

The Cavity QED Induced Thermophotovoltaic Effect

T. V. Prevenslik

14B, Brilliance Court, Discovery Bay,

Hong Kong

Email: bibliotec2002@yahoo.com

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Abstract : Thermophotovoltaic (TPV) devices comprise a heater separated from a photocell by a microscopic gap. As the gap is slowly reduced, the photocell current increases while the temperature drops suggesting an underlying thermal mechanism. Conversely, a non-thermal mechanism is indicated since the current remains in phase with rapid gap changes that are faster than the response time of the heater. Both slow and rapid TPV responses may be explained by a theory based on the modification of thermal blackbody (BB) radiation by the gap as a quantum electrodynamics (QED) cavity, the theory called the cavity QED induced TPV effect. By varying the gap, the electromagnetic (EM) resonance of the QED cavity may be adjusted from infrared (IR) to vacuum ultraviolet (VUV) frequencies. At typical heater temperature, the thermal kT energy of the atoms is emitted in the far IR, but the photocells are only sensitive over a small range of frequencies in the near

IR. Thus, for large gaps having EM resonance beyond the far IR, the gap does not modify the BB spectrum, and therefore the photocell current is negligible because of the lack of near IR photons. But if the gap resonance is adjusted to the near IR, the far IR radiation undergoes a frequency up-conversion to produce near IR photons that dramatically increase the photocell current, the frequency up-conversion a consequence of conserving EM energy within cavity QED constraints. If the gap is adjusted to VUV resonance, the up-conversion of far IR radiation produces VUV photons, but because the photocell is not sensitive to the VUV, no photocell current is observed. Slow TPV gap changes show the heater temperature to drop because the heater cannot supply enough heat to maintain the temperature over the full heater body. But rapid gap changes only require temperature change over atoms in the heater surface and have a fast thermal response.

Keywords: Thermophotovoltaic, Solar cells, Cavity QED.

Introduction

Interest in current enhanced TPV cells finds its origin in the radiative heat transfer between surfaces in close proximity to each other that began over 50 years ago. Proximity effects were thought [1] caused by random thermal fluctuations that induced EM fields, although the physical basis for the thermal fluctuations and the conversion to an electrical current was not identified.

A generalized proximity theory was proposed [2] that assumed electric currents rather than EM fields are induced by the random thermal sources. The fluctuation-dissipation theorem for the thermally induced fluctuating micro-currents was assumed as the source term in Maxwell's equations. Statistical properties were derived that suggested large increases in radiative heat transfer in gaps, although the physical basis for the thermal fluctuations, or how the thermal fluctuations were converted to electrical current was not explained. Indeed, the area of radiative heat transfer across a vacuum gap has a long history of theoretical investigations without clear physical origin.

Increased radiative heat transfer across small gaps was first significantly supported by experimental work [3]. Identical bodies having a thermal wavelength λ_{th} were assumed in close proximity to each other separated by a gap δ . For a gap $\delta > \lambda_{th}$, the thermal flux between the surfaces obeys the Stefan-Boltzmann law. But for $\delta < \lambda_{th}$, the thermal flux was strongly dependent on the gap δ . The generalized theory was found in good agreement for chromium plates at temperatures held at 315 and 145 K over the gap range $1.5 < \delta < 6 \mu\text{m}$.

More recently, the Nyquist formula relating the random fluctuations of noise from electron motion in resistors was related to the radiative heat transfer [4] between the two metal plates. But whether the Nyquist theory applicable to the random motion of electrons in resistors in thermal equilibrium is valid for non-equilibrium radiative heat flow between two plates is

arguable. Current fluctuations were assumed between positive and negative charges parallel to the gap surface, the current inducing a voltage fluctuation dependent on the metal resistance. Since the source of positive and negative charges lacks a physical basis, the voltage fluctuation remains an assumption. The Nyquist voltage was related to the electric field and the EM energy density in the vacuum gap to determine the radiative heat flux between the plates. The measured radiative flux was found to be less than found [2] in the generalized theory.

Enhanced radiative heat transfer of blackbody (BB) radiation has been explained [5] by surfaces in close proximity to each other by evanescent waves leaving the heater and proceeding directly to the photocell that otherwise would undergo total internal reflection [6] at large gaps. The heat transmitted from the high temperature surface was found proportional to the square of the refractive index of the heater above the free space Stefan-Boltzmann law, although the frequency distribution of the evanescent wave is the same as that of the BB radiation. Thus, evanescent waves cannot improve the efficiency of TPV devices with a heater producing BB radiation in the far IR and a photocell sensitive to the near IR. Indeed, the frequency of the evanescent waves can not differ from that of the far IR radiation present in the BB spectrum of the heater at temperature, and therefore evanescent waves cannot explain the increased performance of TPV devices.

Regardless, TPV cell characteristics suggest [7] a thermal origin because the photocell current increases when the gap is

reduced, while the silicon heater temperature decreased. But thermal effects were ruled out because dynamic tests in which the gap was rapidly changed at a frequency of 1 kHz showed the short circuit current to remain in phase with the gap changes. To explain the dynamic response, a mechanism other than evanescent waves or a means of providing an up-conversion of far to near IR frequency is suggested.

Statement of Problem

Generally, the notion that the efficiency of TPV cells can be improved by reducing the gap size between the heater and the photocell is based on radiation tunnelling and evanescent waves.

Indeed, thermal fluctuating fields adjacent the gap surfaces may extract [4] EM energy from these fields if another particle or solid penetrates the fields. But unless the TPV gap is on the order of a few nanometers, radiation tunneling is unlikely to increase [8] the radiative flux across the gap. In fact, the enhanced TPV current is found [7] for micron-sized gaps at near IR resonance, a fact that suggests a mechanism operating in micron sized gaps underlies the observed enhancement of TPV photocell current.

Contrary to the literature [5-7], evanescent waves cannot increase the photocell current in TPV devices because the frequencies of the evanescent waves are not up-converted from far to near IR frequencies, but rather are the same frequency as those of the BB radiation from the heater. Since TPV photocells

are generally sensitive in the near IR and not elsewhere, it can be concluded the observed enhanced TPV current with near IR photocells has nothing to do with evanescent waves. Even plasmon frequencies in the optical range and higher [8] cannot increase the TPV current in the near IR.

Nevertheless, the experimental data [3,4,7] is unequivocal in supporting the argument that gaps increase the radiative heat transfer. Thus, a mechanism other than radiation tunnelling and evanescent waves is necessary to explain the enhanced TPV photocell current. Provided the gap has an EM resonance in the near IR, the proposal here is the far IR radiation from the thermal kT energy of atoms in the heater undergoes a frequency up-conversion to produce higher near IR radiation that can flow across the gap. Thus, gaps by inducing frequency up-conversion of far to near IR radiation are crucial to the enhancement of TPV current.

The purpose of this paper is to propose a theory called the cavity QED induced TPV effect whereby far IR radiation undergoes a frequency up-conversion to the near IR that increases the photocell current. The approach is similar to the cavity QED induced photoelectric effect [9] to explain static electricity by the suppression of IR radiation from micron-sized particles trapped in the gap between contacting surfaces.

Theoretical Background

In TPV devices, the heater is separated from the near IR photocell by a gap δ as shown in Fig. 1.

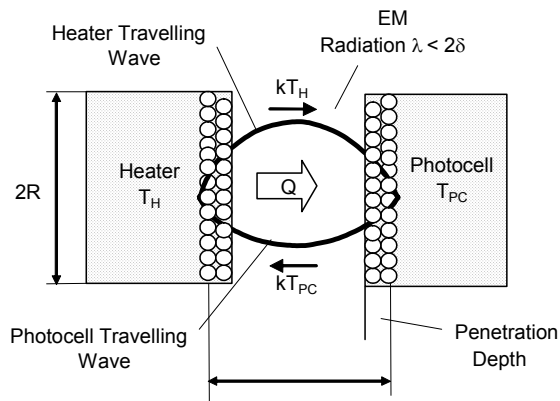


Figure 1. TPV Heater and Photocell – Cavity QED induced TPV Effect.

Atoms of the heater and photocell are shown within the penetration depth ε of the EM wave standing in resonance with the gap. At heater T_H and photocell T_{PC} temperatures, the BB spectrum content is primarily in the far IR. But the gap is a QED cavity having an EM resonance from near IR to VUV frequencies, and therefore the far IR from the atoms in the penetration depth is suppressed by cavity QED. To conserve EM energy, the suppressed far IR radiation lost by the atoms is conserved with an equivalent gain in EM energy in the gap. Since the gap is resonant at frequencies higher than the far IR, the far IR radiation may be said to undergo an up-conversion to the EM gap resonant frequency.

Indeed, frequency up-conversion is an unequivocal consequence of conserving EM energy in the gap within cavity QED constraints. If $T_H > T_{PC}$, heat Q in the form of suppressed IR radiation is transferred between the heater and the photocell by travelling waves that move in opposite directions, i.e., the heater transfers thermal kT_H energy to the photocell while the photocell transfers thermal kT_{TC} energy to the heater. At equilibrium, $T_H = T_{PC}$ and the travelling waves become standing waves without any net transfer of thermal kT energy.

1. BB Radiation.

The cavity QED induced TPV effect finds its origin in the BB radiation from the thermal kT energy of the atoms in the heater. Treating the atom as a harmonic oscillator [10], the average Planck energy E_{avg} is,

$$E_{avg} = \frac{\frac{hc}{\lambda}}{\left[\exp\left(\frac{hc}{\lambda kT}\right) - 1 \right]} \quad (1)$$

where, λ is the wavelength, h is Planck's constant, c is the speed of light, k is Boltzmann's constant, and T is absolute temperature. Fig. 2 shows at temperatures less than 500 K, the Planck energy E_{avg} is significant only at far IR wavelengths. Moreover, Planck energy available in the near IR is negligible for temperatures less than 1000 K.

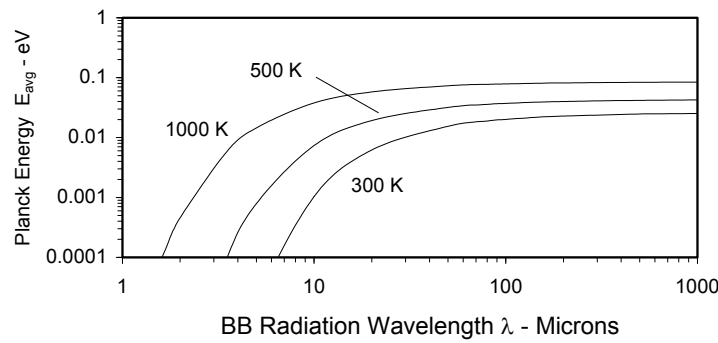


Figure 2. Planck Energy E_{avg} - Harmonic Oscillator at 300, 500, and 1000 K.

2. Cavity QED Constraints on BB Spectra.

In cavity QED, BB radiation having wavelength λ is forbidden [11] in a gap having an EM resonant wavelength λ_g , where $\lambda > \lambda_g$. The EM resonant wavelength λ_g of the gap is, $\lambda_g = 2\delta$. Thus, the gap excludes all BB radiation having $\lambda > 2\delta$, e.g., for a gap $\delta \sim 5 \mu\text{m}$, BB radiation having $\lambda > 10 \mu\text{m}$ is forbidden in the gap.

Alternatively, forbidden EM radiation in a QED cavity is equivalent to the suppression of EM radiation. BB radiation is fully suppressed (Fig. 1 of [12]) in a gap δ if $\delta / \lambda < 0.1$, where λ is the wavelength of the BB radiation. Since Fig. 2 shows most of the thermal kT energy at heater temperatures resides at $\lambda > 10 \mu\text{m}$, and since TPV devices are claimed [8] most efficient in the near IR at wavelengths $\lambda < 2 \mu\text{m}$, or gaps $\delta < 1 \mu\text{m}$, the BB radiation is fully suppressed by cavity QED as $\delta / \lambda < 0.1$.

3. Conservation of EM Energy and Frequency Up-Conversion of BB Radiation.

Frequency up-conversion is consistent with cavity QED constraints in that only EM radiation having wavelengths $\lambda < 2\delta$ is allowed in the gap, and may be expressed by the conservation of EM energy,

$$(\text{Planck Energy gain})_{\text{Gap}} = Q (\text{far IR energy loss})_{\text{Atoms}}$$

$$\text{or, } (E_g \sim \frac{hc}{\lambda_g})_{\text{Gap}} = (m kT)_{\text{Atoms}} \quad (2)$$

where, the photon in the gap having Planck energy E_g is conserved with the suppressed BB radiation from m atoms, i.e., $m = hc / \lambda kT$. For example, if $\delta < 0.10 \mu\text{m}$, then $\lambda_g < 0.20 \mu\text{m}$ and $E_g > 6.2 \text{ eV}$, where $m > 240$.

4. Number of Gap Photons.

The number N_{BB} of BB photons emitted by an atom at temperature having Planck energy E_g ,

$$N_{BB} = \frac{E_{avg}}{E_g} = \frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1} \quad (3)$$

In a population of N_A atoms, assume the frequency up-conversion produces one resonant gap photon having Planck energy E_g for m atoms having thermal kT energy. Thus, the ratio R_g of resonant to thermal photons is,

$$R_g = \frac{\frac{N_A}{m}}{N_A N_{BB}} = \left(\frac{\lambda kT}{hc}\right) \left[\exp\left(\frac{hc}{\lambda kT}\right) - 1 \right] \quad (4)$$

For resonant gap wavelengths λ_g from 1 to 1000 μm , the photon ratio R_g at heater temperatures of 300, 500, and 1000 K is shown in Fig. 3. In the mid IR, the ratio $R_g < 10$. The rapid increase below about 20 μm is caused by the fact that very few thermal photons are available in the BB spectra at heater temperatures.

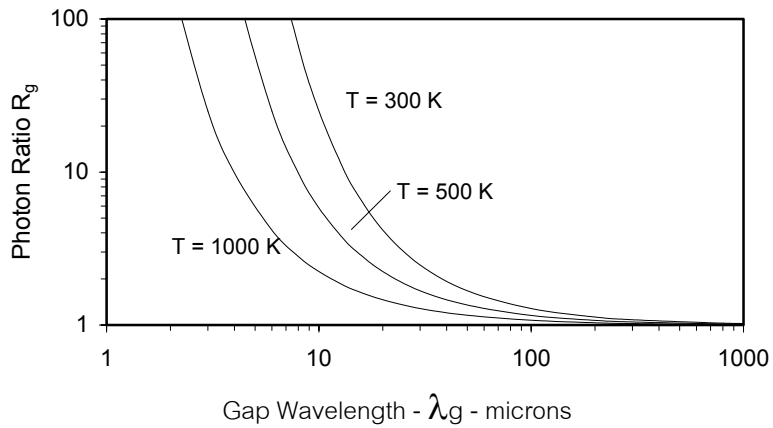


Figure 3. Ratio of Resonant Gap Photos to Thermal BB Photons.

5. Stefan Boltzmann and Cavity QED Induced Heat Flow.

The Stefan-Boltzmann law gives classical radiative heat transfer Q between the heater and photocell,

$$Q = \sigma A [T_H^4 - T_{PC}^4] \quad (5)$$

where, σ is the Stefan-Boltzmann constant and A is the heater area.

However, the Stefan-Boltzmann law for thermal radiation is only valid in large cavities having an EM resonance beyond the far IR. In microscopic gaps having dimension δ shorter than the half wavelength of far IR radiation, the Stefan-Boltzmann law is not valid, i.e., for temperatures from 300 to 1000 K, Fig. 2 shows the Stefan – Boltzmann equation is not applicable for $\lambda_g < 100 \mu\text{m}$, or $\delta < 50 \mu\text{m}$.

Cavity QED induced heat flow Q based on far to near IR frequency up-conversion is proposed for radiative heat flow in gaps $\delta < 50 \mu\text{m}$. EM energy in the form of resonant gap photons at frequency $\nu = c / 2 \delta$ having Planck energy $E_g = h\nu$ transfers thermal kT energy between m_H heater atoms at temperature T_H and m_{PC} photodiode atoms at temperature T_{PC} . The net heat flow Q is,

$$Q = A \left(\frac{1}{\delta^2} \right) \left(\frac{1}{\tau + \delta/c} \right) (E_{gH} - E_{gPC}) = A \left(\frac{1}{\delta^2} \right) \left(\frac{1}{\tau + \delta/c} \right) k(m_H T_H - m_{PC} T_{PC}) \quad (6)$$

where, δ^{-2} is the number of standing photon waves per unit heat flow area, $\delta = c / 2\nu$. Here, $E_{gH} = m_H kT_H$ and $E_{gPC} = m_{PC} kT_{PC}$. The time between successive emission of E_g photons comprises the time δ / c for the EM waves to traverse the gap and the time τ for the atoms to recover from absolute zero. Since $\tau \gg \delta / c$, the heat flux Q ,

$$Q = A \left(\frac{1}{\delta^2} \right) \left(\frac{1}{\tau} \right) mk(T_H - T_C) \quad (7)$$

where, m is the average number of atoms in the standing VUV wave interacting across the gap, $m = \frac{1}{2}(m_H + m_{PC})$.

Heater atoms at absolute zero require time τ to recover temperature prior to the next emission of gap photons. For an initial temperature T_H , the response $T(\epsilon, \tau)$ of an semi-infinite solid to a step change in surface temperature to absolute zero,

$$\frac{T(\epsilon, \tau)}{T_H} = \left[1 - \operatorname{erf} \left(\frac{\epsilon}{2\sqrt{\alpha\tau}} \right) \right] \quad (8)$$

where, ϵ is the penetration depth, $\epsilon = \lambda_g / 4\pi k_{ex}$, k_{ex} is the extinction coefficient. The thermal diffusivity $\alpha = K / \rho C_p$, where K is the thermal conductivity, ρ is the density, and C_p is the specific heat.

Analysis

1. Steady TPV Response.

The steady TPV photocell current I produced from the conversion of radiative heat Q ,

$$I = \eta e \frac{Q}{E_g} = \eta e \dot{N}_g \quad (9)$$

where, η is the quantum efficiency of the photocell in electrons per photon, and e is the electronic charge. The rate \dot{N}_g at which

the number N_g of photons having Planck energy E_g are produced, $\dot{N}_g = Q / E_g$. In terms of the spectral responsivity ξ , the current I is,

$$I = \xi Q \quad (10)$$

where, $\eta = \xi E_g / e$ and ξ is in units of amperes per watt.

Typical TPV photocell spectral responsivity of silicon and germanium are illustrated in Fig. 4. The experimental current response (Fig. 4 of [7]) shows the TPV response to begin at an indicated capacitance $C \sim 50$ pF. For a square 2.2×2.2 mm² heater chip, the area $A \sim 4.8 \times 10^{-6}$ m² and gap $\delta = \epsilon_0 A / C = 0.86$ μ m giving the resonant gap wavelength $\lambda_g = 1.7$ μ m. Fig. 4 shows this is reasonably close to the spectral responsivity of germanium that begins at about $\lambda_g \sim 2$ μ m, and therefore the near IR photocell is taken to have the spectral responsivity ξ of germanium.

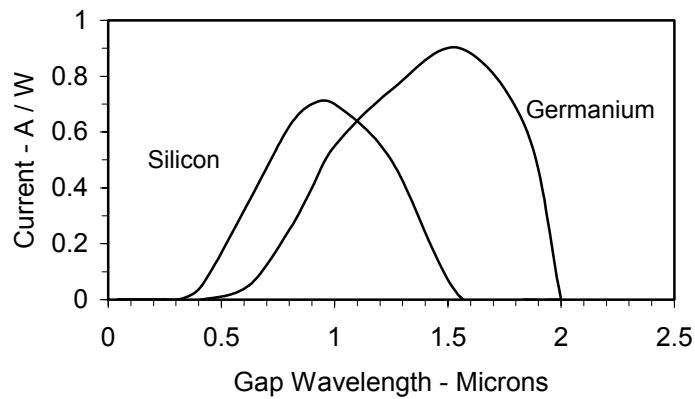


Figure 4. Spectral Responsivity of Silicon and Germanium.

For a circular heater chip [7] having $R \sim 1$ mm, the area $A = 3.1 \times 10^{-6}$ m². Germanium has a peak spectral responsivity $\xi \sim 0.9$ A/W and occurs at $\lambda_g = 1.55$ μ m and $E_g = 0.80$ eV. For temperatures $T_H = 348$ K and $T_{PC} = 300$ K, $m_H = 27$, $m_{PC} = 31$, and $m = 29$ average atoms. Eqn. 7 at the spectral peak gives the heat flow $Q \sim 540$ μ W at a response time $\tau = 225$ ps. Thus, $I \sim 490$ μ A and including the 250 μ A residual gives $I \sim 740$ μ A.

The cavity QED induced heat flow at heater temperatures of 348, 378, and 408 K in relation to experimental data (Fig. 4 of Reference 7) is illustrated in Fig. 5.

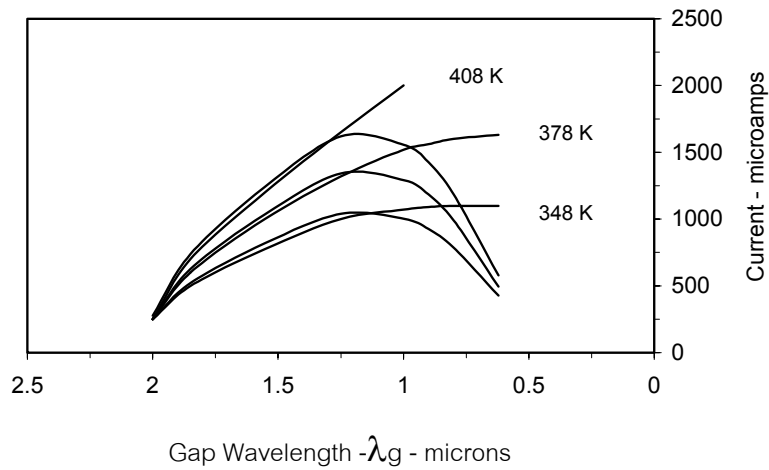


Figure 5. Comparison of cavity QED Induced TPV Effect and Experimental Data.

In the simulation, the number m of atoms supplying thermal kT energy to the resonant standing waves in the gap was maintained constant at $m = 29$. The photocell temperature T_{PC} was held at 300 K. Using Eqns. 7 and 10, the fit of the cavity

QED induced TPV theory to the experimental data was found sensitive to the recovery time τ which for heater temperatures $T_H = 348, 378,$ and 408 K was found to be 260, 300, and 340 ps. For a recovery time of 300 ps, the surface atoms respond at about 3.3 GHz.

For penetration depth $\varepsilon = 20$ and 50 nm, Eqn. 8 gives the recovery time τ shown in Fig. 6. The extinction coefficient k_{ex} ranges from 2.2 to 6.2. For $\varepsilon \sim 20$ to 50 nm, the temperature saturates to 348 K in about 200 to 400 ps, and therefore response times from 260 to 340 ps are reasonable for the silicon heater. In Eqns. 6 and 7, the time $\delta / c \sim 1$ fs. Thus, the term $\delta / c + \tau \sim \tau$, respectively.

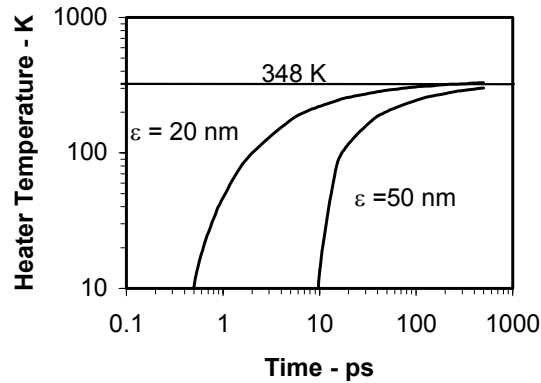


Figure 6. Heater at 348 K – Silicon at Penetration Depth $\varepsilon = 20$ and 50 nm.

The comparison of the cavity QED induced TPV effect and experimental data in Fig. 5 shows good agreement from

$\lambda = 2$ to $1.7 \mu\text{m}$. But for $\lambda = 1.7$ to $0.6 \mu\text{m}$, the agreement is not good. Indeed, the experimental data suggests the response in the visible region is greater than that in the near IR, which for the responsivity germanium in Fig. 4 is difficult to understand.

The likely reason for the disparity between the cavity QED theory and the experimental photocell current is the provision [7] of silicon dioxide spacers ($\sim 1 \mu\text{m}$ high) on the heater surface to control gap spacing, but at the same time keep the heater surface from achieving uniform contact the photocell. Away from the spacers the heater chip deforms with the capacitance increasing as the gap decreases. Over this deformed area, the photocell current decreases because the resonant wavelength λ moves toward the VUV away from the responsivity of the near IR photocell. Even so, the photocell continues to produce a net current because near the spacers, the gap remains at $\delta < 1 \mu\text{m}$ having resonant wavelength $\lambda < 2 \mu\text{m}$ that is in the near IR range of the responsivity of germanium. Indeed, Fig. 5 suggests the spacer height is closer to $\delta \sim 0.85 \mu\text{m}$ because the deviation between QED theory and experiment begins at $\lambda \sim 1.7 \mu\text{m}$, after which the current tends to remain almost constant as the capacitance increases.

That the spacers are the cause of the disparity may be easily confirmed. If the height of the spacers is reduced and in the limit taken to zero, the experimental current should approach the current response predicted by cavity QED induced TPV effect.

2. Dynamic TPV Response.

The dynamic TPV effect was not thought [7] caused by a thermal mechanism because the thermal response of the heater is not fast enough to follow rapid gap changes. Typically, photodiodes exhibit a thermal response at frequencies $f < 100$ Hz. However, experimental data (Fig. 5 of [7]) at 200 Hz suggests the dynamic response time is faster than the thermal response. Hence, the TPV effect was concluded [7] to find its origin in a non-thermal mechanism.

However, the conclusion here differs. In the cavity QED induced TPV effect, the current I only depends on the steady radiative flux Q from the heater to the photocell. Thus, there is no non-thermal mechanism causing the short circuit current to follow the dynamic effect [1] from 200 to 1000 Hz. Rather, the temperature fluctuations of heater surface atoms occur at rates of about 3.3 GHz are far faster than caused by the rapid gap changes. Unlike slow gap changes that depend on the thermal response of the full heater body, rapid gap changes produce temperature changes of surface atoms that are readily compensated by the heater.

The photocell response includes the response time of the photodiode detector as well as the speed of the amplifier, the latter defining the maximum rate at which the data can be collected. Photodiodes are very fast detectors, able to respond to chopping speeds in the MHz range. The amplifier speed at the set gain is far slower, limiting the frequency response to about

1 kHz. Thus, the dynamic TPV effect observed [7] between frequencies of 200 and 1000 Hz is consistent with typical amplifier response.

Conclusions

Cavity QED induced heat flow provides a physical origin to proximity effects in the radiative heat transfer of near IR radiation that is lacking in the fluctuation-dissipation theorem, the Nyquist theory, radiation tunneling, and evanescent waves.

Conservation of EM energy applied to the suppressed far IR radiation from atoms within the penetration depth of resonant EM waves standing across gaps suggests radiative heat flows at the EM resonant frequencies of the gap, say beyond the near IR or even the VUV.

TPV devices illustrate cavity QED induced EM radiation. Far IR radiation from the thermal kT energy of atoms within the penetration depth of standing near IR waves is up-converted to subsequently produce electrical current by the near IR photocell.

Providing spacers to set the nominal gap between the heater and the photocell at the wavelength corresponding to the peak of the near IR spectral responsivity of the semiconductor material optimize the efficiency of the TPV device.

Conversely, selecting a spacer to produce a resonant wavelength in the near IR as opposed to a zero gap where evanescent waves or radiation tunnelling are more likely to be

effective proves the latter have nothing to do with the enhancement of heat transfer or the production of electricity from near IR photocells in microscopic gaps.

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