Chiang Mai J. Sci. 2015; 42(3) : 783-795
http://epg.science.cmu.ac.th/ejournal/
Contributed Paper

Tracking of Moving Object Using Energy of Biorthogonal Wavelet Transform

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Received: 4 July 2013
Accepted: 9 January 2014

ABSTRACT

Moving object tracking in video sequence is one of the challenging problems in computer vision. Tracking of moving object in a video is difficult due to random motion of object. Several orthogonal transform based object tracking algorithms have been proposed in literature but those methods are not able to handle object movement properly in consecutive frames due to use of non-symmetric filters. In this paper, we have proposed a moving object tracking algorithm based on the energy of the biorthogonal wavelet coefficients and the movement of object is estimated on the basis of velocity of object in consecutive frames. The approximate shift-invariance and symmetry properties of biorthogonal wavelet transform (BWT) make it useful for tracking of object in the wavelet domain. The shift-invariance property is useful for translated object modeling whereas symmetry property yields in perfect reconstruction retaining object boundaries. In addition the lifting-scheme based design of biorthogonal filters reduces computational cost. The proposed method matches computed energy of biorthogonal wavelet coefficients of object in the first frame to next consecutive frames. The computation of centroid in n^{th} frame is predicted on the basis of centroids of three previous frames. Experimental results demonstrate the better performance of the proposed method against other state-of-the-art methods.

Keywords: object tracking, shift-invariance, video processing, biorthogonal wavelet transform

1. INTRODUCTION

Tracking of moving objects in a video is one of the crucial problems [1] in computer vision. Tracking of a moving object deals with estimating the location of moving object over the time in a video. This problem have several important applications in various diverse areas like extraction of highlights in a sport video, identifying the faces or parts of faces for video surveillance system [2,3], medical imaging, target detection and interpretation [3] etc. Major difficulties involved in moving object tracking include: rapid appearance changes of object, illumination changes, size and shape changes, occlusion, cluttered background and interaction between multiple objects. The object tracking algorithms can be broadly classified into four groups: region-based [4], contour-based [5], model-based [6] and feature-based [7] algorithms. Some methods use local information captured from sequence of frames that results more accurate detection of object followed by tracking. This local information is usually available in the form of frame differencing and it highlights changing regions in the subsequent frames. For region of interest in the reference frame, the tracker keeps on the track of frame to frame correspondence resulting in a sequence of tracks. This approach of moving object tracking is known as region-based tracking [4,8] and it requires several parameters such as object size, color, shape, velocity, etc. Involvement of several parameters results in high computational cost of region-based tracking methods.

To reduce the computation cost, methods based on single feature of video frame have been proposed [7]. Some heuristic is needed to select suitable feature for tracking purpose. Generally feature-based object tracking methods use color histogram processing in spatial domain [4,8]. These methods are suitable for stationary, low-noisy and partially occluded object videos. Since color histogram works on global color, therefore, the methods based on color histogram yield false tracks when color of background resembles to that of object. Thereafter, mean-shift algorithm with color histogram has been used successfully to track the object in video which has given improved tracking results and proved to be robust on intensity changes [8,10,11]. However, these methods are complex in implementation for handling occlusion. In subsequent years, the improvements in tracking algorithms have been proposed by the use of histogram and spatial information [11]. Bayesian and particle filters in color histogram [9,12,13] and kernel-based tracking [8] are used to balance the computational cost and accuracy of tracking results. Feature based object tracking algorithms are used and applied to perform operations on point, shape or contour in spatial domain [3,7,14]. Other types of tracking algorithms deal with the processing of frequency values of pixel known as transform domain processing [15-21].

In recent past orthogonal wavelet transform based methods [15-21] become popular due to their improved capability of handling occlusion and reduced number of false tracks [3]. Most of the wavelet transforms use orthogonal filters which are not capable to model the image boundaries and hence giving poor tracking results. In addition, the orthogonal wavelet transform based methods use non-symmetric filters except, those which uses Haar wavelet in their filter design. In orthogonal wavelet transform, perfect reconstruction is not possible because of non-symmetric design of wavelet filters and hence boundaries of object can not be handled completely.

In this paper, we have proposed a feature based object tracking algorithm based on biorthogonal wavelet transform, implemented using lifting scheme. Biorthogonal wavelet transform uses compactly supported spline wavelets. With these wavelets symmetry and perfect reconstruction is possible using FIR (Finite Impulse Response) filters, which is impossible for the orthogonal filters (except for the Haar filters). The symmetry means that the filters have linear phase. The biorthogonal family uses separate wavelet and scaling functions for the analysis and synthesis of video frames. The proposed algorithm rely not only on the matching of energy of wavelet coefficient among different frames but also the prediction of displacement of object calculated from Newton's equations of motion that reduces number of false tracks. For performance evaluation, the proposed method is compared with other well established object tracking methods namely, tracking based on Kernel filter [8], tracking based on particle filter [9], method based on Bayesian filter [22], Generalized-kernel based method [23]. Qualitative performance is not enough to judge the quality of any method. Therefore, we have performed quantitative performance comparison of the proposed method with other sate-of-the-art methods. For quantitative measures we have used Euclidean distance, Mahalanobis distance, city-block distance and Bhattacharyya distance.

The rest of paper is organized as follows: Section 2 gives an overview of the biorthogonal wavelet transform, in section 3, the proposed object tracking method along with usefulness of biorthogonal wavelet transform in tracking are given. The experimental results of the work are given in section 4 and finally conclusions are given in section 5.

2. BIORTHOGONAL WAVELET TRANSFORM

In many computer vision applications, we need filters with symmetrical coefficient to achieve linear phase. None of the orthogonal wavelet systems, except Haar, have symmetrical coefficients. Biorthogonal wavelet system can be designed to achieve symmetry property and perfect reconstruction by using two scaling functions and two wavelet functions [24,25]. These two scaling functions ϕ and $\tilde{\phi}$ are used to produce different multiresolution analysis and correspondingly two wavelet functions ψ and $\tilde{\psi}$ are used. $\tilde{\psi}$ and ψ are used in the analysis and synthesis respectively. The scaling functions $\phi, \tilde{\phi}$ and wavelet functions $\psi, \tilde{\psi}$ satisfy the duality condition as-

$$\int \psi_{j.k}(x)\tilde{\psi}_{j^{'},k^{'}}(x)dx = 0 \quad (1)$$

whenever $j \neq j^{'}$, $k \neq k$ and even,

$$\int \phi_{0.k}(x)\tilde{\phi}_{0,k^{'}}(x)dx = 0 \quad (2)$$

whenever $k \neq k^{'}$.

2.1 Lifting Scheme

In Figure 1, let X denote the input signal, and X_{L1} and X_{H1} be the decomposed output signals, where they are obtained through the following three modules of lifting-based one dimensional discrete wavelet transform (DWT) [24,25].

(i). Splitting: In this module, the original signal X is divided into two disjoint parts, i.e., $X_e(n) = X(2n)$ and $X_o(n) = X(2n+1)$ that denote all even-indexed and odd-indexed samples of X, respectively.

(ii). Lifting: In this module, the prediction operation P is used to estimate $X_o(n)$ and $X_e(n)$ and results in an error signal $d(n)$ which represents the detailed part of the original signal. Then we update $d(n)$ by applying it to the update operation U, and the resulting signal is combined with $X_e(n)$ to $s(n)$ estimate, which represents the smooth part of the original signal.

(iii). Scaling: A normalization factor is applied to $d(n)$ and $s(n)$, respectively. In the even-indexed part, $s(n)$ is multiplied by a normalization factor K_e to produce the wavelet subband X_{L1}. whereas in the odd-index part, the error signal $d(n)$ is multiplied by K_o to obtain the wavelet subband X_{H1}.

The outputs X_{L1} and X_{H1} obtained by using the lifting based wavelet transform are the same as those of using the convolution approach for the same input even if they have completely different functional structures. Compared with the traditional convolution-based wavelet transform, the lifting-based scheme has several advantages. First, it makes optimal use of similarities between the highpass and lowpass filters and this leads to reduced computation complexity by a factor of two. Second, it allows a full in-place calculation of the wavelet transform.

3. MATERIALS AND METHOD

3.1 Usefulness of Biorthogonal Wavelet Transform for Object Tracking

On the basis of properties of Biorthogonal Wavelet Transform (BWT), we observed following advantages for moving object tracking in video sequences:

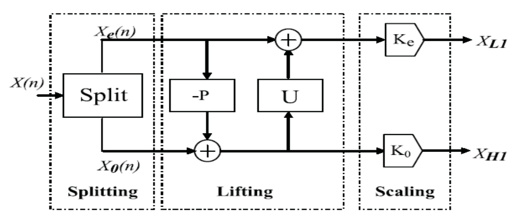


Figure 1. The forward and inverse transform of the lifting scheme.

3.1.1 Availability of linear phase

The filters used in orthogonal wavelet transform do not have the characteristics of linear phase; the phase distortion will lead to distortion of edges of an image. To make up for this shortcoming, the biorthogonal wavelet with linear phase characteristic is introduced which is much desired in object tracking for maintaining the object boundaries.

As shown in Figure 2, the biorthogonal family contains biorthogonal compactly supported spline wavelets. With these wavelets symmetry and perfect reconstruction is possible using FIR (Finite Impulse Response) filters, which is impossible for the orthogonal filters (except for the Haar filters). The symmetry means that the filters have linear phase [26].

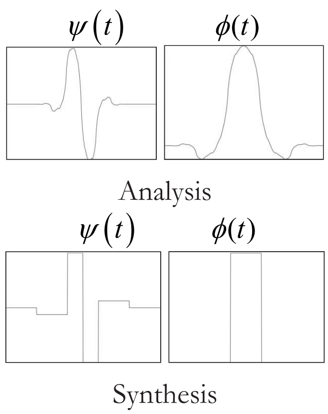


Figure 2. Linear phase in bior 1.3.

This linear phase property of biorthogonal wavelet transform retains the shape of the signal which is desirable for good trackers. Therefore, use of Biorthogonal wavelet transform reduces the false tracks of the moving object in video.

3.1.2 Shift invariance

A transform is said to be shift invariant if a small shift in the input signal causes nearly same shift in transform coefficients. If the transform is shift variant then it yields loss of information at multilevel as well. In a video with moving object, the object may change its location and orientation in different frames. So, it is essential that even after occurring shifts in the object in spatial domain, most of the information remain associated in the transformed domain. The shift invariance property of BWT [27] maintains this information. The rotation of object in different frames keep magnitude and energy of biorthogonal wavelet coefficients approximately same. This property is very much useful for detection of same object in different frames of video for tracking of moving object.

The shift invariance property of Biorthogonal wavelet transform is tested on an image of 256×256 as illustrated in Figure 3. The original image and its shifted version by two pixels are shown in Figure 3(a) and 3(b) respectively. We have computed the BWT of the original image and its translated version. The image is reconstructed by the high pass wavelet coefficients and shown in Figure 3(c) and 3(d). From Figure 3, it can be observed that the biorthogonal wavelet coefficients have corresponding shifts in wavelet domain. It has been found that error in reconstruction, in the form of sum of squares of differences in coefficients energy is of the order of 10^{-10}.

3.2 The Algorithm

A video contains a sequence of consecutive frames. Each frame can be considered as an

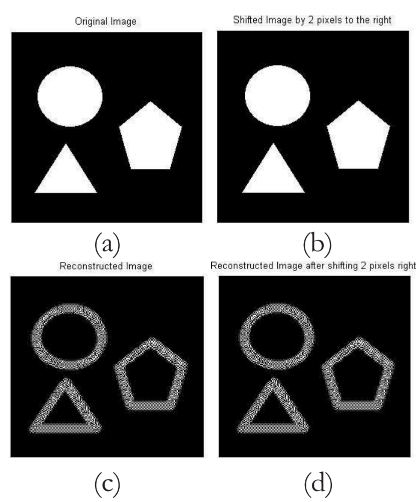


Figure 3. Illustration of shift invariance property of biorthogonal wavelet transform.

image. If the algorithm can track moving object between two consecutive frames then it will be able to track object in video sequence. In the proposed algorithm it is assumed that the frame rate is adequate and the size of the object should not change between adjacent frames. To speed up the performance of the tracker, the object is searched in its neighborhood in all possible directions rather than in entire frame. It is also assumed that the object will never acquire a velocity so that it can escape to its neighborhood. Complete algorithm is given as below -

***Algorithm:** Object Tracking*

1: Initialize frame_num = 1
2: Draw a bounding_box around the object with centroid (c1, c2) and compute energy of its BWT coefficients ξ as

$$\xi = \sum_{(i,j)\in bounding\_box} \left| wcoeff_{i,j} \right|^2$$

// $wcoeff_{i,j}$ are the Biorthogonal Wavelet Transform
// (BWT) coefficients at $(i, j)^{th}$ point
3: for frame_num 2 to end_frame
4: Compute the BWT coefficients of the frame, $wcoeff_{i,j}$
5: Initialize Search_region=16 (in pixels)
6: if frame_num > 4
7: Predict the centroid (c1, c2) of the current frame using centroids of previous four frames and basic Newton's equations of motion.
8: endif
9: for i = -search_region to + search_region do
10: for j = -search_region to + search_region do
11: c1_new = c1+i;
12: c2_new = c2+j;
13: Update bounding_box with centroid (c1_new, c2_new)
14: compute the difference of energy of BWT coefficients of bounding_box, with ξ, say $D_{i,j}$
15: end for
16: end for
17: Select min{$D_{i,j}$} and its index, say (index_x, index_y)
18: c1 = c1+index_x; c2 = c2+index_y;
19: update the bounding_box in the current frame with centroid (c1,c2)and its energy ξ
20: end for

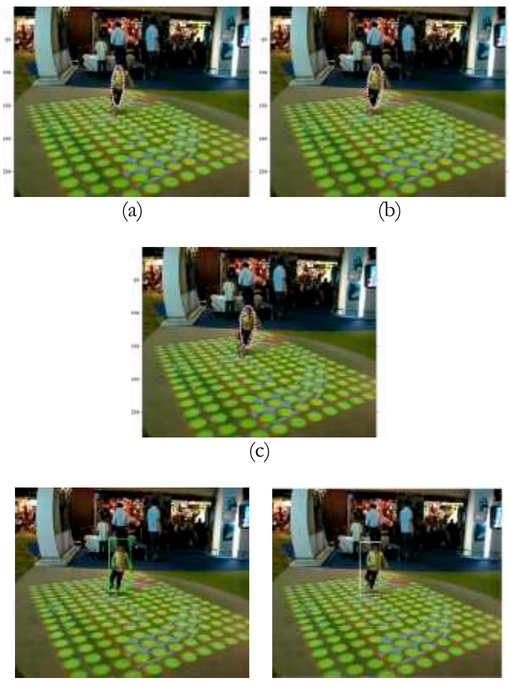
4. RESULTS AND DISCUSSION

This section discusses the experiments performed and results of object tracking obtained by the proposed method. The algorithm described in section 3.2 is implemented using MATLAB and experiments are performed on several video clips of varying object sizes, background, illumination changes etc. The visual results for one representative video clip 'child' of frame size 288 by 352 has been shown in Figure 4. The main challenge of this video is that the child object is moving with varying speed and orientation with the varying background. Size of the object is also changing as the child object bends and moves far from camera. The proposed algorithm automatically tracks the child object successfully without any user intervention. In the first frame the object is segmented and we draw an object window that fully covers the object with centroid (C1,C2) and the energy of biorthogonal wavelet coefficients of the object window is computed. For the second frame and onwards, tracking of the object is performed within the search region which is considered as 16 pixels larger than object window. The search window of the size equal to the object window is considered. The fixed search length 16 pixels of search region allow the search window to move maximum of 16 pixels in all possible four directions. In our experiments we assumed that the desired object can not have sufficient velocity so that it can move more than 16 pixels between two consecutive frames. This is the reason why we have chosen search region greater than 16 pixels of the object window. Also, the reduced search region gives reduced computational cost and will track the object efficiently.

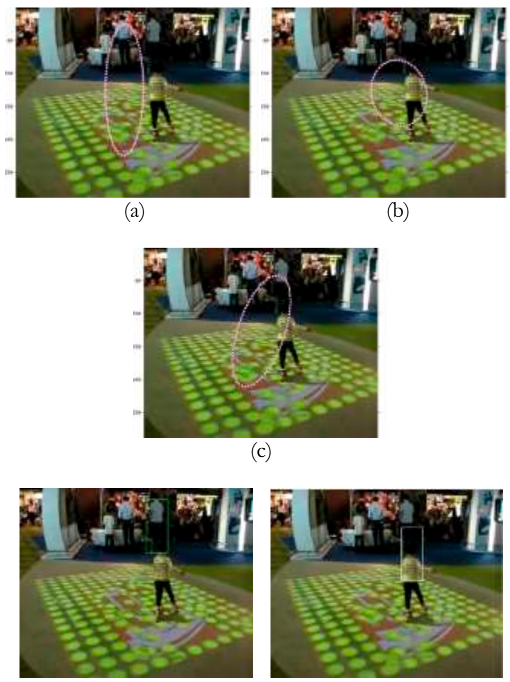
4.1 Qualitative Performance Evaluation

The tracking performance of the proposed method is compared against other well established object tracking methods: tracking based on Kernel filter [8], tracking based on particle filter [9], method based on Bayesian filter [22], Generalized-kernel based tracking method [23], The comparative visual results of the proposed method against above mentioned methods on one representative video clip are shown in Figure 4. The results are shown up to 300 frames for child video clip at a step of 20 frames.

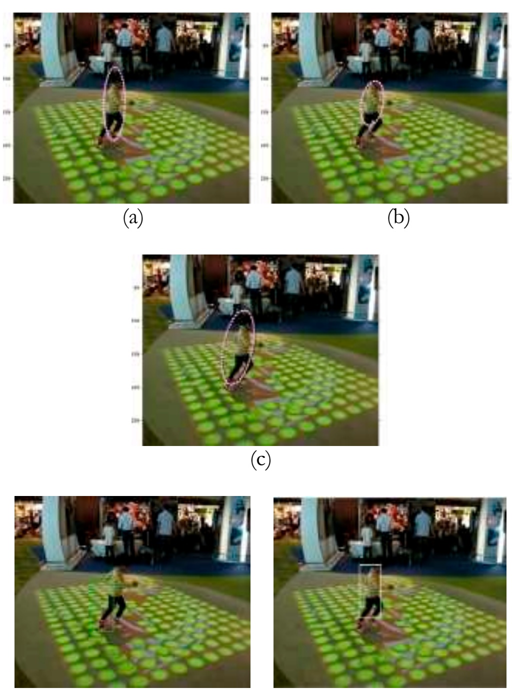
Frame No. 1



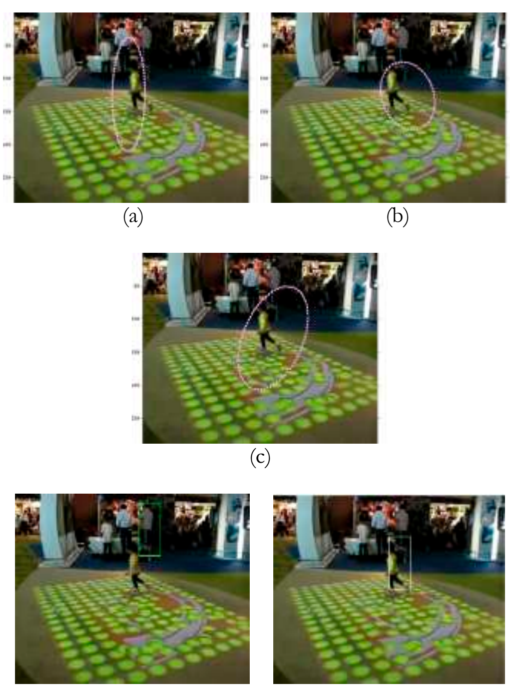
Frame No. 40



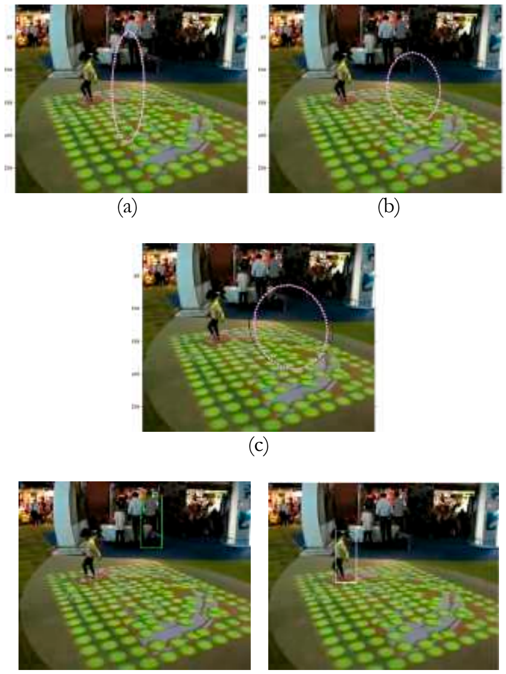
Frame No. 20



Frame No. 60

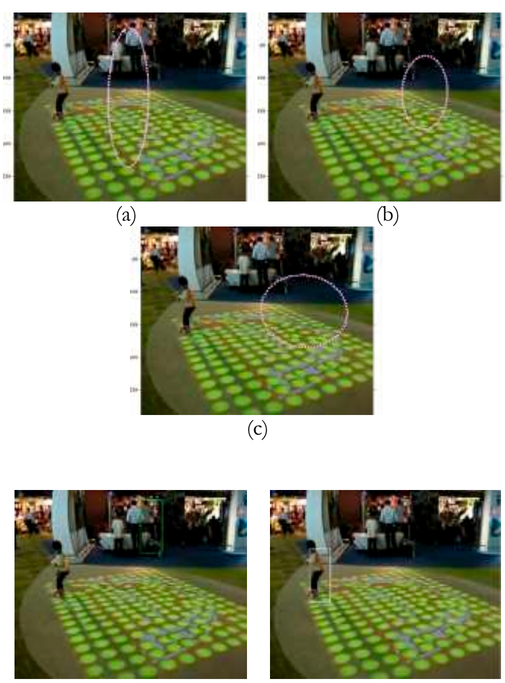


Frame No. 80



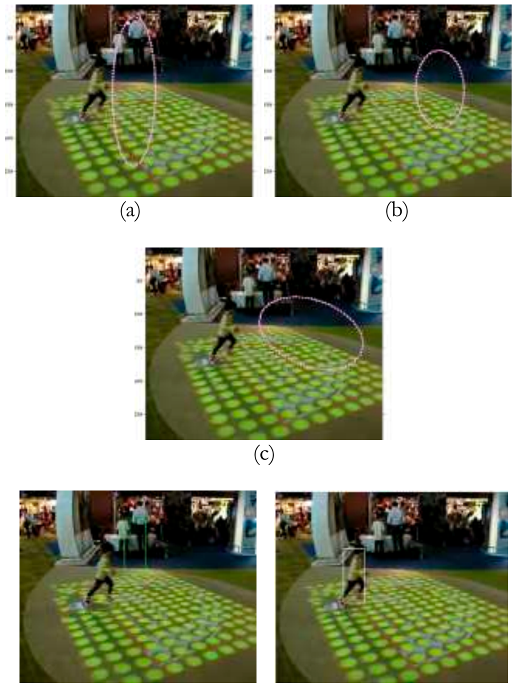
(a) (b) (c) (d) (e)

Frame No. 100



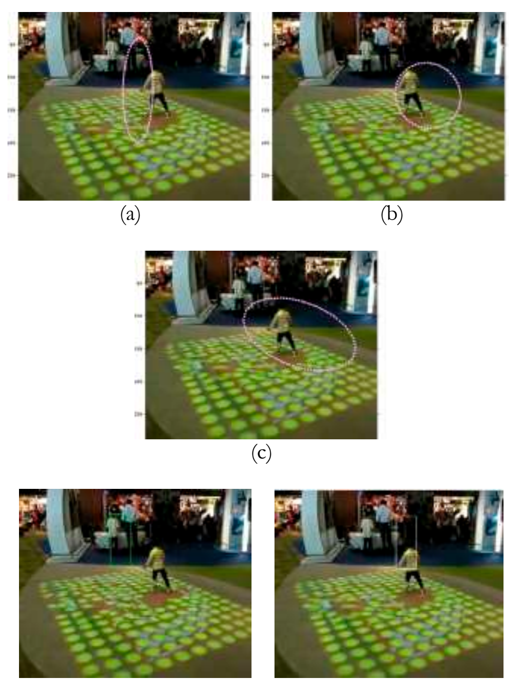
(a) (b) (c) (d) (e)

Frame No. 120



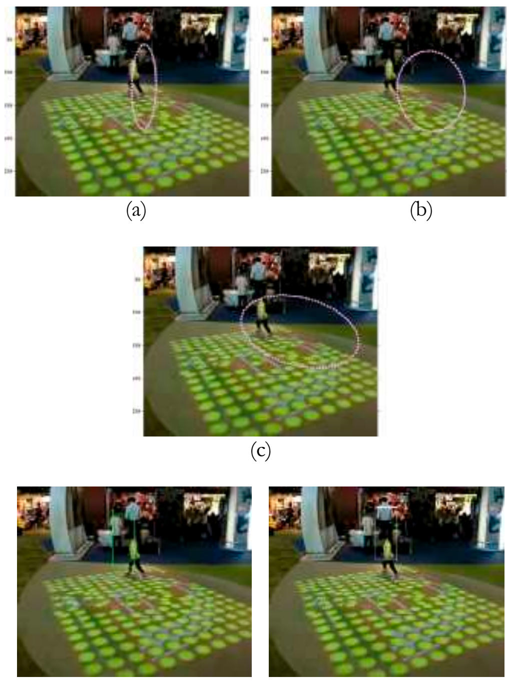
(a) (b) (c) (d) (e)

Frame No. 140



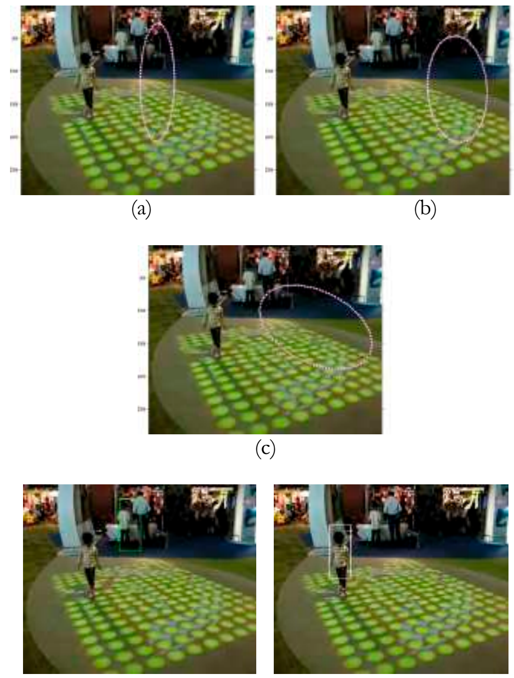
(a) (b) (c) (d) (e)

Frame No. 160



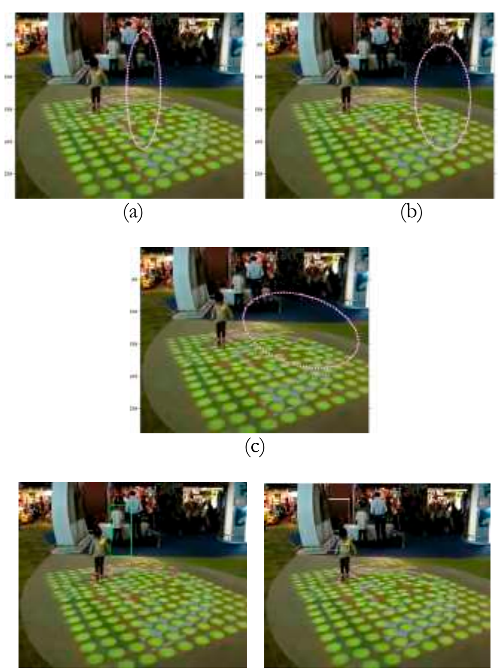
(a) (b) (c) (d) (e)

Frame No. 200



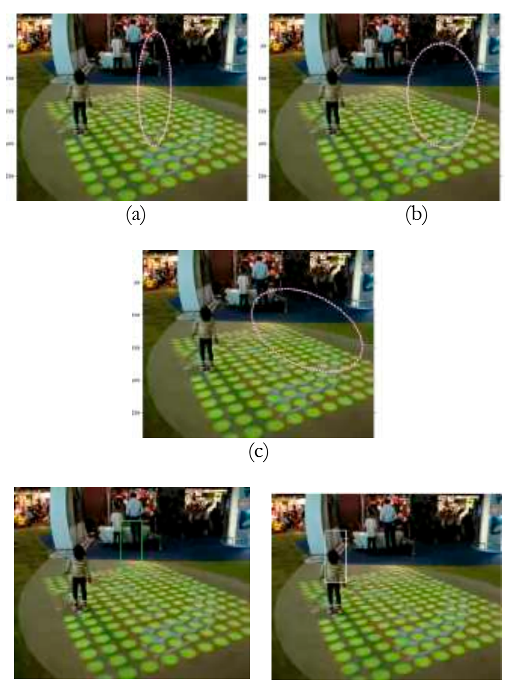
(a) (b) (c) (d) (e)

Frame No. 180



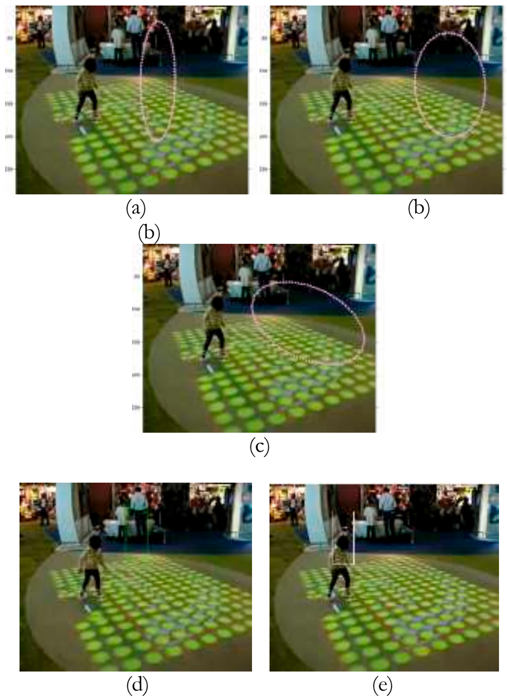
(a) (b) (c) (d) (e)

Frame No. 220

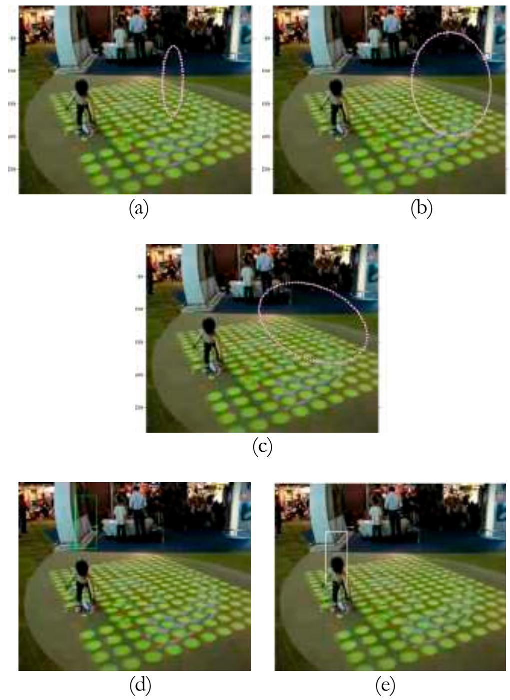


(a) (b) (c) (d) (e)

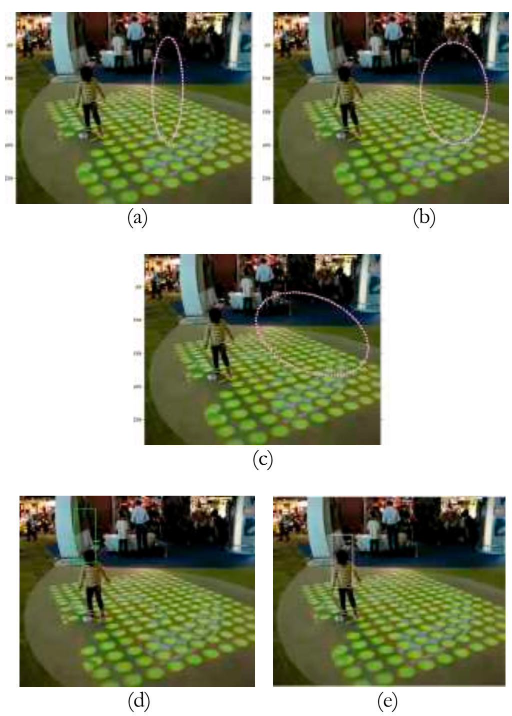
Frame No. 240



Frame No. 280



Frame No. 260



Frame No. 300

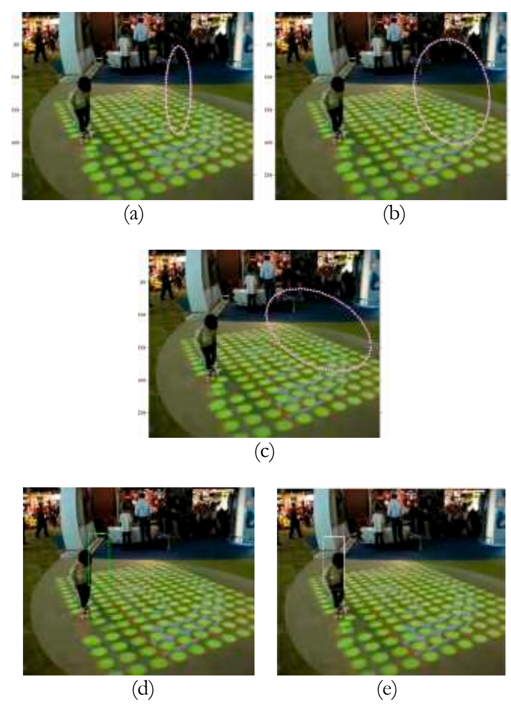


Figure 4. Tracking results of the child object in a video clip for frame no 1 to 300 in steps of 20 frames using methods (a) Kernel based tracking [8] (b) Particle Filter based tracking [9] (c) Bayesian tracking [22] (d) Generalized-kernel tracking [23] (e) The proposed method.

In this video, the child object is changing its position and motion abruptly. The abrupt motion of the child object is very difficult to track. Observing the different frames of this video, we can see that bounding box fully covers the object in first frame. The movement and direction of movement changes abruptly in an arbitrary direction in frame 2 and continues this way and the child object to stops in frame 100. Further we observe that from frame 120 the object reverses its direction of movement and comes at rest in frame 200. The proposed method tracked the object in all these frames accurately. After frame 200, the child object shows some different poses like slight bending etc. and the proposed method still keeps on tracking the object accurately. Moreover, the proposed method tracks the object well in presence of rapid change in the lighting conditions and background. On the other hand tracking based on Kernel Filter [8], tracking based on Particle filter [9], method based on Bayesian Filter [22], are capable of tracking the object in first 20 frames because of its linear and slow motion and as the speed and direction of object changes rapidly, the desired object in the same video is starts slightly out of track up to frame 40. In the frames 60 onwards it is clear that all these three methods fail to track the object. At last, in frame 300 it can be clearly seen from the Figure 4 that the object is totally out of track. In Generalized-kernel based tracking method [23], object is well covered in frame 1 and starts displacing from even frame 20 and there is a total track loss in frame 40. In this method, bounding box remains still and could not recover its track. The extent of track loss is very high in this method. Hence, the suitability of the proposed method can be easily observed because of its tracking performance which is not found in any of the above mentioned other methods. No track loss are found by the proposed method and is able to track the object in all possible conditions like slow object's motion, slight change in object's motion and orientation, rapid change in motion and orientation of object , ceased object, changing light conditions and background etc.

4.2 Quantitative Performance Evaluation

The proposed method is tested on several video clips and compared against the well established tracking methods: kernel-based tracking [8], method based on Particle Filter [9], method based on Bayesian filter [22], Generalized-kernel based tracking method [23]. All the simulations and comparison of the proposed method are performed using MATLAB on a PC with an Intel Pentium Dual 2 GHz processor. Moreover, the performance of the proposed method and its quantitative comparison with other methods has been done on three performance evaluation measures Euclidean distance [1], Mahalanobis distance [1,28],City block distance [1] and Bhattacharyya distance.

4.2.1 Euclidean distance

The Euclidean distance between the computed centroid of tracked object window and actual centroid is defined as-

$$ED = \sqrt{(x_A - x_c)^2 + (y_A - y_c)^2} \tag{3}$$

Figure 5(a) shows the Euclidean distance of tracking algorithms and the proposed method. From Figure 5(a), it is clear that the proposed method has the least Euclidean distance between centroid of tracked bounding box and actual centroid in comparison to other methods. The x-axis shows the ground truth values of centroid of the object in different frames. From Figure 5(a), it is clear that the proposed method has minimum deviation from actual object's centroid values.

4.2.2 Mahalanobis distance

Mahalanobis distance is based on correlations between variables by which different patterns can be identified and analyzed. It varies from Euclidean distance in the sense that it encounters the correlations of the data points. It is defined as a dissimilarity measure between two points X = (x_A, y_A) and Y = (x_C, y_C) with the covariance matrix C.

$$MD = \sqrt{\left(X-Y\right)^{T} C^{-1}\left(X-Y\right)} \quad (4)$$

From Figure 5(b), it is clear that the values of dissimilarity measure are small i.e. the actual centroids and the centroids obtained by the proposed method are almost same and as the speed and other object conditions of the child object change, the Mahalanobis distance remain constant in the proposed method as compared to the other methods used in comparison.

4.2.3 City block distance

In two dimension, the city block distance between (x_A, y_A) and (x_C, y_C) is defined as sum of the absolute value of the difference of the each of the coordinates.

$$CBD = \left|\left(x_A - x_C\right)\right| + \left|y_A - y_C\right| \quad (5)$$

The city block diastance is another distance transform metric. This metric measures the path between the pixels based on the four connected neighborhood. Therefore, this measure is suitable for the detection of object boundaries. From Figure 5(c), it can be observed that this distance for computed object centroids are very close to the actual centroids.

4.2.4 Bhattacharyya distance

The Bhattacharyya distance is another distance transform which is used as separability measure of tracked object region and actual object region. For two classes of actual and computed values, Bhattacharyya distance (BD) is defined as-

$$BD = \frac{1}{8}(mean_c - mean_a)^T \left[\frac{\mathrm{cov}_a + \mathrm{cov}_c}{2}\right]^{-1} \times (mean_c - mean_a) + \frac{1}{2}\ln\frac{\left|\left(\mathrm{cov}_a + \mathrm{cov}_c\right)/2\right|}{\left|\mathrm{cov}_a\right|^{1/2}\left|\mathrm{cov}_c\right|^{1/2}} \quad (6)$$

where, $mean_a$ = mean vector for actual object region, $mean_c$ = mean vector for computed object region, cov_a = covariance matrix for actual object region and cov_c = covariance matrix for computed object region. This performance metric is shown in Figure 5(d).

5. CONCLUSIONS

In the present work, we have exploited the properties of biorthogonal wavelet transform useful for object tracking in video sequences which uses two different filters for decomposition and reconstruction. The Biorthogonal wavelet transform is implemented using lifting-scheme, which helps fast computation. The biorthogonal wavelet coefficients are used for tracking the object. The proposed algorithm is simple to implement since it does not need any parameter except the biorthogonal wavelet coefficients. The experimental results demonstrate the effectiveness of the algorithm even in some complicated situations, such as ceased track, partial occlusion and short occlusion etc. The approximate shift-invariance nature of the biorthogonal wavelet transform helps in accurate tracking of object. The limitations of the proposed algorithm are that the object size, shape should not change much between successive frames. The experimental results show that the proposed algorithm is capable to track the moving object in video sequences with stationary and varying background. Also, the proposed method is capable to track the object of size ranging from very small to the sufficiently large. Experimental results obtained indicate that the proposed method performs well as compared to the other tracking methods like Method based on Kernel Filter [8], Method based on Particle Filter [9], Method based on the Bayesian Filter [22], Generalized-kernel based tracking method [23]. The proposed method is not dependent on other features of object such as size, shape, color, etc.. The proposed method is able to track the object in the video having cluttered background,

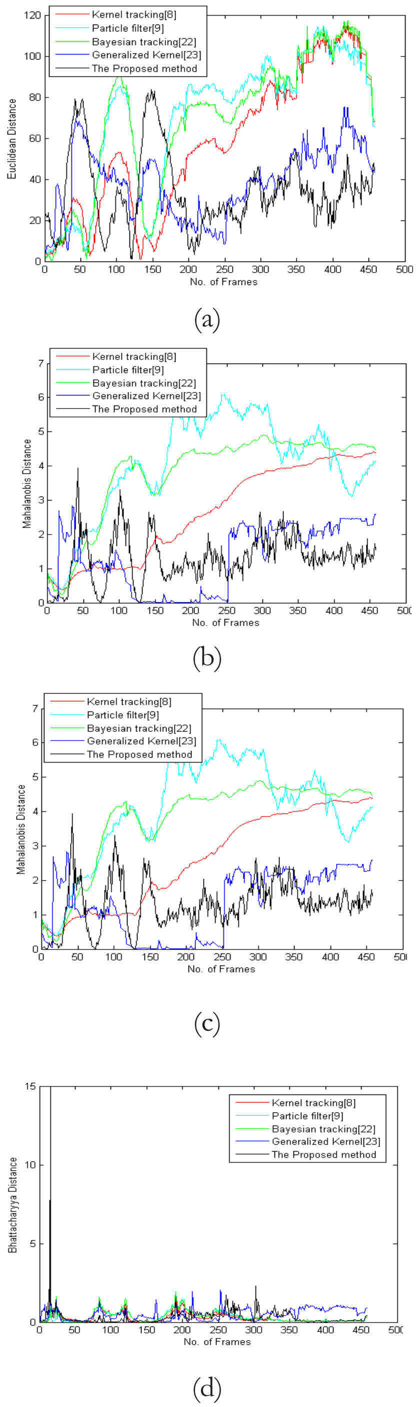


Figure 5. Plot of performance measures in different frames (a) Euclidean distance (b) Mahalanobis distance (c) City block distance (d) Bhattacharyya distance.

changing lighting conditions and varying size object within a certain limit.

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