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Contributed Paper

Meson Spectroscopy: Simple Two-step Potential Model.

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ABSTRACT

The mass spectra and decay widths of γ,ψ,ϕ and ρ resonances in a simple quark-confining, analytically solvable, two-step potential model to study the quark-antiquark system, have been studied. Results are found to be somehow in good agreement with experiments and also with the values predicted by others.

1. INTRODUCTION

A number of inter-quark potential models have been suggested and investigated in the spectroscopy of γ,ψ,ϕ and ρ families [1-5]. Apart from structural details, all these potentials have two broad features in common, which include a coulomb-like singular part and a long range confining part. Many authors such as Sharma P.C *et al.*[6] and Jain *et al.*[7] added a coulombian term to their confining potential to account for the short range behaviour. As the superposition of two potentials dilutes the role of the individual potential, the idea of a two-step potential has been proposed for the $q\overline{q}$ interaction. Purnima *et al.* [8] used the two-step potential in which the coulombian form is used for short range and a linear form for the long range part.

2. MATERIALS, METHODS AND RESULTS

In this note, investigation of a two-step potential in which the short range form is the same, but for the long range, instead of a linear, an oscillator potential has been considered. In the coming calculations, ψ mesons are assumed to be states of a charmed, c-quark and its antiquark, γ assumed bottom, b-quark and its antiquark, ϕ assumed strange, s-quark and its antiquark and lastly the ρ assumed the up, u-quark and its antiquark, all of them bounded by a phenomenologic potential of the form:

$$V_1(r) = \frac{G}{r} + J, \qquad r \le Z \qquad .....1a$$

$$V_2(r) = Ar^2 + B, \qquad r > Z \qquad .....1b$$

Where G and A are constants and J and B are matching parameters. Here Z is the distance at which the forces caused by the coulomb and the oscillator potentials are the same. Physically, Z will be realized as first Bohr radius for the light quark system. At a distance $r = Z$, the formation of an additional $q\overline{q}$ pair takes place out of the original field between q and $\overline{q}$. This new pair of q and $\overline{q}$ after interaction with the original light quark pair forms the heavy quarks, Q and $\overline{Q}$. In order to calculate the mesonic energy levels, appropriate wavefunctions and energy eigenvalues were obtained by solving the Schrödinger equation separately for the potentials V_1 and V_2. All the particles have been treated nonrelativistically. As we all known, the energy eigen-value expression for the potential V_1 is given by [9]

$$E_{nL} = -\frac{\mu G^2}{2n^2} - J \qquad ....\ 2$$

where μ is the reduced mass. The argument may be that the low-lying states of the above mentioned mesons may depend on the coulomb potential (1a), while the higher excitations should depend on the oscillator potential (1b), since the radial separation between the quark-antiquark pair will be larger in this case, $(r > Z)$. So the binding energies for short ranges $n = 1$ and $n = 2$ are calculated by equation 2. Also another argument may be that the two potentials join smoothly at $r = Z$. By equating the potentials (1a) and (1b), and also their first derivatives at $r = Z$, the following relations are obtained:

$$\frac{G}{Z} + J = AZ^2 + B$$

$$Z = \left(\frac{G}{2A}\right)^{\frac{1}{3}} \qquad ....3$$

Equation (3) shows the transition from the first to the second potential. Now we consider the potential V_2 and write its well known energy eigen value expression [9]

$$E_{nH} = (n + 3/2)\omega + B \qquad ....4$$

Where, $\omega = (2A/\mu)^{1/2}$. The binding energies corresponding to $n > 2$ are calculated by this expression, equation 4. The masses of the quark-antiquark bound states are obtained by $M_n = 2m_q + E_n$ (where m_q is the mass of the constituent quark). These masses have been shown in Table 2. Further, the leptonic decay widths of the considered mesons have been calculated using the standard result

$$\Gamma(q\bar{q} \to e^+e^-) = \frac{16\pi e_q^2 \alpha^2}{M_n^2}|\psi_n(0)|^2 \qquad ....5$$

Here, $\alpha = 1/137$ is a fine structure constant. Table 1 below shows the characteristics of the quarks involved.

Table 1. Characteristics of the quarks.

Quark type	Charge, Q	Approximate rest mass *(GeV/c²)*
c	+2/3	1.852
b	-1/3	5.11
s	-1/3	0.2
u	+2/3	0.005
d	-1/3	0.01

$|\psi_n(0)|^2$ is the square of the wavefunction at the origin and M_n is the mass of the meson. The values of $\psi_n(0)$ for the coulomb part is obtained from the standard solution of the Schrödinger equation

$$\psi_{1s}(r) = 2\left(\frac{G}{a_0}\right)^{3/2} e^{-Gr/a_0}$$

$$\psi_{2s}(r) = \left(\frac{G}{2a_0}\right)^{3/2}\left(2 - \frac{G}{a_0}\right)e^{-Gr/a_0} \qquad ....6$$

In equation 6, G and a_0 are constants.

Table 2. Mass (in GeV) of the $q\bar{q}$ bound states.

Meson	State	A	B	C
$\psi(c\bar{c})$	1s	3.096	3.065	3.096
	2s	3.686	3.684	3.686
	3s	4.160	4.040	4.160
	4s	4.415	4.414	4.415
$\gamma(b\bar{b})$	1s	9.434	9.43	9.460
	2s	10.010	9.99	10.020
	3s	10.358	10.32	10.355
	4s	10.608	10.61	10.577
$\phi(s\bar{s})$	1s	1.020	-	1.202
	2s	1.675	-	1.685
	3s	2.032	-	2.055
	4s	2.167	-	-
$\rho(u\bar{u})$	1s	0.770	-	0.770
	2s	1.587	-	1.590
	3s	2.056	-	-
	4s	2.234	-	-

A: Present calculation
B: Prediction of Mundembe *et al.* [12]
C: Available experimental data [11]

Table 3. List of constants used.

Meson	*G(GeV³)*	*A(GeV⁻¹)*	*J(GeV)*	*B(GeV)*	a_0
ψ	0.03	2.7	0.1785	0.436	1.303
γ	0.03	2.7	0.3751	0.963	1.303
ϕ	0.03	2.7	0.1094	0.286	1.303
ρ	0.03	2.7	0.0924	0.112	1.303

In the oscillator part, the expression for the mesons contains the Laguerre polynomial, the value of which cannot be determined for alternate values of n. Thus the following semi-classical formula derived by Quigg and Rosner [10] is used here for the values of the s-wavefunction at the origin for deriving an explicit expression for the leptonic decay widths:

$$|\psi_n(0)|^2 = \frac{(2\mu)^{3/2}}{4\pi^2} E_n^{1/2} \frac{dE_n}{dn} \qquad 7$$

Where E_n is the binding energy of the n^{th} state and dE_n/dn is the derivative of the eigenvalues expression with respect to n. The decay widths calculated have been listed in Table 4; for comparison, the corresponding experimental results have also been included.

Table 4. Leptonic decay widths for the $q\bar{q}$ bound states for the selected mesons (KeV).

Meson	State	A	B	C
$\psi(c\bar{c})$	1s	24.1	26.1	4.8 $\pm$ 0.6
	2s	2.13	2.3	2.1 $\pm$ 0.3
	3s	1.13	0.57	0.75 $\pm$ 0.10
	4s	0.58	0.20	0.44 $\pm$ 0.14
$\gamma(b\bar{b})$	1s	4.5	4.52	1.1 $\pm$ 0.12
	2s	0.5	0.50	0.507 $\pm$ 0.051
	3s	0.316	0.140	0.362 $\pm$ 0.005
	4s	0.22	0.06	0.24 $\pm$ 0.053

A: Present calculation
B: Prediction of Mundembe *et al.* [12]
C: Available experimental data [11]

3. CONCLUSION

In the present work, the mass spectra and leptonic decay widths of various $q\bar{q}$ bound states in a two-step potential have been obtained. The results obtained, including mine (see table 2), clearly show that the two-step potential consisting of a combination of the coulomb and oscillator potentials provides a better agreement with the experimental data [11] on the masses of the mesons chosen than the two-step potential used by Mundembe *et al.* [12]. Further, the results for the leptonic decay widths also show a better agreement with the experimental values than those obtained by Mundembe *et al.* [12]. Though the results of Sharma P.C *et al.* [6] are somewhat better than our results for the lower states, for higher states our results are in better agreement with the experimental data

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