# Blackbody Radiation in Microscopic Gaps

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(Received : 31 January 2004 – Accepted : 15 March 2004)

**Abstract:** Planck's derivation of the radiation law for blackbody (BB) radiation that included the zero point energy (ZPE) was based on an oscillator in thermal equilibrium exchanging discrete quanta of energy linear with frequency. Einstein - Hopf classical theory for a free particle led to the Rayleigh-Jeans law absent the zero point energy. Boyer using classical theory extended Einstein and Hopf's notion of a free particle to include the interaction with the cavity wall to derive Planck's radiation law including the zero point energy. But in microscopic gaps having dimensions less than half the wavelength of the characteristic BB radiation, say in the far infrared (IR), none of prior derivations is valid. To explain why microscopic gaps enhance radiant heat transfer, a new theory of radiative heat transfer based on cavity quantum electrodynamics (QED) is proposed. By this theory, the suppressed long wavelength BB radiation in the gap is conserved with a frequency up-conversion

 

at the gap resonant frequency, thereby allowing the BB radiation to flow across the gap that in prior derivations of the BB radiation is forbidden.

## Introduction and Background

### 1. Boltzmann

Historically, the derivation of the BB spectrum began [1] with Boltzmann's derivation of the total blackbody energy density U and temperature T,

$$U = aT^4 \tag{1}$$

and Wien's displacement law,

$$\rho(\upsilon / T) = \upsilon^3 f(\upsilon / T)$$

and

$$U = \int_0^\infty \rho(\upsilon / T)\, d\upsilon \tag{2}$$

where, $\rho(\upsilon/T)$ is the energy density, and $\upsilon$ is frequency of the BB radiation.

### 2. Einstein and Hopf

By considering the interaction between a free particle and radiation, Einstein and Hopf [3] showed energy equipartition in classical theory led to the Rayleigh-Jeans radiation law.

$$\rho(\upsilon / T) d\upsilon = \left(\frac{8\pi\upsilon^2}{c^3}\right) kT d\upsilon \tag{3}$$

where, c is the velocity of light and k is Boltzmann's constant. The Rayleigh-Jeans derivation is consistent with Wien's displacement law,

$$\rho(\upsilon / T) = \left(\frac{8\pi\upsilon^3}{3c^3}\right) kT \qquad (4)$$

but at increasing frequency the Rayleigh-Jeans derivation diverged, better known as the "ultraviolet catastrophe."

## 3. Planck

About this time, Planck presented [3] the notion that the oscillator could achieve thermal equilibrium with discrete units of energy linear in frequency, the discrete unit of energy to be $h\upsilon$, where h is Planck's constant. Planck assumed that oscillators of frequency $\upsilon$ could only have energy $E_n$,

$$E_n = nh\upsilon, \; n = 1,2,.. \qquad (5)$$

Giving,

$$E_{avg} = \frac{h\upsilon}{\exp\,(h\upsilon / kT) - 1} + \frac{1}{2}h\upsilon \qquad (6)$$

where, ½ $h\upsilon$ is the ZPE.

## 4. Maxwell-Boltzmann and Einstein-Hopf

The Maxwell-Boltzmann distribution gives the average energy $E_{avg}$ per oscillator,

$$E_{avg} = \frac{\sum_n E_n \exp(-E_n / kT)}{\sum_n \exp(-E_n / kT)} \qquad (7)$$

After expansion,

$$E_{avg} = \frac{h\upsilon}{\exp\,(h\upsilon / kT) - 1} \qquad (8)$$

and differs from Planck's derivation of the average oscillator energy by the exclusion of the ZPE. Like the Rayleigh-Jeans law, the ZPE diverges for increasing frequency, and therefore is usually interpreted as non-physical. Applying the Einstein-Hopf formulation over the frequency interval [$\upsilon_1$, $\upsilon_2$ ],

$$U = \left(\frac{8\pi h}{c^3}\right)\int_{\upsilon_1}^{\upsilon_2} \upsilon^2 E_{avg}\, d\upsilon$$

$$U = \left(\frac{8\pi h}{c^3}\right)\int_{\upsilon_1}^{\upsilon_2} \frac{\upsilon^3}{\left[\,\left(\exp(h\upsilon / kT\right) - 1\,\right]} d\upsilon \qquad (9)$$

Over the full range of frequencies,

$$U = \int_0^\infty \rho\left(\upsilon / T\right) d\upsilon = \frac{8\pi^5 (kT)^4}{15(hc)^3} = aT^4 \qquad (10)$$

The flux F radiated,

$$F = \frac{Q}{A} = \frac{c}{4} U = \frac{ac}{4} T^4 = \sigma T^4 \qquad (11)$$

which is the standard form of the Stefan-Boltzmann law. Q is the heat flow, A is the flow area, and σ is the Stefan-Boltzmann constant, $\sigma = 5.67\text{x}10^{-8}$ W/ $m^2$ $K^4$.

## 5. Boyer, Casimir, and Sparnay

More recently, Boyer based on classical theory treated the interaction between the particle and cavity wall [4] and derived the BB radiation spectrum identical to that by Planck including the ZPE. Energy quantization was not invoked, but otherwise the free particle method employed was identical to that of Einstein-Hopf.

Lorentz invariance of the ZPE means that if one observer measures the frequency spectrum of the BB radiation it will be the same as that measured by another observer moving at velocity relative to the first observer. Boyer showed that if the ZPE is linear with frequency, then it is Lorentz invariant. But this may be irrelevant as a zero ZPE is also Lorentz invariant and avoids the difficulty of divergence inherent with a ZPE that is linear with frequency.

To show it is worthwhile to take quantum EM ZPE seriously, Boyer cites Casimir’s derivation of the EM attraction between neutral conducting parallel plates [5] despite the divergence of the energy density at increasing frequency. Casimir’s theory was presumably confirmed by Sparnay [6] who measured an attractive force between neutral plates separated by a microscopic gap. However, Boyer interpreted Sparnay’s experimental data as the affirmation of the existence of the ZPE.

However, if the force measured by Sparnay was caused by, say electrostatic charge, the Casimir effect or the existence of the ZPE would not be affirmed. If so, and in the absence of other supporting experimental data, the ZPE would remain an artifact of quantum theory and disposed of in the usual way on the grounds that although mathematically correct, it is non-physical.

## Problem Definition and Purpose

Derivations of the BB radiation theory have a long history in science. Even to this day difficulties still exist especially for applications where radiative heat transfer occurs across microscopic gaps.

In thermophotovoltaic (TPV) devices, placing a hot surface in close proximity [7] to a photocell is known to enhance heat transfer and produce electricity. Typically, the heater at temperature emits far IR radiation while the photocell is sensitive only to the near IR. How far IR radiation is transmitted across a gap to undergo a frequency up-conversion to near IR wavelengths is not well understood.

Evanescent waves have been proposed [8] to explain the increased heat transfer, but have the same frequency content as that in the BB spectrum of the heater. Absent frequency up-conversion from the far to near IR, evanescent waves in the far IR cannot produce electricity with near IR photocells.

The purpose of this paper is to review conventional BB radiation theory and propose cavity QED induced heat flow [9] as a new theory of radiant heat transfer in microscopic gaps.

## Analysis

### 1. Conventional BB Radiation Theory

Conventional BB radiation theory is based on the Einstein-Hopf derivation that treats the atoms as oscillators with discrete energies linear with frequency as given by the Maxwell-Boltzmann distribution.

Contrary to Planck and Boyer, conventional BB radiation theory treats the ZPE as a constant in the mathematical solution that is rejected on non-physical grounds because like the "ultraviolet catastrophe" the ZPE diverges at increasing frequency. If a zero ZPE is taken, there is no need in the Boyer derivation to justify the Lorentz invariance of a ZPE linear in frequency.

Most arguments in support of the ZPE make the point that it does not matter because the energy differences usually required in quantum mechanics cancel out the ZPE. But these arguments fail. To wit, the thermal kT energy from the atom is emitted on an absolute temperature scale, and therefore the ZPE diverging at increasing frequency would erroneously alter the thermal emission from the surface atoms from that observed.

All that is required in conventional BB radiation theory is for the atoms on the surfaces of the body to emit EM radiation with wavelength following the average Maxwell-Boltzmann energy. Fig. 1 shows that at ambient temperature most of the Planck energy $E_{avg}$ is emitted in the far IR. Since $E_{avg}$ saturates at kT = 0.0258 eV for $\lambda > 100$ μm, over 96% of the kT energy occurs at wavelength $\lambda > 10$ μm.

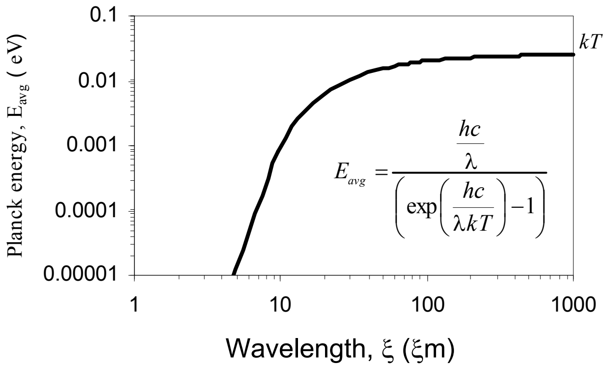


**Figure 1.** Maxwell-Boltzmann Energy $E_{avg}$ at 300 K.

Unlike Einstein-Hopf, Planck and Boyer, the conventional BB theory does not rely on free particles that (1) exchange energy with the radiation field and velocity dependent retarding forces, or (2) momentum exchange with the cavity walls. Indeed, free particles are not necessary. Conventional BB radiation treats the atoms as oscillators that satisfy the Maxwell-Boltzmann distribution.

In conventional BB radiation theory, the radiative flux F between hot $T_H$ and cold $T_C$ bodies is given by the Stefan-Boltzmann equation,

$$F = \frac{Q}{A} = \sigma \left[ T_H^4 - T_C^4 \right] \tag{12}$$

Generally, the Stefan-Boltzmann equation is valid except for microscopic gaps having dimensions less than half the wavelength of the BB radiation at temperature.

## 2. Proposed BB Radiation Theory

How the Stefan-Boltzmann law is modified [9] for radiative heat Q flow in the microscopic gap δ is illustrated in Fig. 2.

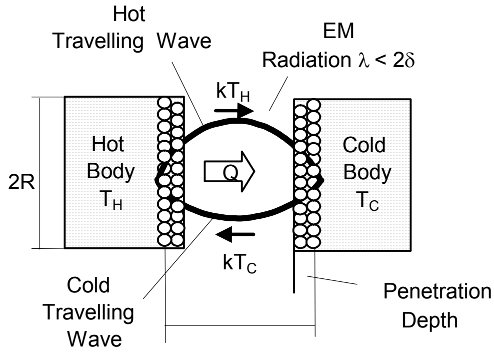


**Figure 2.** BB radiation in a microscopic gap.

In the gap as a QED cavity, the surfaces of the walls are made up of atoms that interact across the gap by standing waves. An important feature of the proposed modification of BB radiation theory is that the standing waves interact over their penetration depth ε to include a number m of atoms below the surfaces of the hot and cold bodies.

## 3. Conservation of Suppressed EM Energy

To illustrate the cavity QED effect, standing EM waves having wavelength λ are shown in Fig. 2. The longest wavelength λ that can stand [10] in the gap is, λ = 2δOnly wavelengths λ < 2 δ are admissible in the gap as a QED cavity.

Indeed, EM energy for wavelengths $\lambda > 2\delta$ is suppressed by cavity QED.

Specifically, if the standing wave resonant frequency in the gap exceeds the principal BB radiation frequency at temperature, the BB radiation is suppressed by cavity QED. At ambient temperature, the principal BB radiation occurs in the far IR, and therefore the BB radiation from atoms within the penetration depth $\varepsilon$ is suppressed for standing wave resonance higher than the far IR, say from the near IR to the VUV.

However EM energy must be conserved, and therefore the suppressed BB radiation loss in the far IR is gained at the near IR or VUV resonant frequency of the gap, i.e., the far IR undergoes a frequency up-conversion to the gap resonance. Frequency up-conversion is consistent with cavity QED constraints in that only EM radiation having wavelengths $\lambda < 2\delta$ may exist inside the gap. For a near IR gap,

$$(\text{near IR energy gain })_{Gap} = (\text{far IR energy loss})_{Atoms}$$

$$\left( E_{\text{near IR}} \sim \frac{hc}{\lambda} \right)_{Gap} = (\text{ m kT })_{Atoms} \qquad (13)$$

where, the near IR photon having Planck energy $E_{nearIR}$ is conserved by the thermal kT energy of m atoms, i.e., m = hc / $\lambda$ kT. For example, in the near IR for $\delta = 0.50\ \mu m$, $\lambda = 1\ \mu m$ and $E_{nearIR} = 1.24$ eV, where m = 48 atoms.

## 4. Heat Flow Mechanism

The extent of the QED cavity in the gap includes surface atoms of the hot and cold bodies within the penetration depth $\varepsilon$ of the standing EM wave as depicted in Fig. 2. However the far IR radiation from these atoms is suppressed by the higher near IR resonant gap. To conserve EM energy, the suppressed far IR radiation lost by these atoms is conserved with an equivalent EM energy gain in the adjacent gap, i.e., the far IR radiation undergoes a frequency up-conversion to the near IR.

The radiant heat Q transfer between hot $T_H$ and cold $T_C$ bodies is the suppressed thermal kT energy carried by travelling waves that move in opposite directions; the hot body transfers thermal $kT_H$ energy to the cold body with thermal $kT_C$ energy transferred to the hot body.

The heat flow Q depends on the density of resonant modes in the gap $\delta$. At resonance, $\delta = c / 2\upsilon$, where, $\upsilon$ is the resonant frequency. Thus, the mode density is $\delta^{-3} = 8\upsilon^3/c^3 \sim 8\pi\upsilon^3/3c^3$ from Eqn. 4. For a gap having flow area A and frequency $\upsilon$, the volume $A\delta$ contains $A/\delta^2 \sim 4A\upsilon^2/c^2$ resonant modes. Thus,

$$Q = A\left(\frac{1}{\delta^2}\right)\left(\frac{1}{\tau + \delta / c}\right) mk\left(T_H - T_C\right) \tag{14}$$

where, m is the average number of atoms in the standing EM wave interacting across the gap, and $\tau$ is the response time of the atoms in the hot and cold surfaces.

## Application and Discussion

The steady TPV photocell current I produced from the conversion of radiative heat Q,

$$I = \xi Q \tag{15}$$

where, $\xi$ is the spectral responsivity in units of A/W. For the purposes here, the typical spectral responsivity of silicon and germanium is shown in Fig. 3.

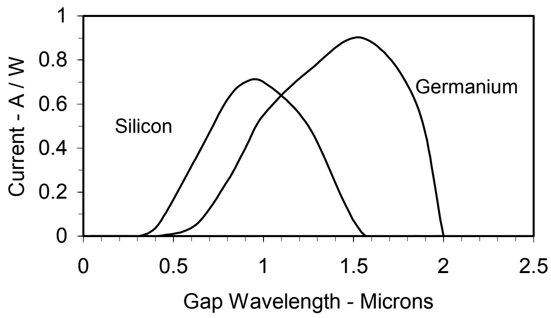


**Figure 3**. Spectral responsivity of near IR materials.

Typical TPV data taken for the purposes of this paper (Fig. 4 of [7]) shows the photocell current I to begin at an indicated capacitance C ~ 50 pF. For a square 2.2 x 2.2 $mm^2$ heater chip, the area $A = 4.8 \times 10^{-6}$ $m^2$ and gap $\delta = \varepsilon_o A/C \sim 0.86$ $\mu$m giving the wavelength $\lambda = 2\delta \sim 1.7$ $\mu$m. The TPV response is consistent with the spectral responsivity of germanium for $\lambda < 2$ $\mu$m.

Germanium has a peak response $\xi \sim 0.9$ A/W and occurs at $\lambda = 1.55$ μm and $E_g = 0.80$ eV. For $T_H = 348$ K and $T_C = 300$K, and m ~ 29 atoms, Eqn. 14 at the spectral peak gives the heat flow Q ~ 540 μW at a response time $\tau = 225$ ps. Thus, the current I ~ 490 μA. Including the 250 μA residual, I ~ 740 μA.

In simulating the TPV current response for cavity QED induced heat flow, Eqns. 14 and 15 were used for heater temperatures of 348, 378, and 408 K, the results plotted in relation to experimental data (Fig. 4 of [7]) in Fig. 4. A constant number m ~ 29 was assumed to supply thermal kT energy to the resonant EM waves standing in the gap. The photocell temperature $T_C$ was held at 300 K.

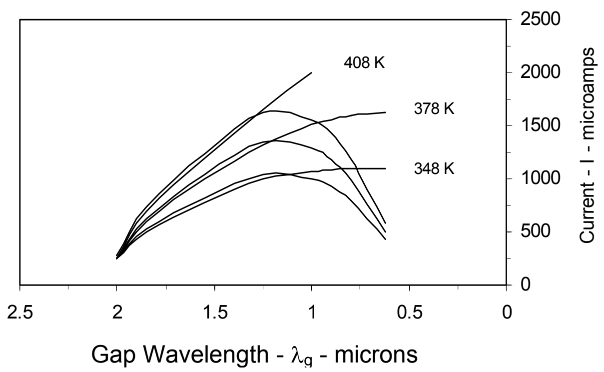


**Figure 4.** Cavity QED induced TPV effect and experiment.

The cavity QED fit to the experimental data was good for $\lambda = 2$ to 1.7 μm, although sensitive to the recovery time $\tau$ which for heater temperatures $T_H = 348$, 378, and 408 K was found to be 260, 300, and 340 ps, respectively. But for $\lambda = 1.7$ to 0.6 μm, the fit is not good. Indeed, the experimental data suggests the response in the visible region is greater than that in the near IR, which for the responsivity of the germanium photocell is difficult to understand.

The likely reason for the disparity between the cavity QED theory and the experimental photocell current is the provision of silicon dioxide spacers (~ 1 μm high) on the heater surface to control gap spacing, but at the same time keep the heater surface from achieving uniform contact the photocell. Away from the spacers the heater chip deforms with the capacitance increasing as the gap decreases. Over this deformed area, the photocell current decreases because the resonant wavelength $\lambda$ moves toward the VUV away from the responsivity of the near IR photocell. But near the spacers, the gap remains at $\delta < 1$ μm having resonant $\lambda < 2$ μm, and therefore the photocell continues to produce a current. Indeed, Fig. 4 suggests the spacer height is closer to $\delta \sim 0.85$ μm because the deviation between QED theory and experiment begins at $\lambda \sim 1.7$ μm. Hence, the current remains almost constant as the capacitance increases.

By reducing the height of the spacers, the experimental current should approach that given by the cavity QED induced TPV effect.

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