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The Selection Method of Fuzzy Composite Operators Based on the Clear Field

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Abstract

Fuzzy comprehensive evaluation is one of the most effective evaluation methods. Fuzzy operations are based on fuzzy composite operators. Through an empirical analysis, we found that, with the same data set, using different fuzzy composite operators might get consistent results or inconsistent results. Therefore, a reasonable selection of fuzzy composite operators is one of the most important issues when conducting a fuzzy comprehensive evaluation. In the extensive literature, there is a clear gap on the general selection method of fuzzy composite operators. This paper analyzes the properties of five typical fuzzy composite operators, and proposes the definitions of the positive field, the negative field, the fuzzy field, the data field and the data point. Furthermore, this paper presents a classification method of the fuzzy composite operators based on the size of clear field and proposes a selection method of fuzzy composite operators based on the data field for Fuzzy evaluation applications.

Keywords: Fuzzy comprehensive evaluation; Fuzzy composite operators; Clear field; Fuzzy field; Data field

1. Introduction

In 1965, Zadeh founded the fuzzy set theory [1]. Based on fuzzy set theory, fuzzy comprehensive evaluation method has been applied in a wide range of fields. However, there are some gaps in the fuzzy set theory. For example, according to Zadeh's fuzzy set theory, the fuzzy sets cannot have the complementary sets. Therefore, the theory was not complete [2]. It makes the fuzzy operators unsystematic and how to use the fuzzy operators is not clear. So far, there is no general fuzzy operator selection method.

Through literature review, the number of research papers on fuzzy comprehensive evaluation is still great and growing year by year (as shown in Figure 1). This fully demonstrates that the evaluation method is the subject of concern. It also shows the importance of fuzzy evaluation.

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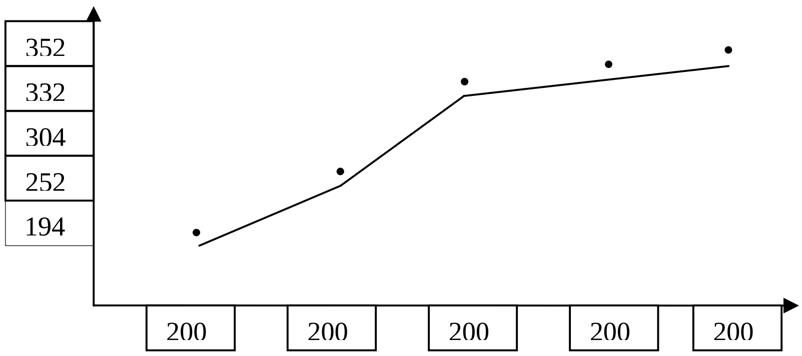


Figure 1 The quantity of fuzzy comprehensive evaluation papers

One stream of research on fuzzy composite operators is the theoretical research. The first fuzzy composite operator is the Zadeh operator, M(∧ , ∨). In 1973, with a logic-based view, Bellman and Giertz studied the axiomatic issues of fuzzy operators [3]. From the perspective of theoretical research on the fuzzy operators, Yager made outstanding contributions. Yager proposed Yager fuzzy operator and the concepts of fuzzy “and” degree and “or” degree [4]. In 1988, he proposed the ordered weighted operator (OWA) [5]. Fuzzy operator theory developed along from the Zadeh operator to the t-norms and s-norms then extended to the generalization operators. Wang and Li studied the clear fields of fuzzy composite operators, playing an important role in promoting the theoretical research on the fuzzy operators [6], Yager and Rybalov conducted an exhaustive study of uninorm operators, spreading t-norms and s-norms [7]. Xiao made an order of the fuzzy operators [8]. Li and colleagues proposed a power mean composite operator [9]. Radzikowska and Kerre defined several fuzzy modal operators and provided characterisations of main classes of binary fuzzy relations by means of these operators [10]. Ye and colleagues presented a comparative analysis of four kinds of fuzzy composite operators, including M (∧ , ∨), M (● , ∨), M (∧ ,) and (M ● ,), in terms of the information usage [11]. These studies got the independent results from different aspects and gained some certain theoretical contributions, but they were unsystematic. Especially, there is a lack of the methods on how to select fuzzy composite operators for evaluation applications.

In contrast to this stream of research, other studies focus on the applications of fuzzy composite operators [12]. In this regard, there are two main areas. One is the fuzzy control. For example, Ying studied the Zadeh operators in fuzzy control design [13]. The other is the fuzzy comprehensive evaluation. For example, Li and Shen used M (●, ⊕) to study the multi-level fuzzy comprehensive evaluation algorithm for security performance, and pointed different definitions of fuzzy operators leading to different fuzzy comprehensive evaluation models [14]. However, it failed to clarify the difference between M (●, ⊕) operator and any other operators. Chen and Chang summarized 11 basic fuzzy composite operators and used these operators to address the complexity in decision making of water resources redistribution in two neighboring river basins [15]. Süer and colleagues adopted 6 different fuzzy operators to study the cell loading problem and finally reported that two fuzzy operators are suitable to reach efficient solutions for the problem [16].

In these extensive studies, there was no explanation on how the authors selected the fuzzy composite operators. General research papers on fuzzy comprehensive evaluation are a lot, but the studies on fuzzy operators are scant. There is a lack of the research on properties of fuzzy composite operators. However, this issue is very important. Due to the lack of research on how to select the fuzzy operators, the applications of fuzzy composite operators are unsystematic. For example, with a data set, Shen and colleague [17] calculated $\tilde{B}_5 \bullet \tilde{R}_5$ = (0.21，0.21，0) using the Zadeh operator (∧， ∨), while the result of using the bounded sum and product operator (,) was (0.76, 0, 0). For these two different results, it is not sure which one is reasonable or they may all be reasonable. This issue is worth to be studied. When selecting the fuzzy composite operators, the reasons need to be addressed. Otherwise, the credibility of the results is doubtful, leading to the false management decision making.

In this paper, we discuss the clear fields and fuzzy fields of five typical composite operators, including Zadeh fuzzy operator（∧，∨), max-product operator（●， ∨), probability sum and product operator ($\bullet, \hat{+}$), bounded sum and product operator （ ， ）, and the Yager operator. Moreover, we proposed a fuzzy composite operator selection method based on data field and clear field.

2. Clear Field of Fuzzy Operator

Fuzzy comprehensive evaluation is based on fuzzy set theory. Before studying the properties and the application methods of fuzzy composite operators, we must firstly understand the mathematical definitions of some main concepts of the fuzzy operators under fuzzy set theory framework, such as the fuzzy composite operators and clear field. This will make the fuzzy operations more reasonable.

2.1 T-norms and T-norms

Let the fuzzy operator $*$ be $[0,1]\times[0,1]\rightarrow[0,1]$. For $x, y, z \in [0,1]$,

（ ） $x * y = y * x$;

（ ） $(x * y) * z = x * (y * z)$;

（ ） $\forall z \in [0,1]$, if $x \le y$， then $x * z \le y * z$;

（ ） $x * 1 = x$ or（ ^） $x * 0 = x$.

If the binary operator $*$ meets the conditions of（ ）、（ ）、（ ）、（ ）, we name it the triangle t-norms or t-norms. If the binary operator $*$ meets the conditions of（ ）、（ ）、（ ）、（ ^）, we name it the triangle t-norms or s-norms. Fuzzy composite operators are composed by t-norms and s-norms. The common fuzzy composite operators have following several kinds:

（1）Zadeh operators (∧， ∨): $x \wedge y \overset{\Delta}{=} \min\{x, y\}$ and $x \vee y \overset{\Delta}{=} \max\{x, y\}$

（2）Max-product operators($\bullet, \vee$): $x \bullet y \overset{\Delta}{=} xy$ and $x \vee y \overset{\Delta}{=} \max\{x, y\}$

（3）Probability sum and product operator ($\bullet, \hat{+}$)： $x \bullet y \overset{\Delta}{=} xy$ and $x \hat{+} y \overset{\Delta}{=} x + y - xy$

（4）Bounded sum and product operators（⊙， ⊕)： $x \odot y \overset{\Delta}{=} \max\{0, x + y - 1\}$ and

$$x \oplus y \overset{\Delta}{=} \min\{x + y, 1\}$$

（5）Yager operator（$\overset{\bullet}{\varepsilon}, \overset{+}{\varepsilon}$）:　$x \overset{\bullet}{\varepsilon} y \overset{\Delta}{=} 1 - \min\left\{1, [(1-x)^p + (1-y)^p]^{\frac{1}{p}}\right\}$

$$x \overset{+}{\varepsilon} y \overset{\Delta}{=} \min\left\{1, [x^p + y^p]^{\frac{1}{p}}\right\} \quad p \in [1, +\infty].$$

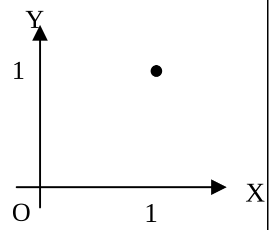
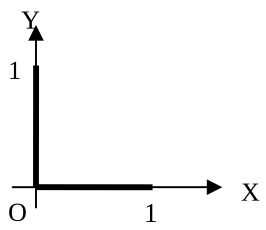
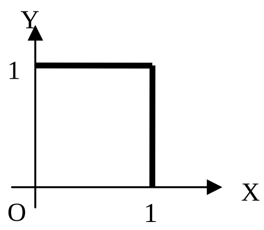
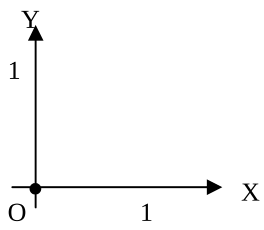
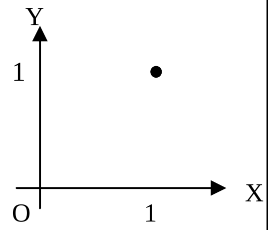
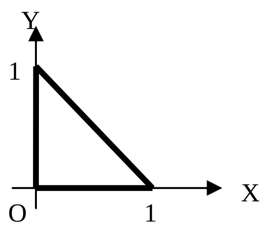
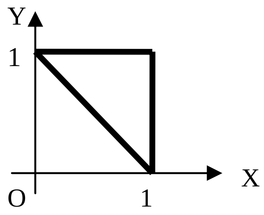
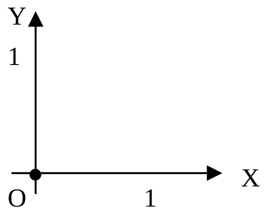
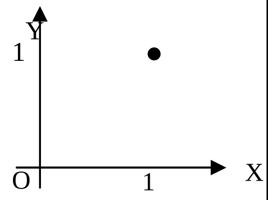
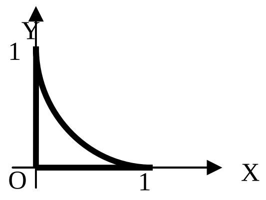
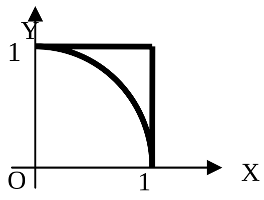
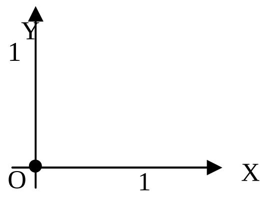
2.2 Clear Field

For the fuzzy operation, there is a data set which makes the operation results clear that it is a Clear field. Let $*$: $[0,1] \times [0,1] \to [0,1]$ be fuzzy operator We define $\sigma(*)$ as the Clear field of fuzzy operator $*$: $\sigma(*) \overset{\Delta}{=} \{(x, y) \in [0,1] \times [0,1] | x * y = 0, \quad x * y = 1\}$

$\sigma(*) \overset{\Delta}{=} \{(x, y) \in [0,1] \times [0,1] | x * y = 1\}$ is Positive field, $\sigma(*) \overset{\Delta}{=} \{(x, y) \in [0,1] \times [0,1] | x * y = 0\}$ is Negative field,

$\sigma(*) = \{(x, y) \in [0,1] \times [0,1], 0 \langle x * y \langle 1\}$ is Fuzzy field.Through fuzzy operation with the data in Clear field, the result is either positive or negative. It's a determinate result. A fuzzy operator's Clear field and Fuzzy field constitute a unit square area. The size of clear field can be reflected by the area value. We record it as $S(*)$, $0 \le S(*) \le 1$. Based on area size above five Fuzzy composite operators graph classification is shown in Table 1.

Table 1 Classification of the Clear fields of fuzzy operators

Fuzzy composite operators	**Positive fields of fuzzy “and” operation**	**Negative fields of fuzzy “and” operation**	**Positive fields of fuzzy “or” operation**	**Negative fields of fuzzy “or” operation**
Zadeh $(\wedge, \vee)$ Max-product $(\bullet, \vee)$ Probability sum and product $(\bullet, \hat{+})$				
Bounded sum and product（ ， ）				
Yager $(\dot{y}, \overset{+}{y})$, Wnen $p = 2$				

3. Application Method of Fuzzy Composite Operators

In the above section, we argue that the clear fields have a negative impact on the fuzzy operations. The larger the area of clear field is, the greater the probability of the computing data falling into the clear field and the smaller the probability of the computing data falling into the fuzzy field which makes the fuzzy operations more inadequate. Therefore, when selecting the fuzzy composite operators, we should avoid or reduce this effect. Clearly, a composite operator with a smaller clear field is more suitable. Data is one of the most basic elements of the evaluation process. Considering the causality between data and fuzzy operation results, if the data is not in the clear field but in the fuzzy field, it will make the causal relationship between data and fuzzy operation results being fully reflected. To some extent, this can weaken the negative impact of the clear fields on the fuzzy operations. In terms of the evaluation data, the less the data belongs to the clear field, the better the evaluation result is.

The objective of studying the selection of fuzzy composite operators is to choose a reasonable fuzzy composite operator according to the evaluation data. First, we should recognize the extent of the data, also known as the data field. Then we should identify the first points of the fuzzy operations, also known as the data point. Second, we should identify the composite operator's clear field of fuzzy "and" operation. That is because the fuzzy "and" operation has priority. Third, we should select the fuzzy composite operator according to the data least subjection principle.

To this end, we specifically addressed as follows.

3.1 Data field and Data point

Let the index weight vector $A=\left(a_1, \quad a_2, \quad ......, \quad a_m\right)$, where $\sum_{i=1}^{m} a_i = 1, a_i \in [0,1]$.

Evaluation matrix $R=\begin{pmatrix} r_{11} & r_{12} & ... & r_{1n} \\ r_{21} & r_{22} & ... & r_{2n} \\ ... & ... & ... & ... \\ r_{m1} & r_{m2} & ... & r_{mn} \end{pmatrix}$, where $r_{jk} \in [0,1]$, $j=1,2,...,m$, $k=1,2,...,n$.

Let $a_s = \min(a_i), a_t = \max(a_i)$, $r_{pq} = \min(r_{jk})$, $r_{mn} = \max(r_{jk})$, we define the area enclosed by the points of (a_s, r_{pq}), (a_s, r_{mn}), (a_t, r_{pq}), (a_t, r_{mn}) as the data field. It is recorded as G. Define $M_{ij}(a_i, r_{ij})$ as the data point in the data field G. The data points belong to the data fields. Each fuzzy evaluation issue has a finite number of the data points. We should identify the locations of the data points and the clear fields of fuzzy composite operators in order to determine the affiliation between the two. And then select the most effective fuzzy composite operator towards the actual evaluation issue.

If the data points fall into a clear field, we can get the "positive" or "negative" results without operations. If we have little data in the fuzzy field, the causal relationship between data and evaluation vector is inadequate. Therefore, when we select the fuzzy composite operators, we should make the data points to avoid the clear field. That is minimizing the number of the data points belonging to the clear field. We should view it as the basic principle of fuzzy composite operator selection.

Let there is k data points in the data field, and $G_1, G_2 ..., G_t$ is the clear fields of t

types of fuzzy composite operators' fuzzy "and" operations. k_i is the number of data points in G_i, where $1 \le i \le t$, $k_i \le k$. If $k_i = \min\{k_1, k_2, ..., k_t\}$, the data points in G_i is the least, we can believe that the data points belonging to the clear field of composite operator *i*' fuzzy "and" operation is the least. Thus, to the fuzzy composite operator *i*, these data points meet the data points least subjection principle. First, analyze the fuzzy evaluation issue and determine the weights, judgment matrix and fuzzy composite operators to be selected. We may choose several commonly used fuzzy composite operators, or define a fuzzy composite operator as an alternative operator.

Second, determine the data fields and data points, while identifying the clear fields of several composite operator options. Determine the affiliation of data points and the clear fields.

Finally, determine which should be selected as the fuzzy composite operator based on data points least subjection principle. The specific steps are as follows:

（1）When there is only one best option in the alternative composite operators, this operator can be viewed as the fuzzy composite operator for the evaluation issue.

（2）When multiple options are considered as the suitable composite operators, we should draw attention on the evaluation issue and the practical effect of the composite operators to make the further selection decision. For example, by calculating, we can obtain the closeness of the composite operators' evaluation vector and the mean evaluation vector.

（3）If there is no suitable alternative operator for the actual evaluation, we need to determine new composite operators for re-selection.

The process of fuzzy composite operators selection is shown in Figure 2.

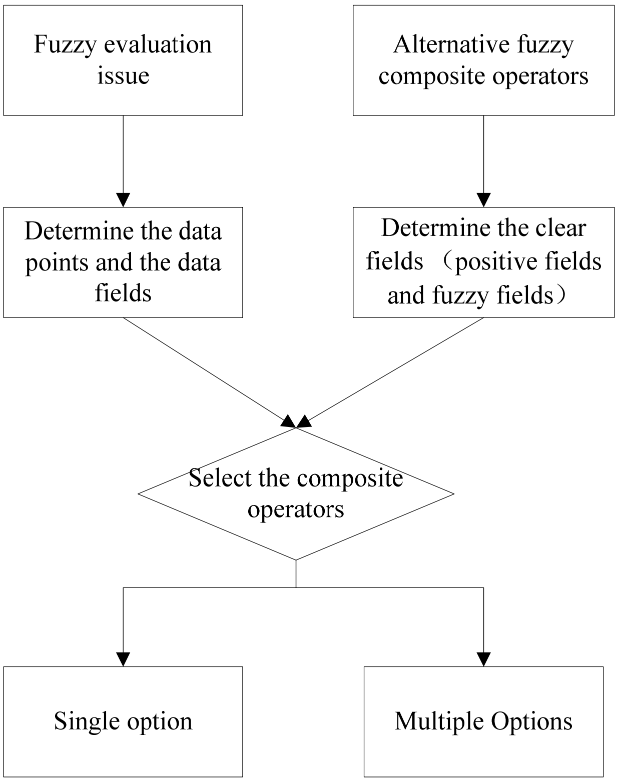


Figure 2 The process of fuzzy composite operator selection

3.2 Empirical Tests

We adopt five different fuzzy composite operators and the data set in Shen and colleagues [17] to conduct a comparative empirical analysis.

$$\tilde{B}_5 * \tilde{R}_5 = (0.11,0.18,0.08,0.04,0.19,0.17,0.1,0.1,0.21) * \begin{pmatrix} 1 & 1 & 1 & 1 & 0.93 & 0.47 & 1 & 1 & 0.61 \\ 0 & 0 & 0 & 0 & 0.07 & 0.53 & 0 & 0 & 0.39 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}^T$$

The Data field is the area enclosed by 4 points, （0.01, 0）,（0.01,1）,（0.21,0）,（0.21,1.
The number of the Data points is 19, distributed as follows.

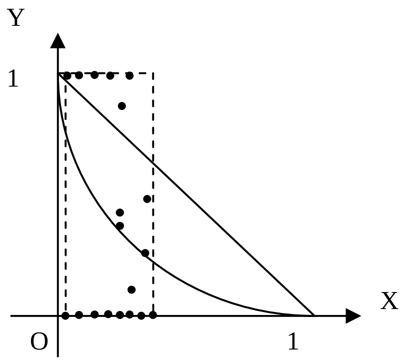


Figure 3 Data points& data field

The negative fields of the composite operators ($\wedge$, $\vee$), ($\bullet$,$\vee$) and ($\bullet$,$\hat{+}$)'s fuzzy "and" operator are the same (see Table 1 and Figure 3), contains eight points all on the boundary line. When $p = 2$, there are nine points in the Negative field of the Yager operators ($\overset{\bullet}{y}, \overset{+}{y}$)'s fuzzy "and" operator, while there are 13 points in the Negative field of the bounded sum and product operator (,)'s fuzzy "and" operator, it is worst. Therefore, the use of ($\bullet$,$\vee$) or ($\bullet$,$\hat{+}$) is quite effective. We respectively conduct the fuzzy operations through the five composite operators. Then we get the evaluation vectors (Table 2).

Table 2 Comparison of the results of the 5 fuzzy composite operators

Zadeh（$\wedge$, $\vee$）	$(0.21 \quad 0.21 \quad 0)$
（$\bullet$,$\vee$）	$(0.18 \quad 0.0901 \quad 0)$
（$\bullet$,$\hat{+}$）	$(0.5069 \quad 0.1757 \quad 0)$
（ , ）	$(0.76 \quad 0 \quad 0)$
Yager（$\overset{\bullet}{y}, \overset{+}{y}$） $p = 2$	$(0.9492 \quad 0.3866 \quad 0)$

First, the composite operators ($\wedge$, $\vee$), ($\bullet$,$\vee$) and ($\bullet$,$\hat{+}$) belong to the same kind of operators. The spearman correlation coefficient of each evaluation vector and the mean evaluation vector (0.5212, 0.1724, 0) is close to 1, and Kendall consistency coefficient is 0.99997. The consistency is very significant. Since $(\wedge, \ \vee)$ loses 50% of the data through the fuzzy "and" operation, the maximum values of the evaluation vector's first and second component value are the same, leading to the same result of fuzzy "or" operation. The order status is different with $(\bullet,\vee)$ and $(\bullet,\hat{+})$. Therefore, $(\bullet,\vee)$ and $(\bullet,\hat{+})$ is better than $(\wedge,\vee)$.

Second, because when $p=2$, the negative field of the Yager operator's fuzzy "and" operation contains 9 data points and the fuzzy field contains the 10 data points. From the perspective of the causal relationship of the data points and the evaluation vectors, the basis of the evaluation vectors is relatively weak. Thus, Yager operators is not the best choice. Similarly, the negative field of the composite operator (,)'s fuzzy "and" operation contains 13 data points, and the fuzzy field contains only 6 data points. Therefore, the basis of the evaluation vectors is inadequate. We can not select (,) as the composite operator.

In summary, $(\bullet,\vee)$ and $(\bullet,\hat{+})$ are able to get more objective results.

If we have to select one from the two, by calculating, we can get the Hamming distance $\rho_{(\bullet,\vee)}$、$\rho_{(\bullet,\hat{+})}$ between $(\bullet,\vee)$ and $(\bullet,\hat{+})$'s evaluation vectors and the mean evaluation vector (0.5212, 0.1724, 0), and then the closeness $N_k(A,B)=1-\rho_k$. By calculating, we get $N_{(\bullet,\vee)}=1-\rho_{(\bullet,\vee)}=0.9177$ and $N_{(\bullet,\hat{+})}=1-\rho_{(\bullet,\hat{+})}=0.9824$. Thus, the best option is $(\bullet,\hat{+})$. The $(\bullet,\vee)$ and $(\bullet,\hat{+})$ composite operators get the same sort of evaluation vector.

The above calculation shows that the same type of fuzzy composite operators is clearly consistent in terms of dealing with the evaluation data. They always get the same result. When dealing with the same data using different types of fuzzy composite operators, there are obvious differences. Therefore, when facing the fuzzy comprehensive evaluation issues, we should identify the types of operators and the characteristic of the data, and effectively consider both of the data and the operators. This can result in the scientific and objective evaluation outcomes.

4. Conclusions

Our analysis indicates the fact that, in the fuzzy operation process, the Clear field has a negative impact on the causal relationship between the data and the results of fuzzy operations. This paper, based on the concept of Clear field, proposes the definitions of Positive field, Negative field and Fuzzy field. Using the ichnography method, the paper showed the Positive field and Negative field of five typical fuzzy composite operators' fuzzy "and" operations and fuzzy "or" operations. We find that some Clear fields of these operators are the same, and some are different. Furthermore, we identify the cause that the fuzzy fields of the fuzzy "and" operations have the negative impact on the causal relationship between the data and results of fuzzy operations.

To avoid the negative impact of the Clear fields, we propose a method that classify the fuzzy composite operators according to the size of fuzzy "and" operators' Negative field. This paper classifies the five typical composite operators into three categories: Zadeh composite operator, bounded sum and product composite operator and composite operator with a parameter. Based on this classification, we demonstrate that the same type of fuzzy composite operators result in the consistent result with the same data. And different types of fuzzy composite operators may lead to different results when dealing with the same data. We should select suitable operators according to the evaluation data.

Based on the causal relationship between the data and fuzzy operation results, we propose the concepts of data field and data points, and demonstrate the process of fuzzy composite operator selection based on the data field and clear field. Then we conduct an empirical test to prove the validity of the method.

This paper recommends that the scholars need to show the basis of fuzzy composite operator selection when conducting fuzzy evaluations and writing related articles. This can ensure that the evaluation process and result is scientific and rationale. In this paper, the specific selection method is given.

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