Research Article

City bus routing model for minimal energy consumption

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Abstract

Chiang Rai has a high percentage of private vehicle usage which is about 90% of all vehicle types. As a result, many places, such as education or business areas, experience a traffic-jam problem. The problem brings about impacts on fuel consumption and environment. Public transport with minimal energy consumption is an alternative way to solve this problem. However, to be effective, characteristics of public transport system such as route network and vehicle scheduling should be well designed. In this paper, we extend the classical Capacitated Vehicle Routing Problem (CVRP) to analyze for appropriate bus routes under a given objective minimization of the fuel consumption by considering velocity, weight and power per weight ratio. We consider single and multi route problem and propose integer linear programming formulations for both cases. Branch and Bound Method was also applied to assist in finding a solution. It was found that Breadth First Search (BFS) and Best Local Bound (BLB) performed better than Depth First Search (DFS) in finding solutions. Also, the model developed by interfacing of GIS (Graphic Information System) modules, MATLAB and GLPK (GNU Linear Programming Tool Kit) can be used with convenience. It appears to have potential for further development for the case of Capacitated Open Vehicle Routing Problem (COVRP) and Multi-Depot Vehicle Routing Problem (MDVRP).

Keywords: bus routing, energy conservation in transport, fuel saving, Capacitated Vehicle Routing Problem (CVRP), Thailand

Introduction

For this research, the well-known Capacitated Vehicle Routing Problem (CVRP) was used and extended to include additional Fuel Consumption Constraints in the case of public transportation and logistics.

Satayopas B. *et al* [1] suggested that the public transport system in Chiang Rai should be promoted and developed to give better service and be more effective. It is important that a new public transport system should be providing wider area coverage in Chiang Rai city. When public transport becomes popular among trip makers, traffic problems will be solved. It is known that many areas are not well covered by public transport at the present time. By way of example, Tambol Rob-Vieng area is serviced by more than one bus route while some other areas have no service at all (see Figure 1). Lacking service of public transport in those areas resulted in an increase in using private vehicles.
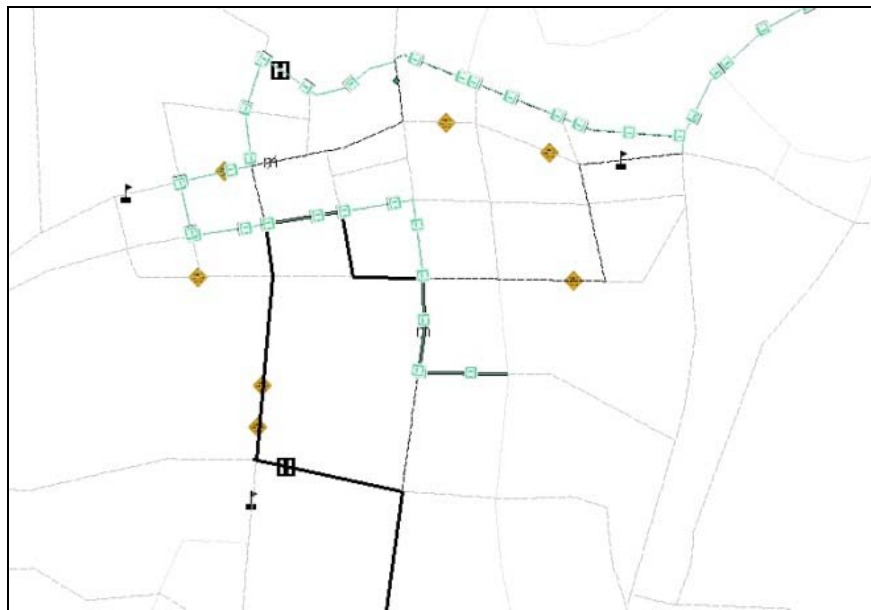

Figure 1. Public Transportation Routes (Rob-Vieng Zone).

Figure 1 shows the 3 bus routes are Rob-Vieng (line), San-Sai (bold line) and Rajabhat (bus symbol line).

CVRP was first defined by Dantzig and Ramser in 1959 [2]. In that study, the authors use distance as a surrogate for the cost function. Imdat Kara and Tolga Bektas [3], in their study, used CVRP and extended to the case where each vehicle is restricted to an additional minimal starting or returning load constraint. This problem formulation is known as the Minimal Load Constrained Vehicle Routing Problem. Two years later, Imdat Kara, Bahar Y. Kara and M. Kadri Yetis [4] introduced a new cost function based on distance and load of the vehicle for the Capacitated Vehicle Routing Problem. A heavily loaded truck will use more fuel than a lightly loaded truck. In that study, the authors use fuel as a surrogate for the cost function, where the fuel consumption is a function of load, distance traveled.

Quite often the design of public transport systems both for serving the suburbs and town area are based on efficient fuel consumption. The purpose of this paper is thus to analyze for appropriate bus routes under a given objective of minimizing fuel consumption by considering velocity, weight and power per weight ratio. The problem was formulated as Integer Linear Programming Formulations (ILPFs) and solved by numerical method.

In section 2, the ILPFs Model is formulated in mathematical form and applied to determine an appropriate bus route passing a selected school (shown in Figure 5).

In section 3, the method is further applied for the design of bus network as a case study of Chiang Rai city as shown in Figure 2.
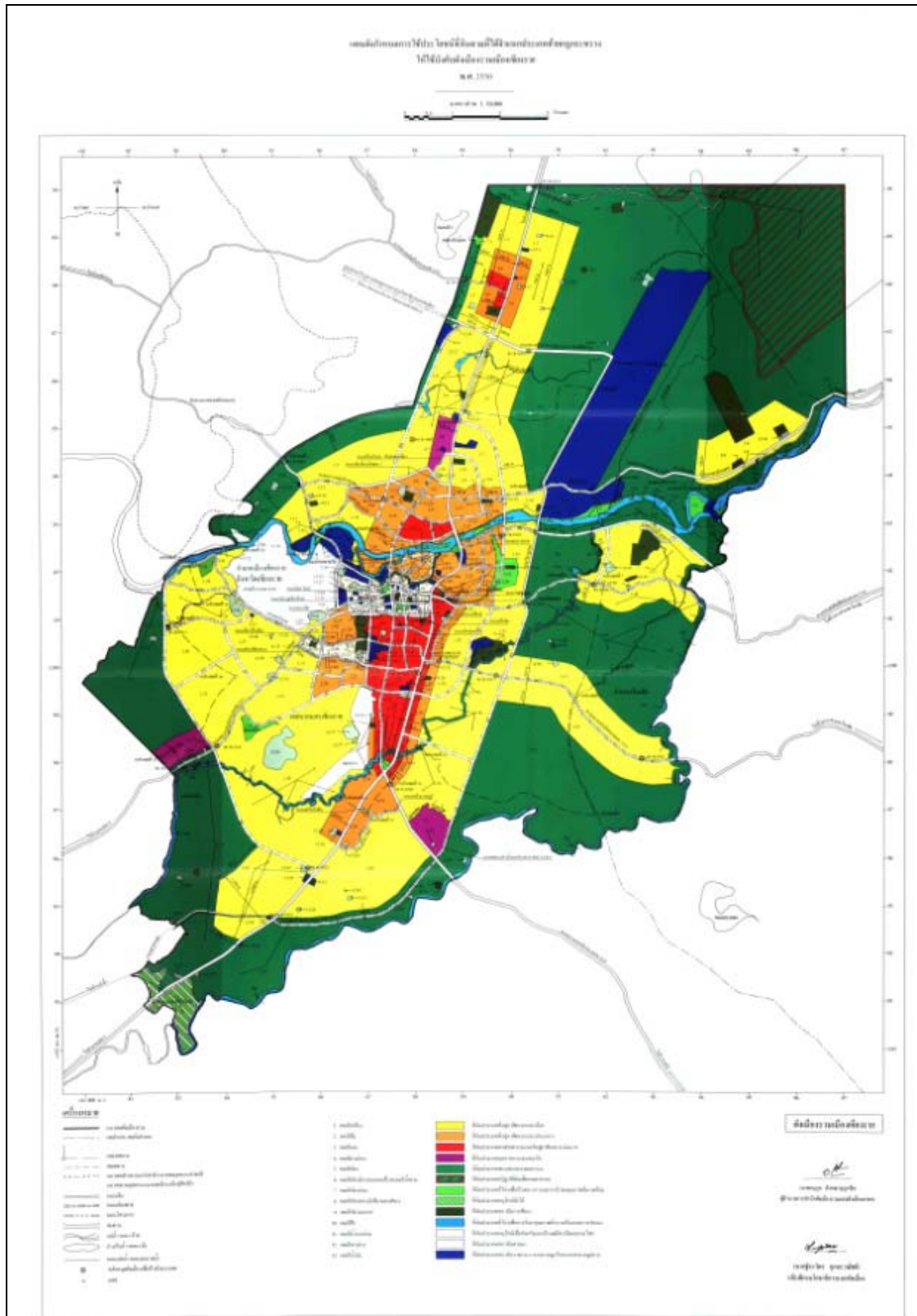

Figure 2. The Study Area: Chiang Rai DTCP Area [4].

Research Methodology

In this section a method of solving using the Branch and Bound Method is described. Branch and Bound is a broad class of algorithms to solve MILPs. Branch and Bound is a divide and conquer approach that reduces the original problem to a series of smaller sub problems and then recursively solves each sub problem.

The static node selection methods employ a fixed rule for selecting the next sub problem to process. A popular static method is Best Local Bound (BLB), which chooses the candidate node with the smallest lower bound. Due to the fathoming rule employed in branch and bound, a Best Local Bound strategy ensures that no sub problem with a low bound above the optimal solution value can ever be processed. Other extreme, Depth First Search (DFS) chooses the next candidate to be a node at maximum depth in the tree and Breadth First Search (BFS) chooses the next candidate to be a node at wide.

Problem formulation in ILPFs
The problem is formally defined on a directed graph G = (V, A) where V = {0, 1, 2, …, n} is the set of nodes (vertices), classical node 0 denote is the depot and the remaining nodes are bus stop. The set $A = \{(i,j) \in N, i \neq j\}$ is a link set. With each link (i,j) is associated a travel cost $c_{i,j}$, defined by travel time $(TravT_{i,j})$ and fuel consumption which can be calculated from link distance ($Dists_{i,j}$), velocity $(v_{i,j})$ and Fuel Consumption Rate $(FuelC_{i,j})$. Link costs are used as Parameters in Objective Function and Constraints.

Set and Parameters:
- N = set of node.
- Route = set of route.
- BS = set of bus stop.
- i = index for traveling from node i
- j = index for traveling to node j
- r = index for the route r
- $Dists_{i,j}$ = distance from node i to j.
- $v_{i,j}$ = average velocity from node i to j.
- MaxDists = max distance allowed each route.
- MaxTravT = max travel time allowed each route.

Decision Variable:
- $x_{i,j,r} = \begin{cases} 1; \\ 0; \end{cases}$
  $\begin{cases} if\ links\ (i,j)in\ rote\ r\ is\ optimal\ solution, \\ \qquad\qquad Otherwise \end{cases}$

- u_i is sequence in which node i is visited.

Function Calculation:
- $TravT_{i,j}$ = travel time from node i to j. It is function of distance and velocity, and used to calculate travel time of each route. Route travel time is not to exceed an upper limit.

In general distance can be calculated from equation (1) (Units: metres and minutes):

$$Distance = velocity \times Time \tag{1}$$

The travel time can be calculated from:

$$TravT_{i,j} = DistS_{i,j} \times \frac{0.06}{v_{i,j}};$$
$$for\ all\ i,j\ in\ N; \tag{2}$$

- $FuelC_{i,j}$ = Fuel Consumption from node i to j can calculate from Fuel Consumption Rate (FCR) and Distance from node i to j as shown in equation 3.

$$FuelC_{i,j} = Dist_{i,j} \times FCR_{i,j};\ \{for\ all\ i,j\ in\ N\} \tag{3}$$

For Capacitated Vehicle Routing Problems where fuel consumption (Travel Cost) from origin (i) to destination (j) in link (i,j) is call ($FCR_{i,j}$). Where $f(.)$ is function of $v_{i,j}$, GVW and PW can be calculated from:

$$FCR_{i,j} = f(v_{i,j}, GVW, PW);\ \{for\ all\ i,j\ in\ N\} \tag{4}$$

where:
$v_{i,j}$:   average velocity form i to j,
GVW   (Gross Vehicle Weight) is weight of
            vehicles add weight of passenger,

PW   (Power per Weight ratio).

Where the U.K. Transport and Road Research Laboratory, TRRL Declared Fuel Consumption rate show in equation (5)

$$FCR_{i,j} = \left(-48.6 + \frac{903}{v_{i,j}} + 0.0143\ v^2 + 69.2\sqrt{GVW} - 2.4PW\right) * 1.13 \tag{5}$$

This study selected mini bus as the public transport service in Chiang Rai city. The variables and parameters related to the equation (5) are as follows.

Parameter GVW can be calculated from:
- Vehicle weight   = 7,000
                                                                    kg.
- Weight of passenger (24 x 80   = 1,920
kg.)   kg.
- Gross Vehicle Weight (GVW)   = 8,920
                                                                    kg.

Parameter Power per Weight   = 89
Ratio[13]   BHP

Parameter of average velocity in a link (i,j) was surveyed and collected by GPS Garmin Model 60CSx.

The relationship between fuel consumption rate and vehicle speed can be calculated for empty and fully loaded minibus. The graph shows fuel consumption in km/litre and is plotted against the speed varying between 1 – 70 km/hr as shown in Figure 3. In the graph, the red-dotted curve (the lower curve) is for fuel consumption rate of fully loaded minibus and the black-dotted curve (the upper curve) is for fuel consumption rate in case of empty minibus. From the graph, it found that the average velocity at 32 km/hr. is the highest average fuel consumption rate (5.0000 km/litre and 6.0269 km/litre).
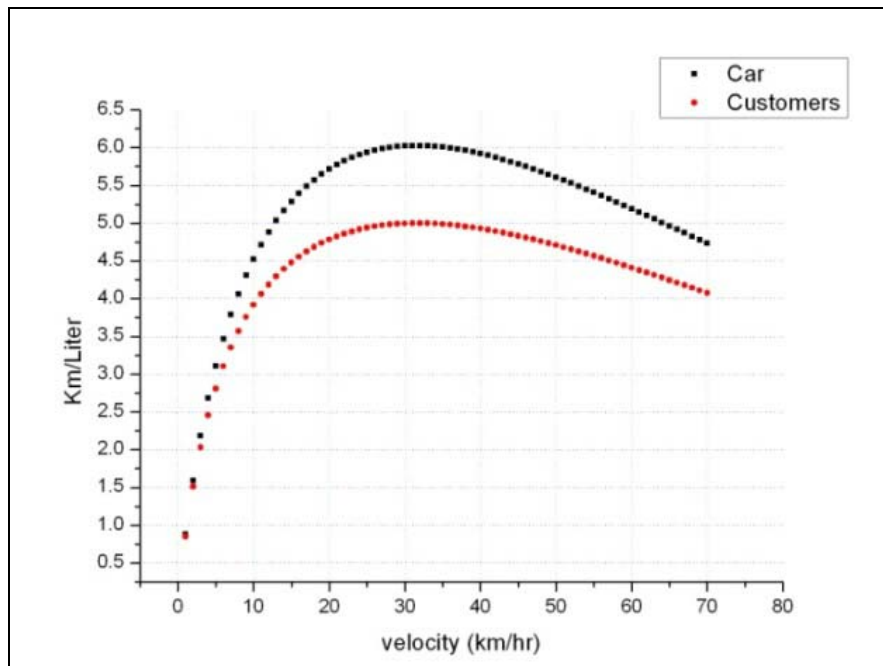

Figure 3. Fuel Consumption Rate.

The problem of CVRP can be stated as finding routes that satisfy with minimum cost under the following conditions; (1) each route starts and ends at the depot, (2) each bus stop is served by at least one route and other nodes (not bus stop) is either served or not served, and (3) the total distance or travel time of each route does not exceed the distance limit or travel time limit. Figure 4 shows the stated problem.
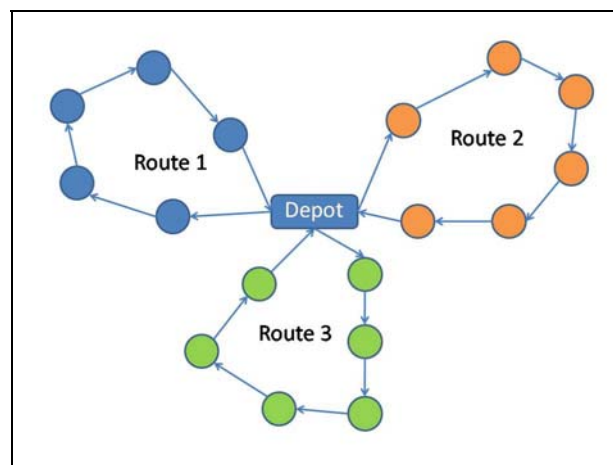

Figure 4. Format of CVRP.

Problem Formulation

The problem can be formulated mathematically as:

Objective Function
In the above model, the objective function (6) of this study is set to minimize total fuel consumption of overall route which is as follows:

$$\sum_{r \in Route} \sum_{i \in N} \sum_{j \in N} FuelC_{i,j} x_{i,j,r} \qquad (6)$$

Description Constraint
Constraint (7). Number of route constraints; each route starts at depot and number of routes does not exceed a given limit.

$$\sum_{r \in Route} \sum_{j \in N} x_{Depot,j,r} \leq |Route| \qquad (7)$$

Constraint (8). Nodes (Bus Stop) constraint; state that every Bus Stop should be visited at least once (≥ 1) and state that other node (not Bus Stop node) should be visited or not visited (≥ 0).

$$\sum_{r \in Route} \sum_{\substack{i \in N \\ i \neq j}} x_{i,j,r} \geq \begin{cases} 1; for\ all\ j \in BusStop \\ 0; for\ all\ j \notin BusStop \end{cases} \qquad (8)$$

Constraint (9). Links balance constraint; this is the flow conservation constraints on each individual link. This is to be satisfied at all nodes (Depot, Bus Stop and non Bus Stop).

$$x_{i,j,r} = x_{j,k,r}; \begin{cases} i,j,k \in (1,2,\dots,n); \\ r \in 1,2,\dots,route; \end{cases} \qquad (9)$$

Constraint (10). Capacity constraint; the distance or travel time of any bus route (r) does not exceed limits.

$$\sum_{i \in N} \sum_{\substack{j \in N \\ i \neq j}} \begin{Bmatrix} Dists_{i,j} \\ TravT_{i,j} \end{Bmatrix} x_{i,j,r} \leq \begin{Bmatrix} MaxDists \\ MaxTravT \end{Bmatrix};$$
$$\{r \in 1,2,\dots,route;\} \qquad (10)$$

Constraint (11). Sub Tour Elimination constraint; ensuring the solution contains no illegal sub tours. In this study use declare sub tour elimination in format of MTZ [5-8].

$$u_i - u_j + n \sum_{r \in Route} x_{i,j,r} \leq (n-1) \qquad (11)$$

Constraint (12). Decision variable; $x_{i,j,r}$ is binary variable, whether route r traveled form node i to j.

$$x_{ijr} = \begin{cases} 1; \\ 0; \end{cases} \quad i \neq j$$

(12)

$$\begin{cases} if\ link(i,j)\ in\ route\ r\ is\ optimal\ solution \\ \qquad\qquad Otherwise \end{cases}$$

Constraint (13). Decision Variable; u_i is sequence in which node i is visited.

$$u_i \geq 0, real;$$

(13)

Results

E equations are coded in section 2, (ILPFs) in GMPL format (GNU Mathematical Programming Language) [11] and solved by using GLPK (GNU Linear Programming tool Kit) version 4.38 [12] cooperate of Microsoft Visual C++ 2008 Express Edition [17]. The program was run on an Intel Core2 Dual running at 1.6 Ghz, RAM 1,024 Mb.

In this study, the problem size is 57 nodes and 89 links (1 direction) for test model. Comparison of Branch and Bound Method by BLB, BFS and DFS showed that BLB and BFS performed better than DFS in finding a solution (shown in Table 1).

Table 1. Computational comparison.

	Computation Time (sec.)	Iteration	Memory (mb)
BLB (GLPK)	0.3910	1,413	1.6927
BFS (GLPK)	0.3910	1,413	1.6927
DFS (GLPK)	588.9060	1,314,377	1.7706
SYMPHONY	144.0000	3,126	-

Results from Branch method (BLB, BFS and DFS) gave the best route with travel time 10.302 minutes, travel distance of 4,943.69 metres and fuel consumption of 658.7437 millilitres.
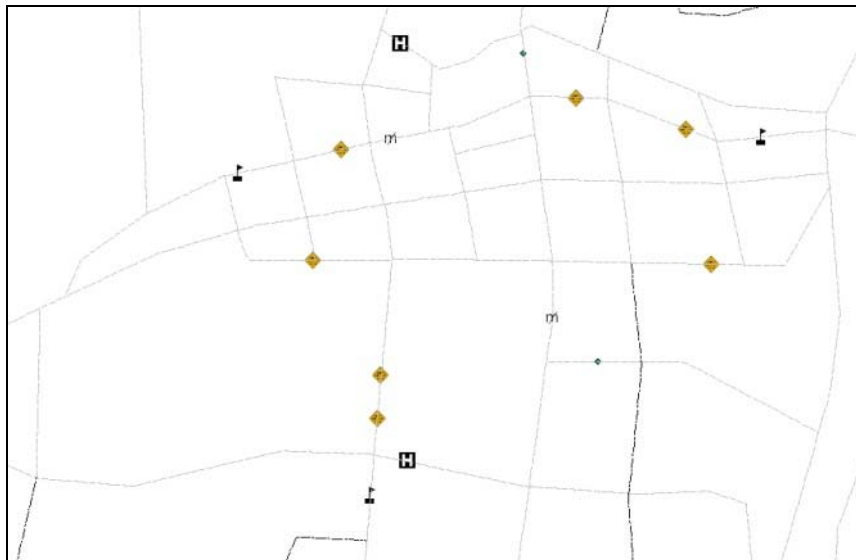

Figure 5. Bus Stop locations.

Table 2. Solution of bus route (1 route problem).

From	To	Time	Distance	Fuel
39	38	0.584	292.16	38.5131
38	31	0.347	231.58	28.5582
31	24	0.494	329.33	40.6126
24	25	0.736	245.49	36.5359
25	26	0.369	123.14	18.3268
26	36	0.641	341.92	44.3457
36	42	0.424	226.40	29.3632
42	43	0.382	190.79	25.1503
43	1	0.573	286.67	37.7894
1	49	0.366	183.04	24.1287
49	2	0.515	171.83	25.5732
2	51	0.506	168.62	25.0955
51	13	0.429	228.64	29.6538
13	12	0.506	168.82	25.1252
12	11	0.384	128.15	19.0724
11	47	0.332	221.16	27.2732
47	50	0.569	569.16	65.3484
50	41	1.406	468.68	69.7530
41	39	0.736	368.11	48.5250
Total		10.302	4,943.69	658.7437

The model was also tested using the SYMPHONY Program and it was found that SYMPHONY gave the same results as GLPK. Route detail is shown in Table 2 and Figure 6.
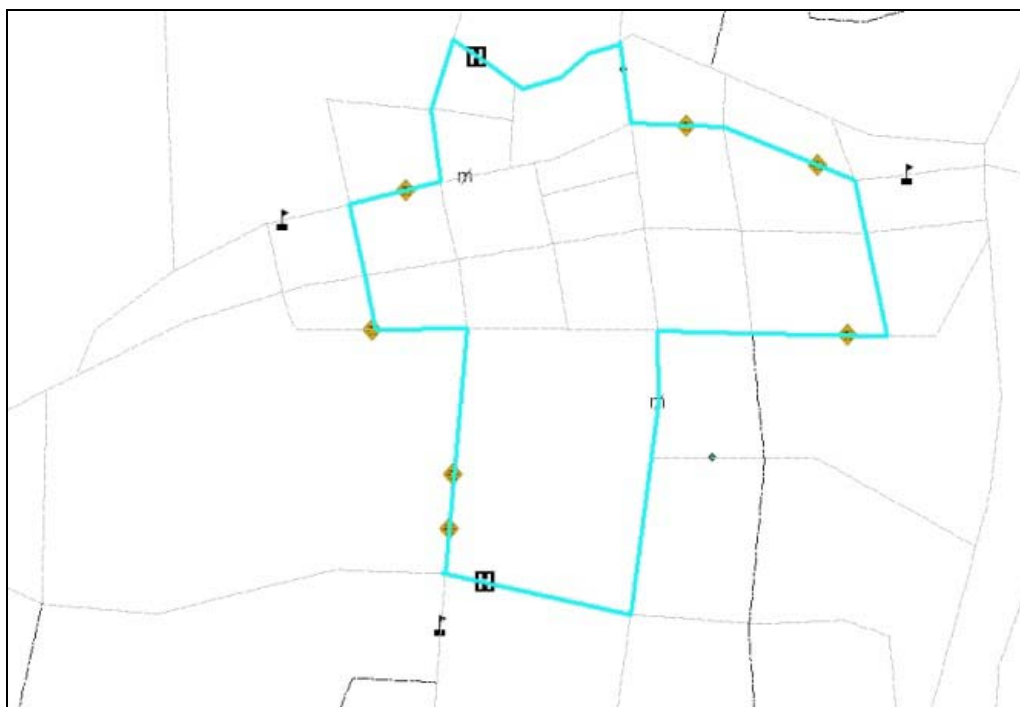
Figure 6. Solution of bus route (1 route problem).

For the problem of finding routes with travel time not exceeding 10 minutes, BLB and BFS methods result in 2 routes with BLB run time of 598.1720 seconds and BFS of 604.9840 seconds and travel time of 5.836 minutes and 8.346 minutes, respectively (shown in Table 4 and Figure 10). However, for run time of 6,000 seconds, DFS method does not converge to solution (as shown in Table 3).

Figures 7 – 9 show how the solution is approached for BLB, BFS and DFS method. It was shown that BLB and BFS perform better than DFS.

Table 3. Computation comparison.

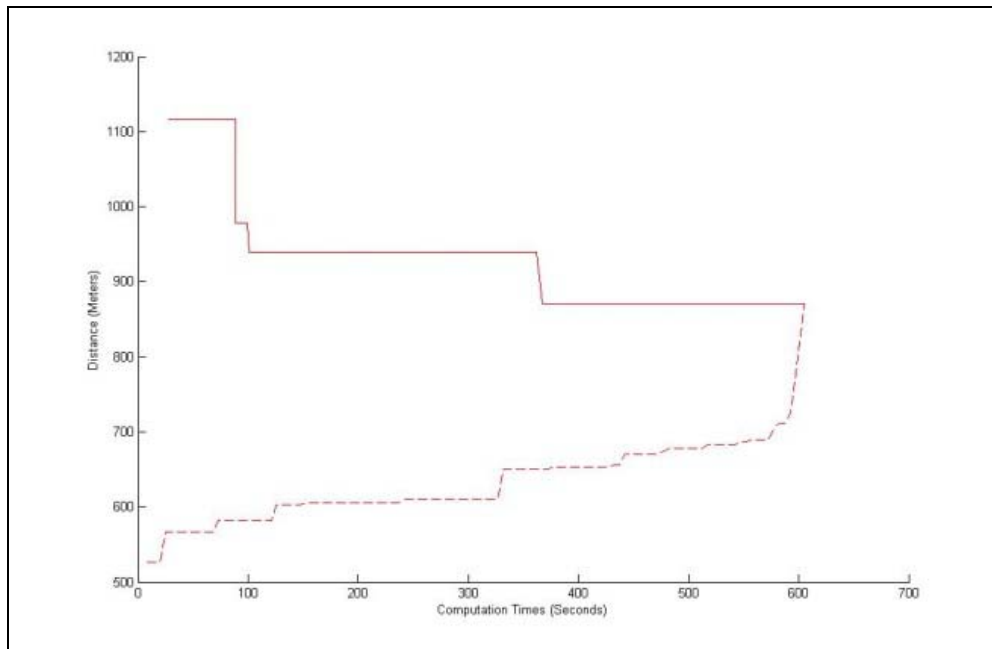|	Computation Time (sec.)	Iteration	Memory (mb)
BLB	598.1720	1,555,231	37.6525
BFS	604.9840	1,716,021	30.4206
DFS	6,000.0001*	21,728,679*	1.7706*

Figure 7. Computing Time of Breadths First Search Method.

Figure 7 shows computing time ability in the case of BFS, the upper line shows the integer solution, while the lower line (dashed line) shows linear solution. The blank between integer and linear solutions is the gap.
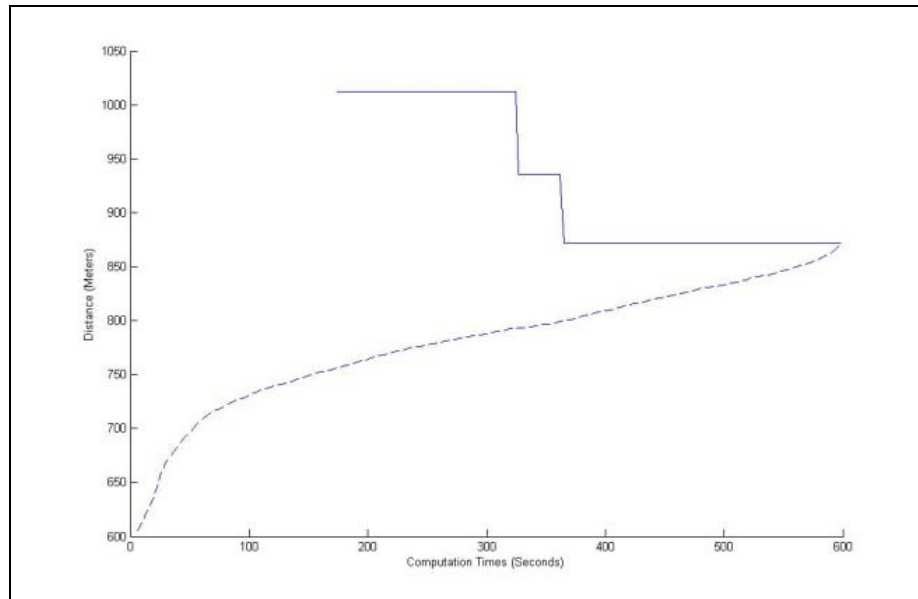
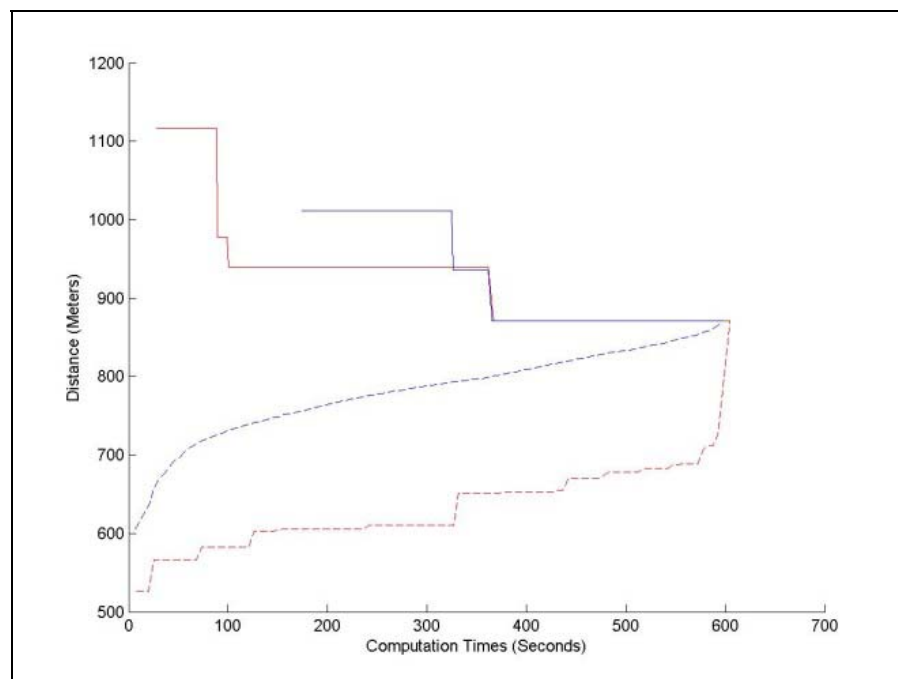**Figure 8. Computing Time of Best Local Bound Method.**

Figure 9. Comparison of Computing Time Between BFS & BLB Methods.

In comparing BLB and BFS (Shown in Figure 9) it was found that BLB can find solution faster than BFS and BLB's reduced gap reduce rate is better than BFS.
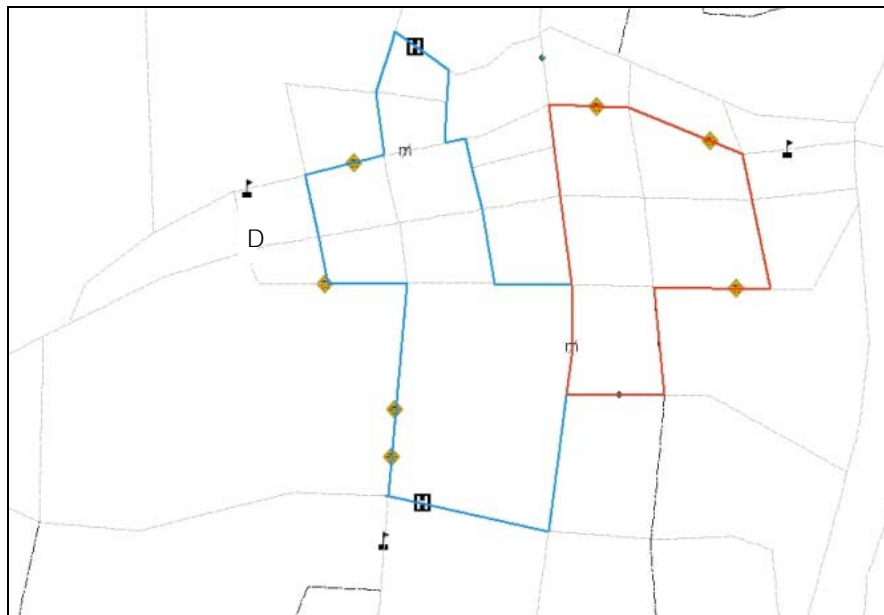
Figure 10. Solution of bus route with travel time not exceeding the limit (10 minutes).

Table 4. Summary of Fuel Consumption.

Route no.	Travel Time (Minutes)	Distance (Metres)	Fuel Consumption (Millilitres)
1	5.836	2,608.7100	354.3984
2	8.346	3,833.9800	516.9400
Total	14.1819	6,442.6900	871.3384

Conclusion

In this study the classical Capacitated Vehicle Routing Problem (CVRP) was used to analyze for appropriate bus routes under a given objective to minimize the fuel consumption by considering velocity, weight and power per weight ratio.

Both single and multi-route cases were considered and the Integer Linear Programming Formulations (ILPFs) adopted. It was found that the BFS and the BLB can find the solution faster than the DFS.

The model was also developed with interfacing the GIS (Graphic Information System) modules, MATLAB and GLPK (GNU Linear Programming Tool Kit) and has potential for further development for the case of Capacitated Open Vehicle Routing Problem (COVRP) and Multi-Depot Vehicle Routing Problem (MDVRP).

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