - R_{pk}S_x\right) \leq 0$ After simplifying condition I, we get

$$\Rightarrow \left(\frac{\rho S_y - RS_x}{S_x}\right) \leq R_{pk} \leq \left(\frac{RS_x - \rho S_y}{S_x}\right)$$

Condition II: $\left(\rho S_y - RS_x + R_{pk}S_x\right) \geq 0$ and $\left(\rho S_y - RS_x - R_{pk}S_x\right) \geq 0$ After solving condition II, we get

$$\Rightarrow \left(\frac{RS_x-\rho S_y}{S_x}\right) \leq R_{pk} \leq \left(\frac{\rho S_y-RS_x}{S_x}\right)$$

Hence, $\Rightarrow MSE\left(\hat{\hat{\bar{Y}}}_{pj}\right) \leq MSE\left(\hat{\bar{Y}}_r\right) \Rightarrow \left(\frac{\rho S_y-RS_x}{S_x}\right) \leq R_{pk} \leq \left(\frac{RS_x-\rho S_y}{S_x}\right)$

or $\left(\frac{RS_x-\rho S_y}{S_x}\right) \leq R_{pk} \leq \left(\frac{\rho S_y-RS_x}{S_x}\right)$.

On the same lines we will show that the proposed estimators of class-II are more efficient than the usual ratio estimators. The only difference in the proof that instead of $R_{pk}$ we will mention $R_{pl}$.

## 4. DISCUSSION

The performance of the proposed modified ratio estimators and the existing modified ratio estimators is evaluated by using the four populations. They are: The population 1 is the closing price of the industry ACC in the National Stock Exchange from 2, January 2012 to 27, February 2012, population 2 and population 3 are taken from Singh and Chaudhary [7] page 177, population 4 are taken from Murthy [4] page 228 The characteristics of the four populations are given below in table 1, whereas the constants, the biases and MSEs of the existing and proposed modified ratio estimators are given in tables 2-8.

The percentage relative efficiencies (PREs) of the proposed estimators (p) with respect to the existing estimators (e) can be computed as

$$PRE(e,p) = \frac{MSE(e)}{MSE(p)} * 100 \qquad (2)$$

The PREs of the population 1 is given in table 9-13 for the new modified proposed estimators of class-I and II. The PREs of the other three populations i.e. population 2, population 3 and population 4 can also be found by using the expression given in equation (2).

The information contained in Table 7 and 8 discloses that the constants, biases and MSEs for the proposed ratio estimators are smaller as compared with the usual ratio estimator and the existing ratio estimators. Moreover, these values even decrease with increase in the decile orders. From Table 9 and 11 it becomes evident that the PREs of the proposed class-I and class-II ratio estimators with regards to the existing ones are much higher, which indicates that they are more efficient. From Table 10 and 12 it can be seen that the PREs of the proposed class-I and II ratio estimators with regards to those proposed in Subramani and Kumarapandiyan [12] are better and more efficient. From Table 13 it can be observed that PREs of the proposed class-II estimators with regards to the proposed class-I estimators are much higher, which shows that they are more efficient for population 1. The PREs of the class-I estimators with respect to the class-II estimators can also be found by using the other three populations i.e. population 2, population 3 and population 4.

The comparison of proposed modified ratio estimators of class-I and II and the existing modified ratio estimators is also shown by graphically. From Figures 1-2, it can be seen that the proposed estimators of class-I and II have a lesser values of biases as compared to the existing ratio estimators. It can also be observed that the proposed estimators of class-I and II have a smaller values of MSEs as compared to the existing ratio estimators (cf. Figures 3-4)

## CONCLUSION

In this paper, we have proposed a class of modified ratio estimators using known values of population deciles, coefficient of variation and population correlation coefficient by using the information on the study variable and the auxiliary variable. It is observed that the mean squared errors of the suggested estimators based on the deciles, population correlation coefficient and coefficient of variation of the auxiliary variable are less than the usual ratio estimator and the mean squared errors of the existing modified ratio estimators for all the four known populations considered for the numerical study (see from table. 2-8). We have also observed from tables 9-12 that our new modified proposed estimators are more efficient than the existing estimators. Also, we know that the parameters like the mean, coefficient of skewness and coefficient of kurtosis are affected by the extreme values in the population, while deciles are robustness to extreme values. Hence, we strongly recommend our proposed modified ratio estimators over the existing modified ratio estimators for the use of practical applications.

## ACKNOWLEDGEMENTS

The authors are thankful to the editor anonymous reviewers for their constructive comments that helped in significantly improving the previous version of the paper. The author Muhammad Riaz is also indebted to King Fahd University of Petroleum and Minerals Dhahran Saudi Arabia for providing excellent research facilities.

**Table 1**. Characteristics of the populations.

| Parameter | Population 1 | Population 2 | Population 3 | Population 4 |
|---|---|---|---|---|
| $N$ | 40 | 34 | 34 | 80 |
| $n$ | 20 | 20 | 20 | 40 |
| $\bar{Y}$ | 5141.5363 | 856.4117 | 856.4117 | 5182.637 |
| $\bar{X}$ | 1221.6463 | 208.8823 | 199.4412 | 1126.463 |
| $\rho$ | 0.9244 | 0.4491 | 0.4453 | 0.9413 |
| $S_y$ | 256.1464 | 733.1407 | 733.1407 | 1845.659 |
| $C_y$ | 0.04982 | 0.8561 | 0.8561 | 0.354193 |
| $S_x$ | 102.5494 | 150.5059 | 150.2150 | 845.6097 |
| $C_x$ | 0.0839 | 0.7205 | 0.7531 | 0.7506772 |
| $\beta_2$ | -1.5154 | 0.0978 | 1.0445 | -0.063386 |
| $\beta_1$ | 0.3761 | 0.9782 | 1.1823 | 1.050002 |
| $D_1$ | 1111.8150 | 70.3 | 60.60 | 369.7 |
| $D_2$ | 1119.48 | 76.8 | 83.00 | 460.4 |
| $D_3$ | 1139.20 | 108.2 | 102.70 | 597 |
| $D_4$ | 1159.84 | 129.4 | 111.20 | 676.8 |
| $D_5$ | 1184.2250 | 150.0 | 142.50 | 757.5 |
| $D_6$ | 1252.55 | 227.2 | 210.20 | 850.2 |
| $D_7$ | 1307.100 | 250.4 | 264.50 | 1484.5 |
| $D_8$ | 1345.72 | 335.6 | 304.40 | 1810 |
| $D_9$ | 1366.7850 | 436.1 | 373.20 | 2500 |
| $D_{10}$ | 1389.300 | 564.0 | 634.00 | 3485 |

**Table 2**. MSE, bias and constant of the usual ratio estimator.

| Estimator | Constant | | | | Bias | | | | MSE | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Population | | | | | | | | | | | |
| | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| $\hat{\bar{Y}}_r$ | 4.209 | 4.100 | 4.294 | 4.601 | 0.0971 | 4.2704 | 4.9407 | 60.8770 | 1187.5 | 10539.3 | 10960.8 | 189775.1 |

**Table 3**. MSE, biases and constants of existing modified ratio estimators by Kadilar and Cingi [2].

| Estimators | Constant | | | | Bias | | | | MSE | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Population | | | | | | | | | | | |
| | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| $\hat{\bar{Y}}_1$ | 4.209 | 4.100 | 4.294 | 4.601 | 0.9057 | 9.1539 | 10.0023 | 36.5063 | 4895.6 | 16673.5 | 17437.7 | 193998.1 |
| $\hat{\bar{Y}}_2$ | 4.208 | 4.086 | 4.278 | 4.598 | 0.9056 | 9.0911 | 9.9272 | 36.4577 | 4894.9 | 16619.6 | 17373.3 | 193746.2 |
| $\hat{\bar{Y}}_3$ | 4.214 | 4.098 | 4.272 | 4.601 | 0.9080 | 9.1454 | 9.8983 | 36.5104 | 4907.2 | 16666.1 | 17348.6 | 194019.4 |
| $\hat{\bar{Y}}_4$ | 4.209 | 3.960 | 4.279 | 4.650 | 0.9058 | 8.5387 | 9.9303 | 37.2861 | 4896.0 | 16146.6 | 17376.0 | 198039.9 |
| $\hat{\bar{Y}}_5$ | 4.272 | 4.097 | 4.264 | 4.601 | 0.9331 | 9.1420 | 9.8646 | 36.5117 | 5036.3 | 16663.3 | 17319.8 | 194026.4 |

**Table 4**. MSE, biases and constants of existing modified ratio estimators by Kadilar and Cingi [3].

| Estimators | Constant | | | | Bias | | | | MSE | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Population | | | | | | | | | | | |
| | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| $\hat{\bar{Y}}_6$ | 4.206 | 4.091 | 4.284 | 4.597 | 0.9044 | 9.1147 | 9.9578 | 36.4453 | 4888.5 | 16639.9 | 17399.5 | 193682.3 |
| $\hat{\bar{Y}}_7$ | 4.171 | 4.088 | 4.281 | 4.596 | 0.8896 | 9.0995 | 9.9432 | 36.4251 | 4812.7 | 16626.9 | 17387.1 | 193577.6 |
| $\hat{\bar{Y}}_8$ | 4.208 | 4.069 | 4.258 | 4.598 | 0.9056 | 9.0149 | 9.8348 | 36.4546 | 4894.9 | 16554.4 | 17294.2 | 193730.5 |
| $\hat{\bar{Y}}_9$ | 4.211 | 4.011 | 4.285 | 4.662 | 0.9067 | 8.7630 | 9.9597 | 37.4882 | 4900.2 | 16338.7 | 17401.1 | 199087.0 |
| $\hat{\bar{Y}}_{10}$ | 4.214 | 4.096 | 4.244 | 4.601 | 0.9082 | 9.1349 | 9.7711 | 36.5106 | 4908.1 | 16654.2 | 172239.7 | 194020.7 |

**Table 5**. MSE, biases and constants of existing modified ratio estimators by Yan and Tian [14].

| Estimators | Constant | | | | Bias | | | | MSE | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Population | | | | | | | | | | | |
| | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| $\hat{\bar{Y}}_{11}$ | 4.207 | 4.081 | 4.269 | 4.597 | 0.9052 | 9.0688 | 9.8847 | 36.4383 | 4892.7 | 16600.5 | 17337.0 | 193645.9 |
| $\hat{\bar{Y}}_{12}$ | 4.223 | 4.098 | 4.275 | 4.601 | 0.9118 | 9.1452 | 9.9143 | 36.5102 | 4926.5 | 16666.0 | 17362.3 | 194018.4 |

**Table 6**. MSE, biases and constants of existing modified ratio estimators by Subramani and Kumarapandiyan [12].

| Estimators | Constant | | | | Bias | | | | MSE | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Populations | | | | | | | | | | | |
| | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| $\hat{\bar{Y}}_{D1}$ | 2.203 | 3.068 | 3.293 | 3.464 | 0.2483 | 5.1243 | 5.8836 | 20.6939 | 1515.0 | 13222.5 | 13910.4 | 112048.6 |
| $\hat{\bar{Y}}_{D2}$ | 2.196 | 2.998 | 3.032 | 3.266 | 0.2466 | 4.8938 | 4.9874 | 18.3960 | 1506.7 | 13025.0 | 13142.8 | 100138.9 |
| $\hat{\bar{Y}}_{D3}$ | 2.178 | 2.701 | 2.834 | 3.007 | 0.2425 | 3.9725 | 4.3582 | 15.5954 | 1485.6 | 12236.1 | 12604.0 | 85624.8 |
| $\hat{\bar{Y}}_{D4}$ | 2.159 | 2.532 | 2.757 | 2.874 | 0.2383 | 3.4902 | 4.1230 | 14.2457 | 1464.1 | 11823.0 | 12402.5 | 78629.5 |
| $\hat{\bar{Y}}_{D5}$ | 2.137 | 2.386 | 2.505 | 2.751 | 0.2335 | 3.1010 | 3.4027 | 13.0514 | 1439.4 | 11489.7 | 11785.7 | 72439.9 |
| $\hat{\bar{Y}}_{D6}$ | 2.078 | 1.964 | 2.091 | 2.622 | 0.2208 | 2.1003 | 2.3709 | 11.8559 | 1374.0 | 10632.6 | 10902.1 | 66244.4 |
| $\hat{\bar{Y}}_{D7}$ | 2.033 | 1.865 | 1.846 | 1.985 | 0.2114 | 1.8934 | 1.8484 | 6.7952 | 1325.5 | 10455.5 | 10454.6 | 40016.2 |
| $\hat{\bar{Y}}_{D8}$ | 2.003 | 1.573 | 1.700 | 1.765 | 0.2051 | 1.3472 | 1.5673 | 5.3722 | 1293.1 | 9987.7 | 10213.8 | 32641.5 |
| $\hat{\bar{Y}}_{D9}$ | 1.986 | 1.328 | 1.496 | 1.429 | 0.2018 | 0.9601 | 1.2133 | 3.5224 | 1276.0 | 9656.2 | 9910.7 | 23054.5 |
| $\hat{\bar{Y}}_{D10}$ | 1.977 | 1.108 | 1.028 | 1.124 | 0.1998 | 0.6686 | 0.5727 | 2.1783 | 1266.0 | 9406.6 | 9362.1 | 16088.9 |

**Table 7**. MSE, biases and constants of new modified ratio estimators of class-I.

| Estimators | Constant | | | | Bias | | | | MSE | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Population | | | | | | | | | | | |
| | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| $\hat{\bar{Y}}_{p1}$ | 2.121 | 2.344 | 2.552 | 3.411 | 0.2300 | 2.9912 | 3.5341 | 20.0707 | 1421.1 | 11395.7 | 11898.3 | 108818.5 |
| $\hat{\bar{Y}}_{p2}$ | 2.114 | 2.254 | 2.220 | 3.208 | 0.2284 | 2.7676 | 2.6727 | 17.7480 | 1413.1 | 11204.2 | 11160.5 | 96780.7 |
| $\hat{\bar{Y}}_{p3}$ | 2.095 | 1.904 | 1.991 | 2.944 | 0.2245 | 1.9741 | 2.1511 | 14.9429 | 1392.7 | 10524.6 | 10713.8 | 82243.2 |
| $\hat{\bar{Y}}_{p4}$ | 2.076 | 1.723 | 1.907 | 2.808 | 0.2204 | 1.6169 | 1.9722 | 13.6016 | 1372.0 | 10218.7 | 10560.6 | 75291.5 |
| $\hat{\bar{Y}}_{p5}$ | 2.054 | 1.578 | 1.649 | 2.684 | 0.2158 | 1.3553 | 1.4746 | 12.4208 | 1348.2 | 9994.6 | 10134.4 | 69171.7 |
| $\hat{\bar{Y}}_{p6}$ | 1.995 | 1.198 | 1.275 | 2.553 | 0.2036 | 0.7818 | 0.8824 | 11.2447 | 1285.5 | 9503.5 | 9627.3 | 63076.7 |
| $\hat{\bar{Y}}_{p7}$ | 1.951 | 1.117 | 1.079 | 1.917 | 0.1946 | 0.6800 | 0.6320 | 6.3378 | 1239.1 | 9416.3 | 9412.9 | 37646.0 |
| $\hat{\bar{Y}}_{p8}$ | 1.920 | 0.896 | 0.969 | 1.700 | 0.1886 | 0.4369 | 0.5103 | 4.9819 | 1208.2 | 9208.1 | 9308.6 | 30618.7 |
| $\hat{\bar{Y}}_{p9}$ | 1.904 | 0.726 | 0.825 | 1.370 | 0.1854 | 0.2869 | 0.3696 | 3.2380 | 1191.9 | 9079.7 | 9188.1 | 21580.8 |
| $\hat{\bar{Y}}_{p10}$ | 1.895 | 0.585 | 0.528 | 1.073 | 0.1836 | 0.1862 | 0.1510 | 1.9867 | 1182.4 | 8993.4 | 9000.9 | 15095.6 |

**Table 8**. MSE, biases and constants of new modified ratio estimators of class-II.

| Estimators | Constant | | | | Bias | | | | MSE | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Population | | | | | | | | | | | |
| | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| $\hat{\bar{Y}}_{p1}$ | 0.355 | 2.795 | 3.060 | 3.201 | 0.0065 | 4.2530 | 5.0783 | 17.6739 | 271.8 | 12476.2 | 13220.7 | 96397.0 |
| $\hat{\bar{Y}}_{p2}$ | 0.353 | 2.715 | 2.766 | 2.979 | 0.0064 | 4.0132 | 4.1497 | 15.3043 | 271.4 | 12270.9 | 12425.4 | 84116.2 |
| $\hat{\bar{Y}}_{p3}$ | 0.348 | 2.385 | 2.550 | 2.697 | 0.0062 | 3.0981 | 3.5284 | 12.5432 | 270.4 | 11487.2 | 11893.3 | 69806.4 |
| $\hat{\bar{Y}}_{p4}$ | 0.342 | 2.205 | 2.467 | 2.555 | 0.0060 | 2.6466 | 3.3027 | 11.2627 | 269.4 | 11100.5 | 11700.0 | 63170.1 |
| $\hat{\bar{Y}}_{p5}$ | 0.335 | 2.053 | 2.204 | 2.427 | 0.0058 | 2.2962 | 2.6341 | 10.1574 | 268.2 | 10800.4 | 11127.5 | 57441.3 |
| $\hat{\bar{Y}}_{p6}$ | 0.319 | 1.634 | 1.790 | 2.294 | 0.0052 | 1.4535 | 1.7375 | 9.0772 | 265.3 | 10078.7 | 10359.6 | 51843.3 |
| $\hat{\bar{Y}}_{p7}$ | 0.306 | 1.539 | 1.555 | 1.670 | 0.0048 | 1.2901 | 1.3123 | 4.8079 | 263.3 | 9938.8 | 9995.4 | 29716.9 |
| $\hat{\bar{Y}}_{p8}$ | 0.298 | 1.269 | 1.419 | 1.465 | 0.0045 | 0.8775 | 1.0920 | 3.7015 | 262.0 | 9585.5 | 9806.8 | 23983.0 |
| $\hat{\bar{Y}}_{p9}$ | 0.294 | 1.052 | 1.232 | 1.163 | 0.0044 | 0.6026 | 0.8238 | 2.3322 | 261.3 | 9350.0 | 9577.1 | 16886.1 |
| $\hat{\bar{Y}}_{p10}$ | 0.291 | 0.864 | 0.822 | 0.898 | 0.0043 | 0.4062 | 0.3670 | 1.3919 | 260.9 | 9181.8 | 9185.9 | 12013.1 |

**Table 9**. The percentage relative efficiency of existing estimators versus the proposed class-I.

| Existing Estimators | Proposed Estimators | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\hat{\bar{Y}}_{p1}$ | $\hat{\bar{Y}}_{p2}$ | $\hat{\bar{Y}}_{p3}$ | $\hat{\bar{Y}}_{p4}$ | $\hat{\bar{Y}}_{p5}$ | $\hat{\bar{Y}}_{p6}$ | $\hat{\bar{Y}}_{p7}$ | $\hat{\bar{Y}}_{p8}$ | $\hat{\bar{Y}}_{p9}$ | $\hat{\bar{Y}}_{p10}$ |
| $\hat{\bar{Y}}_{1}$ | 344.5 | 346.5 | 351.5 | 356.8 | 363.1 | 380.8 | 395.1 | 405.2 | 410.8 | 414.0 |
| $\hat{\bar{Y}}_{2}$ | 344.4 | 346.4 | 351.5 | 356.8 | 363.1 | 380.8 | 395.0 | 405.2 | 410.7 | 414.0 |
| $\hat{\bar{Y}}_{3}$ | 345.3 | 347.3 | 352.3 | 357.7 | 364.0 | 381.7 | 396.0 | 406.2 | 411.7 | 415.0 |
| $\hat{\bar{Y}}_{4}$ | 344.5 | 346.5 | 351.5 | 356.9 | 363.1 | 380.9 | 395.1 | 405.2 | 410.8 | 414.1 |
| $\hat{\bar{Y}}_{5}$ | 354.4 | 356.4 | 361.6 | 367.1 | 373.5 | 391.8 | 406.4 | 416.9 | 422.6 | 425.9 |
| $\hat{\bar{Y}}_{6}$ | 344.0 | 346.0 | 351.0 | 356.3 | 362.6 | 380.3 | 394.5 | 404.6 | 410.2 | 413.5 |
| $\hat{\bar{Y}}_{7}$ | 338.7 | 340.6 | 345.6 | 350.8 | 357.0 | 374.4 | 388.4 | 398.4 | 403.8 | 407.0 |
| $\hat{\bar{Y}}_{8}$ | 344.4 | 346.4 | 351.5 | 356.8 | 363.1 | 380.8 | 395.0 | 405.2 | 410.7 | 414.0 |
| $\hat{\bar{Y}}_{9}$ | 344.8 | 346.8 | 351.8 | 357.2 | 363.5 | 381.2 | 395.5 | 405.6 | 411.1 | 414.4 |
| $\hat{\bar{Y}}_{10}$ | 345.4 | 347.3 | 352.4 | 357.7 | 364.0 | 381.8 | 396.1 | 406.2 | 411.8 | 415.1 |
| $\hat{\bar{Y}}_{11}$ | 344.3 | 346.3 | 351.3 | 356.6 | 362.9 | 380.6 | 394.8 | 405.0 | 410.5 | 413.8 |
| $\hat{\bar{Y}}_{12}$ | 346.7 | 348.6 | 353.7 | 359.1 | 365.4 | 383.2 | 397.6 | 407.8 | 413.3 | 416.7 |

**Table 10**. Percentage relative efficiency of Subramani and Kumarapandiyan [12] estimators versus proposed class-I.

| Population | Proposed Estimator / Existing Estimators | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\hat{\bar{Y}}_{p1}/\hat{\bar{Y}}_{D1}$ | $\hat{\bar{Y}}_{p2}/\hat{\bar{Y}}_{D2}$ | $\hat{\bar{Y}}_{p3}/\hat{\bar{Y}}_{D3}$ | $\hat{\bar{Y}}_{p4}/\hat{\bar{Y}}_{D4}$ | $\hat{\bar{Y}}_{p5}/\hat{\bar{Y}}_{D5}$ | $\hat{\bar{Y}}_{p6}/\hat{\bar{Y}}_{D6}$ | $\hat{\bar{Y}}_{p7}/\hat{\bar{Y}}_{D7}$ | $\hat{\bar{Y}}_{p8}/\hat{\bar{Y}}_{D8}$ | $\hat{\bar{Y}}_{p9}/\hat{\bar{Y}}_{D9}$ | $\hat{\bar{Y}}_{p10}/\hat{\bar{Y}}_{D10}$ |
| 1 | 106.0 | 106.6 | 106.7 | 106.7 | 106.8 | 106.9 | 107.0 | 107.0 | 107.1 | 107.1 |

**Table 11**. Percentage relative efficiency of existing estimators versus proposed class-II.

| Existing Estimators | Proposed Estimators | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\hat{\bar{Y}}_{p1}$ | $\hat{\bar{Y}}_{p2}$ | $\hat{\bar{Y}}_{p3}$ | $\hat{\bar{Y}}_{p4}$ | $\hat{\bar{Y}}_{p5}$ | $\hat{\bar{Y}}_{p6}$ | $\hat{\bar{Y}}_{p7}$ | $\hat{\bar{Y}}_{p8}$ | $\hat{\bar{Y}}_{p9}$ | $\hat{\bar{Y}}_{p10}$ |
| $\hat{\bar{Y}}_{1}$ | 1800.9 | 1803.6 | 1810.5 | 1817.4 | 1825.3 | 1845.3 | 1859.5 | 1868.7 | 1873.4 | 1876.2 |
| $\hat{\bar{Y}}_{2}$ | 1800.6 | 1803.4 | 1810.3 | 1817.2 | 1825.0 | 1845.0 | 1859.2 | 1868.4 | 1873.2 | 1875.9 |
| $\hat{\bar{Y}}_{3}$ | 1805.1 | 1807.9 | 1814.8 | 1821.7 | 1829.6 | 1849.6 | 1863.8 | 1873.1 | 1877.8 | 1880.6 |
| $\hat{\bar{Y}}_{4}$ | 1801.0 | 1803.8 | 1810.7 | 1817.6 | 1825.4 | 1845.4 | 1859.6 | 1868.8 | 1873.6 | 1876.3 |
| $\hat{\bar{Y}}_{5}$ | 1852.6 | 1855.5 | 1862.6 | 1869.7 | 1877.7 | 1898.3 | 1912.9 | 1922.4 | 1927.3 | 1930.1 |
| $\hat{\bar{Y}}_{6}$ | 1798.3 | 1801.0 | 1807.9 | 1814.8 | 1822.6 | 1842.6 | 1856.8 | 1866.0 | 1870.7 | 1873.5 |
| $\hat{\bar{Y}}_{7}$ | 1770.4 | 1773.1 | 1779.9 | 1786.7 | 1794.4 | 1814.0 | 1828.0 | 1837.0 | 1841.7 | 1844.4 |
| $\hat{\bar{Y}}_{8}$ | 1800.6 | 1803.4 | 1810.3 | 1817.2 | 1825.0 | 1845.0 | 1859.2 | 1868.4 | 1873.1 | 1875.9 |
| $\hat{\bar{Y}}_{9}$ | 1802.6 | 1805.3 | 1812.2 | 1819.2 | 1827.0 | 1847.0 | 1861.2 | 1870.4 | 1875.2 | 1877.9 |
| $\hat{\bar{Y}}_{10}$ | 1805.5 | 1808.2 | 1815.2 | 1822.1 | 1829.9 | 1850.0 | 1864.2 | 1873.4 | 1878.2 | 1880.9 |
| $\hat{\bar{Y}}_{11}$ | 1799.8 | 1802.6 | 1809.5 | 1816.4 | 1824.2 | 1844.2 | 1858.4 | 1867.6 | 1872.3 | 1875.1 |
| $\hat{\bar{Y}}_{12}$ | 1812.2 | 1815.0 | 1821.9 | 1828.9 | 1836.8 | 1856.9 | 1871.2 | 1880.4 | 1885.2 | 1888.0 |

**Table12**. Percentage relative efficiency of Subramani and Kumarapandiyan [12] estimators versus proposed class-II.

| Population | Proposed Estimator / Existing Estimators | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\hat{\bar{Y}}_{p1} / \hat{\bar{Y}}_{D1}$ | $\hat{\bar{Y}}_{p2} / \hat{\bar{Y}}_{D2}$ | $\hat{\bar{Y}}_{p3} / \hat{\bar{Y}}_{D3}$ | $\hat{\bar{Y}}_{p4} / \hat{\bar{Y}}_{D4}$ | $\hat{\bar{Y}}_{p5} / \hat{\bar{Y}}_{D5}$ | $\hat{\bar{Y}}_{p6} / \hat{\bar{Y}}_{D6}$ | $\hat{\bar{Y}}_{p7} / \hat{\bar{Y}}_{D7}$ | $\hat{\bar{Y}}_{p8} / \hat{\bar{Y}}_{D8}$ | $\hat{\bar{Y}}_{p9} / \hat{\bar{Y}}_{D9}$ | $\hat{\bar{Y}}_{p10} / \hat{\bar{Y}}_{D10}$ |
| 1 | 557.3 | 555.1 | 549.4 | 543.5 | 536.7 | 517.9 | 503.5 | 493.6 | 488.3 | 485.2 |

**Table13**. Percentage relative efficiency of proposed class-I versus proposed class-II.

| Population 1 | Proposed Estimators Class-I / Proposed Estimators Class-II | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\hat{\bar{Y}}_{p1}$ | $\hat{\bar{Y}}_{p2}$ | $\hat{\bar{Y}}_{p3}$ | $\hat{\bar{Y}}_{p4}$ | $\hat{\bar{Y}}_{p5}$ | $\hat{\bar{Y}}_{p6}$ | $\hat{\bar{Y}}_{p7}$ | $\hat{\bar{Y}}_{p8}$ | $\hat{\bar{Y}}_{p9}$ | $\hat{\bar{Y}}_{p10}$ |
| $\hat{\bar{Y}}_{p1}$ | 522.8 | 523.6 | 525.6 | 527.5 | 529.9 | 535.7 | 539.7 | 542.4 | 543.9 | 544.7 |
| $\hat{\bar{Y}}_{p2}$ | 519.9 | 520.7 | 522.6 | 524.5 | 526.9 | 532.6 | 536.7 | 539.4 | 540.8 | 541.6 |
| $\hat{\bar{Y}}_{p3}$ | 512.4 | 513.2 | 515.1 | 517.0 | 519.3 | 525.0 | 528.9 | 531.6 | 533.0 | 533.8 |
| $\hat{\bar{Y}}_{p4}$ | 504.8 | 505.5 | 507.4 | 509.3 | 511.6 | 517.2 | 521.1 | 523.7 | 525.1 | 525.9 |
| $\hat{\bar{Y}}_{p5}$ | 496.0 | 496.8 | 498.6 | 500.4 | 502.7 | 508.2 | 512.0 | 514.6 | 516.0 | 516.7 |
| $\hat{\bar{Y}}_{p6}$ | 473.0 | 473.7 | 475.4 | 477.2 | 479.3 | 484.5 | 488.2 | 490.6 | 492.0 | 492.7 |
| $\hat{\bar{Y}}_{p7}$ | 455.9 | 456.6 | 458.2 | 459.9 | 462.0 | 467.1 | 470.6 | 472.9 | 474.2 | 474.9 |
| $\hat{\bar{Y}}_{p8}$ | 444.5 | 445.2 | 446.8 | 448.5 | 450.5 | 455.4 | 458.9 | 461.1 | 462.4 | 463.1 |
| $\hat{\bar{Y}}_{p9}$ | 438.5 | 439.2 | 440.8 | 442.4 | 444.4 | 449.3 | 452.7 | 454.9 | 456.1 | 456.8 |
| $\hat{\bar{Y}}_{p10}$ | 435.0 | 435.7 | 437.3 | 438.9 | 440.9 | 445.7 | 449.1 | 451.3 | 452.5 | 453.2 |

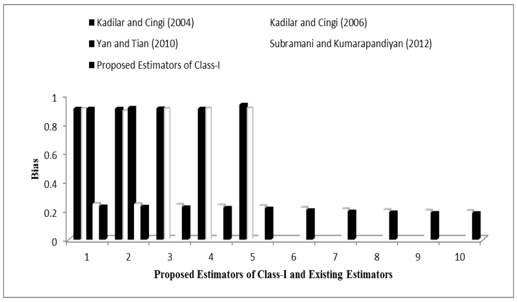


**Figure 1**. Biases of the proposed estimators of class-I and existing estimators.

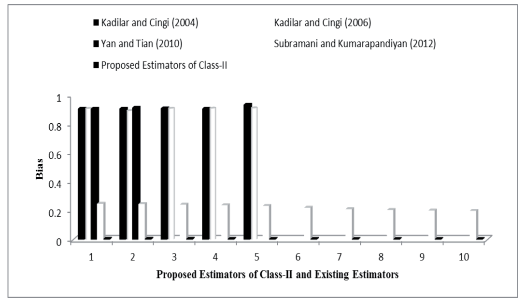


**Figure 2**. Biases of the proposed estimators of class-II and existing estimators.

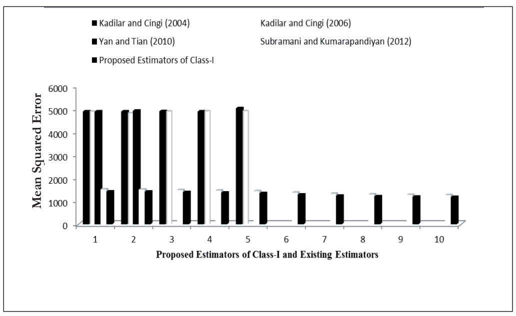


**Figure 3**. Mean squared error of the proposed estimators of class-I and existing estimators.

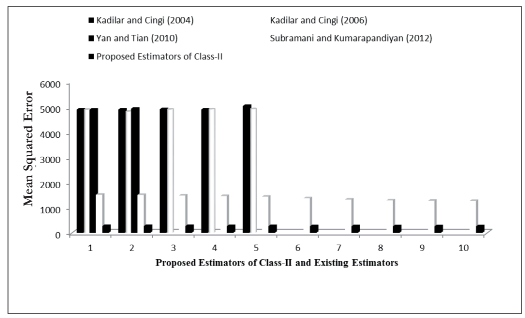


**Figure 4**. Mean squared error of the proposed estimators of class-II and existing estimators.

## REFERENCES

[1] Cochran W. G., *Sampling Techniques*, 3rd Edn., Wiley Eastern Limited, India, 1940.

[2] Kadilar C. and Cingi H., *Appl. Math. Comp.,* 2004; **151**: 893-902. DOI:10.1016/S0096-3003(03)00803-8.

[3] Kadilar C. and Cingi H., *Hacet. J. Math. Stat.,* 2006; **35**: 103-109.

[4] Murthy M.N., *Sampling Theory and Methods*, Statistical Publishing Society, India, 1967.

[5] Prasad B., *Comm. Statist. Theory Methods,*1989; **18**: 379–392.

[6] Rao T.J., *Comm. Statist. Theory Methods,*1991; **20**: 3325–3340. DOI.org/10.1080/03610929108830705.

[7] Singh D. and Chaudhary F.S., *Theory and Analysis of Sample Survey Designs*, New Age International Publisher, India, 1986.

[8] Singh H.P. and Tailor R., *Stat. Tran.*, 2003; **6**: 555-560.

[9] Singh H.P., Tailor R., Tailor R. and Kakran M.S., *J. Indian Soc. Agric. Stat.,* 2004; **58**: 223-230.

[10] Sisodia B.V.S. and Dwivedi V.K., *J. Indian Soc. Agric. Stat.,* 1981; **33**: 13-18.

[11] Sohel, R. Md, S.U.D. Habshah M. and Imon A.H.M.R., *Chiang Mai J. Sci.,* 2012; 39**(3)**: 478-485.

[12] Subramani J. and Kumarapandiyan G., *Int. J. Stat. Appl.*, 2102; **2**: 101-107.

[13] Upadhyaya L.N. and Singh H.P., *Biometric J.*, 1999; **41**: 627-636 DOI: 10.1002/(SICI)1521-4036(199909)41:5<627::AID-BIMJ627>3.0.CO;2-W.

[14] Yan Z. and Tian B., of auxiliary variable *ICICA.*, 2010, **106**, pp. 103–110.