Thermal Design Algorithm of Three-Stream Plate-and-Frame Heat Exchangers with Two Thermal Communications

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Abstract

To fully achieve the capability of a plate heat exchanger (PHE), this paper presents a new design structure as a single-phase three-stream plate-and-frame heat exchanger with two thermal communications. In this article, one hot stream transfers heat to two colder streams in which there is no direct heat transfer between two cold streams. Six-port plates are discussed for their use in the new structure. A thermal design methodology on the basis of minimum number of channels is proposed, and the extension of pinch analysis is undertaken to complete also the design of heat recovery between the streams.

1. Introduction

High level of energy recovery in micro fabricated plants can be achieved by the use of compact heat exchangers. This type of exchangers offers a number of advantages over shell-and-tube heat exchangers, where energy conservation is a main concern. Apart from their light weight and low volume, they can serve more effectively for the heat transfer with a close temperature approach. Compact heat exchangers of the plate type have geometrical characteristics that make them suitable for the incorporation of a number of streams into the same unit. With this feature, they offer a number of potential benefits over conventional two-stream heat exchangers. These are lower capital, energy and installation costs, reduced weight, space and support structure, improved safety and finally improved process design and plant layout [1] .

A number of published papers are available for the design algorithm of multistream plate-fin type. Recently, a new design algorithm for single-phase multistream plate-fin exchangers was proposed for counter-current flow configuration. The design based on the appropriate exchanger sectoring with process integration by pinch analysis. Uniform heat loads per channel and uniform UA values were the chosen basis for the sizing of the units [2] . However, the use of plate-fin heat exchangers in the process industry has been somewhat limited due to their complex structure. In the process where the ease of cleaning is required (e.g. general feed processing, pharmaceutical industries, paper mills and petrochemical plants), the plate-and-frame type is most suitable.

The use of multistream plate-and-frame heat exchangers (MPHE) can be currently found in sterilization and pasteurization processes of thermally sensitive fluids such as cow milk. Several reports confirm more than 90% of heat recovery in such processes, when process fluids are assigned to exchange heat in a single frame [1,3]. A few papers discussed on the design of a three-stream plate heat exchanger and its connection with pinch analysis for arrangement of process heat integration. The process composite curves provided valuable information for stream matching in block diagram. The studies based on common flow configurations that can be

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organized via the plate packs where general four-port plates were used [4-5]. Due to limitation of the number of ports, each block in the diagram can only involve an energy transfer between two streams (e.g. one thermal communication). However, dividing plates are used to permit a number of streams to enter or exit the plate pack.

A new design structure as a three-stream plate heat exchanger with two thermal communications is proposed in this paper. A case of one hot stream transfers heat to two colder streams is scrutinized. Six-port plates are introduced for their use in the new structure, which causes the unit more compact due to no need of any dividing plate. The thermal design methodology on the basis of minimum number of channels is proposed for counter-current flow configuration.

2. Plate Heat Exchanger (PHE)

Plate heat exchangers basically comprise a stack of corrugated-metal plates fitted together into a frame. The frame consists of an upper and lower carrying bar supported the plates see Figure 1. Each plate normally has four corner ports which in pairs function as inlets and outlets for both the heating and cooling medium.

PHEs are especially preferable to conventional shell-and-tube heat exchangers in the following situations: the operation which requires a large change in temperature of one or both fluids, the temperature

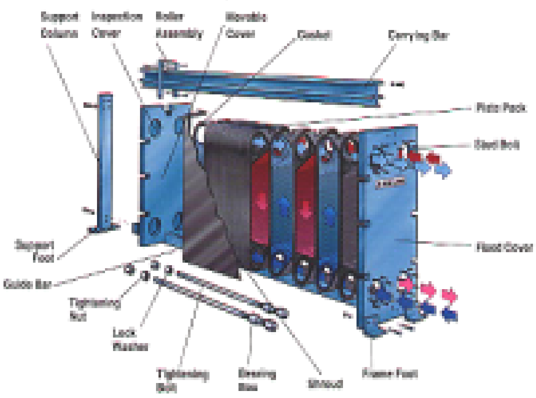

Figure 1 The plate heat exchanger [6].

approach is very small, there is a temperature cross, i.e. the outlet temperature of the hot side is colder than that of the cold fluid, and the installation space or weight is a major consideration [7].

3. Design Methodologies of a PHE

Generally for a single-pass counter-current flow, there are two design methodologies for a two-stream PHE [8]. The first is the Effective Mean Temperature Difference, ΔT_m method. This method requires an estimate of the overall heat transfer coefficient to be introduced into the basic equation (1)

$$\dot{Q} = UA\Delta T_{LM} \tag{1}$$

where

$$\Delta T_{LM} = \frac{\left(T_{h,in} - T_{c,out}\right) - \left(T_{h,out} - T_{c,in}\right)}{\ln\left[\left(T_{h,in} - T_{c,out}\right) / \left(T_{h,out} - T_{c,in}\right)\right]} \tag{2}$$

A number of iterations are required to determine the number of channels for the selected plate in order to perform a specified duty. The second method is the Effectiveness and the Number of Transfer Unit (ε - NTU) method. The method exploits an asymptotic ε - NTU formula for a single-pass PHE as follows:

$$\varepsilon = \frac{\exp\left[\left(1 - C^*\right)NTU_{min}\right] - 1}{\exp\left[\left(1 - C^*\right)NTU_{min}\right] - C^*} \tag{3}$$

where

$$C^* = C_{min} / C_{max} \tag{4}$$

$$NTU_{min} = UA / C_{min} \tag{5}$$

Both methods can be applied when a PHE has a large number of channels. Since when a PHE has the number of channels less than 50 channels, the end effects (from two end channels) are the important parameters. This can effect the fluid temperature in the inner passages. The design procedure for two-stream PHEs by these methods is too long to repeat here, but can be seen in the manual of any four-port PHE type.

4. Three-Stream PHEs

Most PHE applications in process industries involve transfer of thermal energy between two fluids through one thermal communication in two-stream PHEs. However, in some processes the transfer of heat often takes place among three or more streams. The reason for bringing more than two streams into a single frame might be different in different applications. For example, sterilization and pasteurization processes are normally carried out at high level of energy recovery, the processes are then dominated by requirements for very small temperature differences within a compact structure. Two process streams (hot and cold fluids) and two utility streams (hot and cold utilities) are usually incorporated in a single frame for a general ultra-high-temperature (UHT) process for milk and milk products [9].

For a plate pack fabricated by four-port plate type, a number of streams can be incorporated in a single frame via the dividing plates with special blocking ports, see Figure 2. The plate packs being separated by this technique are considered to be adiabatic, because of the large thermal resistance caused by the thickness of the dividing plate and the two air gaps between the end plates and the dividing plate [10]. The prevalent applications of process integration (pinch) technology can be applied to match streams for this multistream unit in order

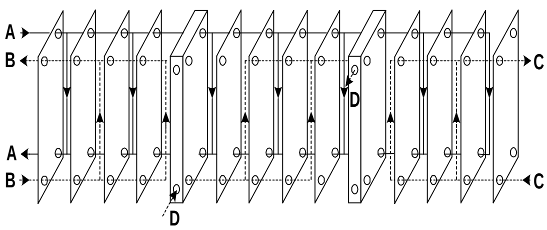

Figure 2 The plate packs separated by dividing plates.

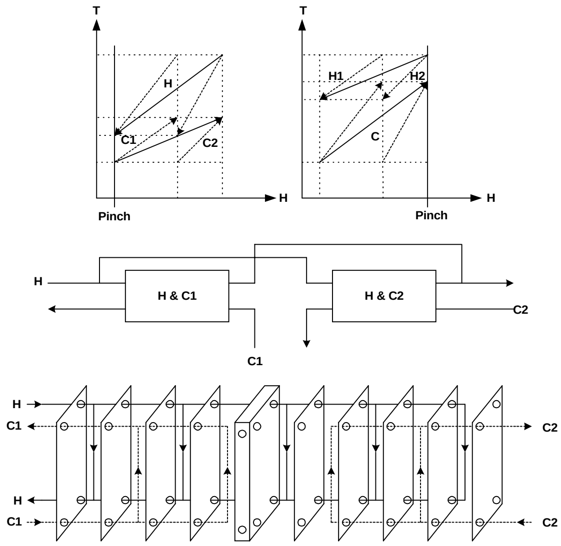

Figure 3 A multistream PHE with one thermal communication heat transfer.

to achieve the capital and energy targets of heat exchanger networks [5, 10].

To apply a pinch analysis toward the design of a multistream PHE, composite curves are necessary tools. The curves represent the enthalpy change requirements of the hot and cold streams over specific temperature ranges. At some point, the curves reach their closest temperature approach, this location is known as the 'pinch' [11]. Three principle rules for stream matching in order to achieve the minimum energy targets must be followed, these are no heat across the pinch, no cold utility usage above the pinch and no hot utility usage below the pinch.

Normally, by seeing the shapes of the composite curves, the heat transfer block diagram can be thermally designed by exploiting the principle rules. For example in Figure 3, the two cold streams (C1 and C2) are designed to receive heat with one hot stream (H) and vice versa. From the Figure, the block diagram is generated with respect to only one hot and one cold stream in each single block. The single hot stream needs to be split to match with the other two cold streams in the region where their temperature ranges overlap. This will make all the heat transfer blocks satisfy the minimum temperature approach of the network. All the single blocks in this case are performing one thermal communication heat transfer. A multistream unit can be configured as in the same Figure. A dividing plate must be used to separate two plate packs (the two heat transfer blocks). If a single-pass counter flow is used, the splitting of the hot stream channels can be estimated from the ratio of heat capacity flowrates of the two cold streams [5].

From the point of view, the pinch location in Figure 3 can be either on the extremely left or the right side of the composite curves. Therefore, there is no need to omit any heat interaction effects between the blocks. The blocks can be thermally connected by taking out the dividing plate. However, some of the middle plates may see the different in the overall heat transfer coefficients, and the overall duty of the unit can be slightly changed. To overcome this problem, a new design structure as a three-stream plate heat exchanger with two thermal communications is proposed. In this structure, six-port plates are introduced for their use in a case of one big hot stream transferring heat to two small cold streams, see Figure 4. In the figure, the flow pattern of all streams is single-pass counter-current flow. The hot stream occupies half of the total number of channels, and the two cold streams share the rest with desired proportions of cold channels, such as 'HC1HC2HC2HC1HC2HC2HC1'. The new structure causes the unit more compact due to no need of any dividing plate, and variety of flow configurations can be further adjusted for each stream without the difficulty of port arrangement.

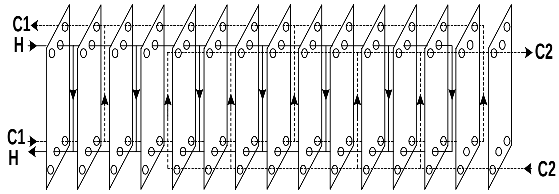

Figure 4 A multistream PHE by six-port plates.

5. Thermal Design Methodology

The thermal design methodology for the new structure is proposed for a problem with one hot stream and two cold streams comprising counter-current flow configuration. In the design of this structure, some consideration must be given that two cold streams enter and leave the unit at the same temperatures. Since the unit is of the plate-and-frame type, the number of heat transfer plates (or number of channels) conveys an accurate idea of the size of the unit.

The design basis of the new structure, is that the unit must result in the effective use of temperature driving forces throughout the exchanger. Bringing this basis via composite curves at the same enthalpy interval, the two cold streams enter at the same temperature and are heated over the same temperature span. Stream temperature profiles are related to heat load, overall heat transfer coefficient and selected heat transfer surface area by:

$$\Delta_{LM} = \dot{Q}/UA \tag{6}$$

Thus uniform hot and cold temperature fields are achieved if $\dot{Q}/UA$ is constant for all heat transfer plates in the unit. With respect of the heat load constraint per plate and each plate seeing the same heat transfer surface area, the more restricted criteria is U uniform for all heat transfer plate. This can be presented with the following important conclusion : the only means available for the achievement of U uniformity is through the allocation of channels for two cold streams.

To evaluate the thermal efficiency of an existing PHE or to design the PHE for a specified duty, the total number of required channels is used to represent the PHE size in this work. For a single-pass counter-flow configuration with equal numbers of hot and cold channels, the number of required channels can be approximated by using the ε-NTU relationships as follows:

$$NTU_{cc} = \frac{\varepsilon_{cc}}{1-\varepsilon_{cc}} \quad \text{for } C^* = 1 \tag{7}$$

$$NTU_{cc} = \frac{1}{1-C^*}\ln\left(\frac{1-C^*\varepsilon_{cc}}{1-\varepsilon_{cc}}\right) \quad \text{for } C^* \neq 1 \tag{8}$$

The number of heat transfer units can also be given by:

$$NTU_{cc} = \frac{UA_T}{C_{min}} = \frac{U(2n-1)wL_{ch}}{C_{min}} \tag{9}$$

where n is the number of hot channels, which is also the number of cold channels. Hence, the total number of channels is 2n, and the number of heat transfer plates is 2n-1. The substitution of equation (9) into equations (7) and (8) gives:

$$U(2n-1)wL_{ch} = \frac{C_{min}\varepsilon_{cc}}{1-\varepsilon_{cc}} \quad \text{for } C^* = 1 \tag{10}$$

$$U(2n-1)wL_{ch} = \frac{C_{min}}{1-C^*}\ln\left(\frac{1-C^*\varepsilon_{cc}}{1-\varepsilon_{cc}}\right) \quad \text{for } C^* \neq 1 \tag{11}$$

Using the correlation for the film heat transfer coefficient recommended by Cooper [12],

$$Nu = 0.28\,Re^{0.65}\,Pr^{0.4} \tag{12}$$

the overall heat transfer coefficients for clean conditions are as following.

$$\frac{1}{U} = \frac{n^{0.65}}{Const_H} + \frac{a}{\lambda_p} + \frac{m^{0.65}}{Const_{C1}} \tag{13}$$

$$\frac{1}{U} = \frac{n^{0.65}}{Const_H} + \frac{a}{\lambda_p} + \frac{(n-m)^{0.65}}{Const_{C2}} \tag{14}$$

where m is the number of cold channels occupied by cold stream C1, and then n-m is the number of cold channels occupied by cold stream C2. The term 'Const' can be seen as follows:

$$Const = 0.28\frac{\lambda}{D_e}\left(\frac{D_e \dot{M}_T}{wb\mu}\right)^{0.65}\left(\frac{C_p\mu}{\lambda}\right)^{0.4} \tag{15}$$

Multiplying equation (13) by (2n-1)wL_{ch}, and rearranging provides the following equation.

$$U(2n-1)wL_{ch} = \frac{(2n-1)wL_{ch}}{\frac{n^{0.65}}{Const_H} + \frac{a}{\lambda_p} + \frac{m^{0.65}}{Const_{C1}}} \tag{16}$$

Equating equation (16) with equations (10) and (11) results:

$$\frac{(2n-1)wL_{ch}}{\frac{n^{0.65}}{Const_H} + \frac{a}{\lambda_p} + \frac{m^{0.65}}{Const_{C1}}} = \frac{C_{min}\varepsilon_{cc}}{1-\varepsilon_{cc}} \quad \text{for } C^* \tag{17}$$

$$\frac{(2n-1)wL_{ch}}{\frac{n^{0.65}}{Const_H} + \frac{a}{\lambda_p} + \frac{m^{0.65}}{Const_{C1}}} = \frac{C_{min}}{1-C^*}\ln\left(\frac{1-C^*\varepsilon_{cc}}{1-\varepsilon_{cc}}\right) \quad \text{for } C^* \neq 1 \tag{18}$$

For sizing problem, if the desired effectiveness is specified with all of the plate and stream properties, equations (17) and (18) can be used as the design equations to obtain the number of required channels. However, unlike two-stream PHEs, one more equation must be used to make the zero degree of freedom. According to the restricted criteria for the design, which is U uniform for all heat transfer plate, therefore equation (13) must be equal to equation (14) to obtain:

$$\frac{n^{0.65}}{Const_H} + \frac{a}{\lambda_p} + \frac{m^{0.65}}{Const_{C1}} = \frac{n^{0.65}}{Const_H} + \frac{a}{\lambda_p} + \frac{(n-m)^{0.65}}{Const_{C2}} \tag{19}$$

$$\text{or} \quad \frac{m^{0.65}}{Const_{C1}} = \frac{(n-m)^{0.65}}{Const_{C2}} \tag{20}$$

Unfortunately, the system of equations, i.e. equations (17) and (20), or equations (18) and (20), are both highly non-linear. An efficient method to solve for values of n and m is to apply the numerical iterative techniques, which are widely available in the spreadsheet package programs such as the Microsoft Excel Solver or MATLAB. With the use of these programs, the solutions of the system of non-linear equations can be found rapidly.

6. Case Study

The hot fluid H, initially at 75 °C is fed into the three-stream PHE, comprising APV Junior plates [10], at a heat capacity flowrate of 160 W/K and flows through half of the total channels in counter-current flow to the two cold fluids (C1 and C2), at an inlet temperature 20 °C, flowing at a total heat capacity flowrate of 160 W/K (80 W/K for each). The required outlet temperature of the hot stream is 25 °C, whereas the cold streams should be 70 °C. The properties of fluids at the mean temperature, 50 °C are in Table 1.

The objective of the design is the determination of the number of channels for the selected plate in order to perform a specified duty by using the design equations (equations (17), (18) and (20)). There are four cases to be investigated depending upon the corporation of the cold side, which can be shown in Table 2.

The first two cases are set up in order to meet two objectives. The first is to verify the use of the design equations, since the obtained results of these cases should see the same number of channels for the two cold streams. The second objective is to investigate the effects of the different values of the term 'Const' of the two cold streams on the required total number of channels. For the third case, all of the three streams are different types, but the total

Table 1 The properties of fluids at mean temperature

Stream	C_p (J/kg K)	λ (W/m K)	μ (Pa s)	ρ (kg/m^3)
Stream H	3,800	0.5	0.0012	1,050
Stream C1	4,180	0.622	0.00074	994.5
Stream C2	4,200	0.68	0.00035	975

Table 2 The conditions of the four studying cases

Case 1	Both of the cold streams are C1 type.
Case 2	Both of the cold streams are C2 type.
Case 3	One cold stream is C1, another stream is C2.
Case 4	One cold stream is C1, another stream is C2.*

* The heat capacity flowrate of the hot fluid H has changed to 150 W/K, and the outlet temperature of the cold streams is about 66.87 °C.

Table 3 The solutions of the case studies

Case	Number of channels*			
	2n	n	m	n-m
1	380	190	95	95
2	288	144	72	72
3	324	162	67	95
4	146	73	30	43

*The terms 2n, n, m, and n-m are the total number of channels, the number of channels for hot side, the number of channels for one cold stream, and the number of channels for another cold stream respectively.

heat capacity flowrates of the hot and cold sides are the same ($C^* = 1$), and for the last case those values are different ($C^* = 0.9375$). The effects of the different values of the term 'Const' of the two cold types on the proportion of the numbers of cold channels can be investigated.

By solving the design equations (17) and (18) together with the constraint of uniform U, i.e. equation (20), the solutions of the required channels for hot and cold sides can be illustrated in Table 3. From the results in Table 3, three topics can be discussed:

1) To obtain the number of required channels for a three-stream PHE with two communications, the system of equations (17) or (18) and (20) can be solved numerically by using the Microsoft Excel Solver. The positive values of n and m are obtained from solving each equation in turn. A graphical

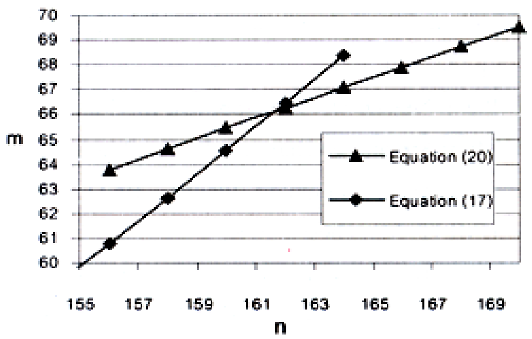

Figure 5 Finding the solutions by graphical method.

method is employed to find out a pair of those values that satisfies by both of the equations, see for example in Figure 5. The real numbers of n and m must be adjusted to their next integer values, since the channel number must have been an integer.

2) For the cases 1 and 2, the obtained numbers of channels for the two cold streams are the same, since the two cold streams are of the same type and having equal heat capacity flowrate. For the total number of channels, the case 1 needs 92 channels more than that of the case 2. This can be explained by the term 'Const' of the cold side, since this term can be arranged to give the following relationship for the film coefficient and the number of channels namely:

$$\alpha_{C1} = \frac{Const_{C1}}{m^{0.65}} \quad \text{and} \quad \alpha_{C2} = \frac{Const_{C2}}{m^{0.65}} \tag{21}$$

The value of $Const_{C1}$ is 7582.9 and is less than the value of $Const_{C2}$ at 9634.7. If the value of $Const_H$ is the same at 9472.7, the case 2 then needs less channels than the case 1 for the same duty due to a higher film heat transfer coefficient of the cold side.

3) For the results of the case 3, the numbers of channels for the two cold streams are different. This can be explained from the different values of the term 'Const' which has affected directly at the equation (20), namely:

$$\frac{m^{0.65}}{(n-m)^{0.65}} = \frac{Const_{C1}}{Const_{C2}} \tag{22}$$

The cold stream that has a larger value of Const, will need to distribute through more channels in order to maintain the uniform value of the overall heat transfer coefficient. For the case 4, the heat capacity flowrate of the hot side has decreased, so the value of C^* is less than 1, and the equation (18) will be used to estimate the number of channels instead of equation (17), and the total number of channels is less than that of the case 3 due to a smaller duty.

7. Conclusions

A new design structure as a single-phase three-stream plate-and-frame heat exchanger with two thermal communications is presented in this article. The six-port plates can be used to form a plate pack that can incorporate three streams without the use of a dividing plate. To estimate the required numbers of channels for each stream, the relationship of the heat exchanger effectiveness and the number of transfer unit for counter-flow exchanger can be employed, in which the constraint of uniform U must be followed. The results from the case studies show that the term 'Const' which consists of the stream properties and the plate dimensions, affects directly on the total number of channels and the channel proportions occupied by two cold streams. However, this paper only presents the case of one hot stream and two cold streams, the opposite of one cold stream exchanging heat with two hot streams can be described in the same manner, which can also result in the thermal design methodology on the basis of minimum number of channels.

8. Acknowledgments

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Nomenclature

A	heat transfer area, m^2
a	plate thickness, m

b	mean plate (flow) gap, m
C	heat capacity flowrate, W/K
C_p	specific heat capacity, J/kg K
C	heat capacity flowrate ratio
Const	constant term in equations (13) and (14)
D_e	hydraulic diameter, m
L_{ch}	length of plate channel, m
m	number of cold stream 1 channels
n	number of hot stream channels
NTU	number of transfer units
Nu	Nusselt number
Pr	Prandtl number
$\dot{Q}$	heat duty, W
Re	Reynolds number
T	temperature, K (°C)
ΔT_{LM}	log mean temperature difference, K (°C)
U	overall heat transfer coefficient, W/m^2 K
w	plate width, m
α	film heat transfer coefficient, W/m^2 K
ε	heat exchanger effectiveness
λ	thermal conductivity, W/m K
λp	plate thermal conductivity, W/m K
μ	fluid viscosity, N s/m^2

Subscripts

C1	cold stream C1
C2	cold stream C2
H	hot stream H
c	cold stream
h	hot stream
cc	counter current
in	inlet
out	outlet
max	maximum
min	minimum
T	total

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