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Research Article

# Analysis of a two-phase induction motor using dynamic model based on Matlab/Simulink

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## Abstract

This paper sets forth the development of the stationary  $(\alpha\beta)$  reference frame model of an asymmetrical two-phase induction motor. The aim of this research was to develop and design an asymmetrical two-phase induction motor suitable for transient and steady-state analysis. The system has been simulated to verify its capability such as input phase voltages, stator and rotor currents, electromagnetic torque and rotor speed under conditions of no-load and full-load test. The performance of an asymmetrical two-phase induction motor under these various conditions is simulated using Matlab/Simulink and the simulation results demonstrate the feasibility of the proposed scheme.

**Keywords:** stationary reference, dynamic model, simulation, small power applications, Thailand.

## Introduction

In small power applications, asymmetrical two-phase induction motors fed by single-phase supply have been widely used in electric machines in home appliances and industrial applications requiring less than 5 kW [1]. Single-phase induction motors with main and auxiliary winding can be viewed as two-phase machines, since these winding mechanisms are displaced ninety degrees apart from each other. Therefore, two-phase induction motors have a configuration identical to single-phase induction motors, but the input voltage applied to the stator winding terminals is independently controlled so that a two-phase voltage is supplied.

In recent years, several methods that use models for simulation of two-phase induction motors have been proposed. The dynamics of single/two phase induction motors have been studied and

developed with Matlab software [2]. The simulation model showing start-up transients and fault transients of the different motors are compared for performance. In Simulink development of a two-phase induction motor model, Krause's model was analyzed on a two-phase induction motor with known parameters [3]. The saturation phenomenon for main magnetic flux and its implications on the function characteristics of a two-phase induction motor were studied. It offers good transient and steady-state performance. However, this technique has limitations in being computationally intensive.

This paper presents the design and simulation of an asymmetrical two-phase induction motor using a dynamic model with Matlab/Simulink. The proposed method is applicable to both transient and steady-states of the two-phase induction motor under conditions of no-load and full-load test. This method is based on the analysis of the stationary ( $\alpha\beta$ ) reference frame model of an asymmetrical two-phase induction motor with a general two single-phase supply connected to its terminal. Following this, mathematical modelling of the asymmetrical twophase induction motor is undertaken based on a stationary reference frame. The dynamic simulation model using Matlab/Simulink programs is then developed and the results and performance evaluated.

#### **Mathematical Model**

The two-phase induction motor is composed of two asymmetrical windings. Therefore, the auxiliary winding usually has fewer turns than the main winding and is displaced at ninety electrical degrees between these winding [4, 5]. Fig. 1 shows the schematic view of a two-phase induction motor, illustrating that the auxiliary ( $\alpha$ ) windings and main ( $\beta$ ) windings are not identical sinusoidal distributed windings, but are arranged in space quadrature.



Figure 1. Asymmetrical two-phase induction motor.

The equivalent circuits representing the asymmetrical two-phase induction motor in stationary  $(\alpha\beta)$  reference frame are shown in Fig. 2. The dynamic model equation of asymmetrical two-phase induction motor can be written as  $\alpha\beta$  reference frame variables. Components stator and rotor voltage of the two-phase induction motor can be expressed as follows:



(a) Auxiliary winding in  $\alpha$  - axis.



(b) Main winding in  $\beta$  - axis.

# Figure 2. Equivalent circuit of an asymmetrical two-phase induction motor in the stationary ( $\alpha\beta$ ) reference frame.

$$v_{s\alpha} = R_{s\alpha}i_{s\alpha} + \frac{d}{dt}\psi_{s\alpha}, \qquad (1)$$

$$v_{s\beta} = R_{s\beta}i_{s\beta} + \frac{d}{dt}\psi_{s\beta}, \qquad (2)$$

$$v_{r\alpha} = 0 = R_{r\alpha}i_{r\alpha} + \frac{d}{dt}\psi_{r\alpha} + a\omega_r\psi_{r\beta}, \qquad (3)$$

$$v_{r\beta} = 0 = R_{r\beta}i_{r\beta} + \frac{d}{dt}\psi_{r\beta} - \frac{1}{a}\omega_r\psi_{r\alpha}.$$
 (4)

The components of stator and rotor flux linkages equations can also be expressed as:

$$\psi_{s\alpha} = L_{s\alpha}i_{s\alpha} + L_{m\alpha}i_{r\alpha}, \qquad (5)$$

$$\psi_{s\beta} = L_{s\beta} i_{s\beta} + L_{m\beta} i_{r\beta}, \qquad (6)$$

$$\psi_{r\alpha} = L_{m\alpha} i_{s\alpha} + L_{r\alpha} i_{r\alpha}, \qquad (7)$$

$$\psi_{r\beta} = L_{m\beta}i_{s\beta} + L_{r\beta}i_{r\beta}.$$
 (8)

Using equation (5)-(8), as for the stator and rotor currents equations are given by:

$$i_{s\alpha} = \frac{L_{r\alpha}\psi_{s\alpha} - L_{m\alpha}\psi_{r\alpha}}{L_{s\alpha}L_{r\alpha} - L_{m\alpha}^2},$$
(9)

$$i_{s\beta} = \frac{L_{r\beta}\psi_{s\beta} - L_{m\beta}\psi_{r\beta}}{L_{s\beta}L_{r\beta} - L_{m\beta}^2},$$
(10)

$$i_{r\alpha} = \frac{L_{s\alpha}\psi_{r\alpha} - L_{m\alpha}\psi_{s\alpha}}{L_{s\alpha}L_{r\alpha} - L_{m\alpha}^2},$$
(11)

$$i_{r\beta} = \frac{L_{s\beta}\psi_{r\beta} - L_{m\beta}\psi_{s\beta}}{L_{s\beta}L_{r\beta} - L_{m\beta}^2}.$$
 (12)

The equation of electromagnetic torque produced by the machine is then given by the equation:

$$T_e = p_p \left( L_{m\beta} i_{s\beta} i_{r\alpha} - L_{m\alpha} i_{s\alpha} i_{r\beta} \right), \tag{13}$$

and the mechanical dynamic is modeled by the equation

$$J\frac{d}{dt}\omega_r = T_e - T_L. \tag{14}$$

where  $v_{s\alpha}, v_{s\beta}, v_{r\alpha}, v_{s\beta}$  are the stator and rotor voltages,  $i_{s\alpha}, i_{s\beta}, i_{r\alpha}, i_{s\beta}$  are the stator and rotor currents,  $\psi_{s\alpha}, \psi_{s\beta}, \psi_{r\alpha}, \psi_{s\beta}$  are the stator and rotor flux linkages,  $R_{s\alpha}, R_{s\beta}, R_{r\alpha}, R_{s\beta}$  are the stator and rotor resistances,  $L_{s\alpha}, L_{s\beta}, L_{r\alpha}, L_{s\beta}$  are the stator and rotor inductances,  $L_{m\alpha}, L_{m\beta}$  are the magnetizing inductances,  $\omega_r$  is the electrical rotor angular speed,  $T_e$  is the electromagnetic torque,  $T_L$  is the load torque, J is the rotor moment of inertia,  $\frac{d}{dt}$  is the differential operator and a is the main per auxiliary winding turns ratio.

#### **Modelling Using Matlab/Simulink**

Matlab/Simulink models were developed to examine the asymmetrical two-phase induction motor. The equation from (1)-(4) and (9)-(14) have been implemented in Simulink using a different block. In this paper the step by step modelling of the asymmetrical two-phase induction motor has been described.

Figs. 3 and 4 show the subsystem block of stator and rotor voltages and currents, respectively. The electromagnetic torque and rotor speed determined is as shown in Fig. 5. Finally in Fig. 6 implementation of the Simulink model of asymmetrical two-phase induction motor has been shown.



Figure 3. Stator and rotor voltage equations.



Figure 4. Stator and rotor current equations.



Figure 5. Electromagnetic torque and rotor speed equations.



Figure 6. Implemented Simulink model as proposed in this paper.

#### **Simulation Results**

In this section, some simulation results showing excellent performance of the proposed system are presented. The simulation model has been developed in Matlab/Simulink environment. The parameters of the asymmetrical two-phase induction motor are then calculated from short circuit and no-load measurements. Throughout the main and auxiliary winding resistances are measured using DC tests, while the other parameters are estimated using the stand-still and synchronous speed tests [1]. The motor parameters are tabulated in Table 1 and the ratings are 110 V, 60 Hz, 1/4 HP, and four-pole [2]. In all simulated cases, the load torque is fixed at 1.0096 Nm.

#### Table 1. Two-phase induction motor parameters

$R_{s\alpha} = 7.14 \ \Omega$	$L_{s\alpha} = 0.2549 \text{ H}$	$L_{m\alpha} = 0.2464 \text{ H}$
$R_{s\beta} = 2.02 \ \Omega$	$L_{s\beta} = 0.1846 \text{ H}$	$L_{m\beta} = 0.1772 \text{ H}$
$R_{r\alpha} = 5.74 \ \Omega$	$L_{r\alpha} = 0.2542 \text{ H}$	$J = 2.92 \times 10^{-3} \text{ kg-m}^2$
$R_{r\beta} = 4.12 \ \Omega$	$L_{r\beta} = 0.1828 \text{ H}$	a = 1.18

The simulation results for the first method, which uses dynamic modelling, is shown in Fig. 7. In Fig. 7 (a), the main and auxiliary supply voltage are identical in amplitude at 60 Hz and two phase quadrature (sine and cosine wave forms) voltages were selected according to  $v_{aux.} = av_{main}$  [6]. The supply voltage  $v_{aux.}$  was fixed at  $1.18v_{main}$  since the winding turn ratio *a* was estimated to be approximately 1.18. The simulation results of the dynamic response under no-load is shown in Fig. 7 (b) and (d). It can be seen that the stator and rotor currents decrease into steady-state. At zoom pictures the stator and rotor currents. It can be seen that the stator and rotor currents is well quadrature shown in Figs 7 (c) and (e). As a result, in Figs. 7 (f)-(g) it is seen that the electromagnetic torque response is generated in transient and steady-state and rotor speed is constant at 1800 rpm.



Figure 7. Simulation results of the main and auxiliary two-phase induction motor ( $v_{main} = 110V_{rms}$  and  $v_{aux.} = a^*110V_{rms}$ ) at no-load condition.

(a) supply voltage (b) stator current (c) stator current at zoom (d) rotor current (e) rotor current at zoom (f) electromagnetic torque and (g) rotor speed.



Figure 8. Simulation results of the main and auxiliary two-phase induction motor ( $v_{main} =$  $v_{aux.} = 110 V_{rms}$ ) at no-load condition. (a) supply voltage (b) stator current (c) stator current at zoom (d) rotor current (e) rotor current at zoom (f) electromagnetic

torque and (g) rotor speed.

Fig.8 (a) shows the simulation supply voltage for 60 Hz two phase quadrature voltages with equal magnitude of  $110V_{rms}$  at no-load. Under these conditions the auxiliary winding current is no longer in quadrature, and has small magnitude. This is because the larger back-emf developed in the auxiliary winding prevents a useful winding current flowing when the applied phase voltage magnitude is too low as shown in Figs 8 (b)-(c). Figs 8 (d)-(e) show transient and steady-state rotor current, respectively. In Figs. 8 (f)-(g) shows the transient responses of the electromagnetic torque and rotor speed is 1800 rpm at no-load.

The transient and dynamic response are presented in Fig 9, when applied supply voltage  $(v_{aux.} = av_{main})$  is changed values of the load torque at 1.0096 Nm (full-load) as shown in Fig. 9 (a). In Figs 9 (b)-(e), shows the transient and dynamic response of the stator and rotor winding currents. Fig. 9 (f)-(g) shows the transient responses of the rotor speed and the electromagnetic torque when the electromagnetic torque is changed. It is noted here that the smooth and stable of speed at 1765 rpm.

The supply voltage, the stator and rotor currents waveform, the electromagnetic torque and rotor speed obtained with the scheme are shown in Fig 10 (a)-(g), respectively. The rotor speed is 1765 rpm as shown in Fig. 10 (e) and the reference load torque is 1.0096 Nm. Fig. 10 (a) shows the steady-state response of the motor when it is supplied with a two-phase balanced sinusoidal supply. The simulation results of the dynamic response under full-load is shown in Fig. 10 (b) and (d). It can be seen that the high ripple stator and rotor currents. Zoom on pictures the stator and rotor currents, it can be seen that the stator and rotor currents is well quadrature shown in Figs 10 (c) and (e). The electromagnetic torque responses of the two-phase induction motor when it has two asymmetrical winding are shown in Fig 10 (f). In this case, the torque ripple under full-load is 1 Nm (rate torque).

Although the supply input voltage  $(v_{aux.} = av_{main})$  has better performance than the two phase quadrature supply voltages with equal magnitude  $(v_{aux.} = v_{main})$ , in this research work asymmetrical two-phase induction motors were used since they are commercially available over the counter.



Figure 9. Simulation results of the main and auxiliary two-phase induction motor ( $v_{main} =$  $110V_{rms} \text{ and } v_{aux.} = a*110V_{rms} \text{) at full-load condition.}$ (a) supply voltage (b) stator current (c) stator current at zoom (d) rotor current (e) rotor current at zoom (f) electromagnetic

torque and (g) rotor speed.



Figure 10. Simulation results of the main and auxiliary two-phase induction motor ( $v_{main} = v_{aux.} = 110 \text{ V}_{rms}$ ) at full-load condition.

(a) supply voltage (b) stator current (c) stator current at zoom (d) rotor current (e) rotor current at zoom (f) electromagnetic torque and (g) rotor speed.

# Conclusions

This paper has presented the developed and designed dynamic modelling of the asymmetrical two-phase induction motor using Matlab/Simulink. The paper has proposed model analyses the transient and steady state characteristics with variable two phase quadrature supply voltages. The simulation results have shown that the transient and steady-state of the supply voltage, the stator and rotor winding currents, the electromagnetic torque and rotor speed under no-load and full-load. The model gives good dynamic and steady-state response performance of the asymmetrical two-phase induction motor.

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